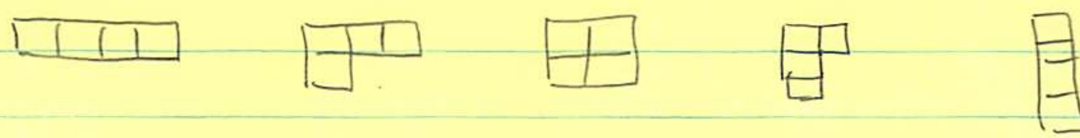
Lecture 13 what we did

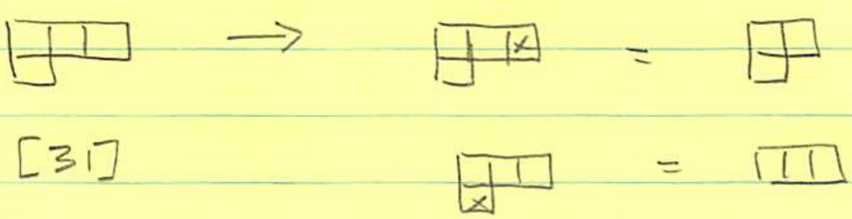
Went back to Permutation Group and the use of Partitions in calculating classes for $S_n$. — and a graphical method.

For $S_4$ the partitions are represented in Young Tableaux

[4] [31] [22] [211] [1111]



and how to find the $S_3$ content of $S_4$. For example



[21]

[31] [3]

$S_3$ partitions.

...which I found laboriously by using the character table and the "old" way.

# partitions = # classes = # IRR

learn from diagramatic playing

Listed the rules for "Standard Arrangement" of Young Tableaux — plus

- rules on how to count the IRR

$S_3$: $\boxed{1|2|3}$ $\quad \begin{array}{|c|}\hline 1\\ \hline 2\\ \hline 3\\ \hline\end{array}$ $\quad \begin{array}{|c|c|}\hline 1&2\\ \hline 2\\ \cline{1-1}\end{array}$ $\quad \begin{array}{|c|c|}\hline 1&3\\ \hline 2\\ \cline{1-1}\end{array}$ etc.

$\Rightarrow$ 4 IRR

For SU(n) -- magic occurs w/ some additional rules to account for possibility of same particle in different states

$\xi^i \equiv \boxed{i}$ $\qquad \eta^j \equiv \boxed{j}$ $\Big\}$ link of basis states of IRR to diagrams of IRR

product state

$$\xi^i\eta^j = \boxed{i} \otimes \boxed{j} = \boxed{\;|\;} \oplus \cancel{\begin{array}{|c|}\hline \\ \hline \\ \hline\end{array}}$$

$\downarrow$

count

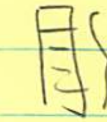
$\boxed{1|1}$ $\quad \boxed{1|2}$ $\quad \boxed{2|2}$

$\xi^1\eta^1$ $\quad \xi^1\eta^2$ $\quad \xi^2\eta^2$

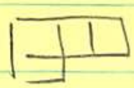
$\eta^1\xi^2$

Then described the rules for calculating the dimensionalities ...

Also, the conjugate states —

since "$\xi \bar{\xi} = 1$", the conjugate states are those that create all columns with  $n$ boxes, eg —

 for SU(3)

 is conjugate.

For the "fundamental" states —

SU(2): the $\underline{2}$ $\square \Rightarrow$  so $\square$ also for conjugate $\underline{2}^*$

SU(3): the $\underline{3}$ $\square \Rightarrow$  so  for $\underline{3}^*$

SU(4): the $\underline{4}$ $\square \Rightarrow$  so  for $\underline{4}^*$

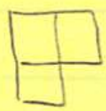
etc.

Lecture 13

9) The SU(n-1) subgroups of SU(n)

Consider all allowed positions in a Y.D. with the number "n"

Eliminate those boxes → leaving the SU(n-1) submultiplets.

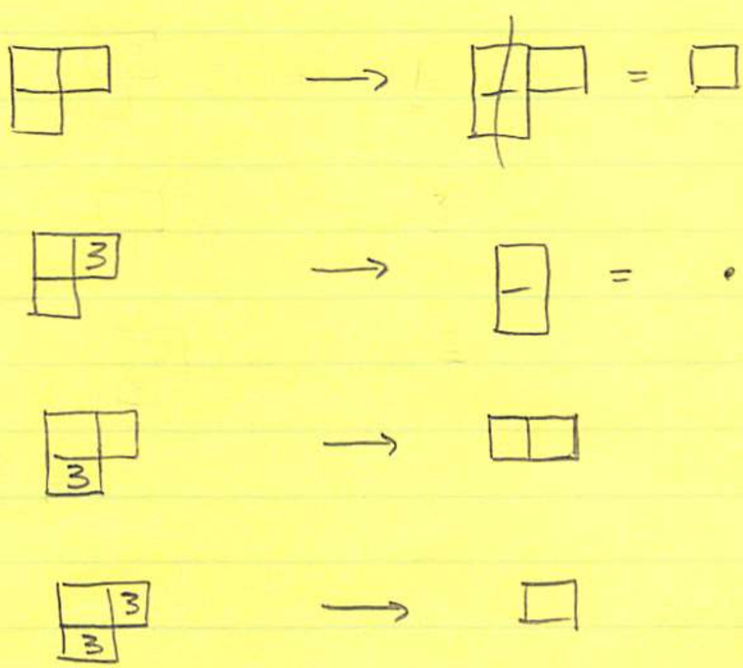


for SU(3)

| | | | | |
|---|---|---|---|---|
| 11<br>2 | 12<br>2 | 13<br>2 | 22<br>3 | ~~33<br>3~~ |
| 11<br>3 | 12<br>3 | 13<br>3 | 23<br>3 | |

it's the the $\underline{8}$, octet.

As an SU(3) Tableaux | As an SU(2) Tableaux



$\underline{2}$

$\underline{1}$

$\underline{3}$

$\underline{2}$

$\underset{SU(3)}{\underline{8}} \longrightarrow \underline{3} \oplus \underline{2} \oplus \underline{2} \oplus \underline{1}$ of SU(2)

Some History.

One of the clever ideas of Heisenberg in 1932 was to try to understand $\beta$ decay and nuclear stability

Exchange Force

P (e)    ( ) P    $Q = +1$

P ( )  e  ( ) P    $= +1$

P ( )    (e) P

sometimes it escapes

But then the neutron was discovered, and Fermi used it and built on another idea of Heisenberg –

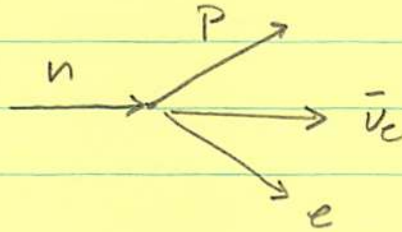
$$m(\text{neutron}) = 1.675 \times 10^{-27}\ \text{kg} \qquad Q = 0$$
$$m(\text{proton}) = 1.673 \times 10^{-27}\ \text{kg} \qquad Q = e$$

He likened them to be very similar → in fact 2 different states of the same particle

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$ → basis vectors in an internal SU(2) representation of what Wigner termed "isospin", I

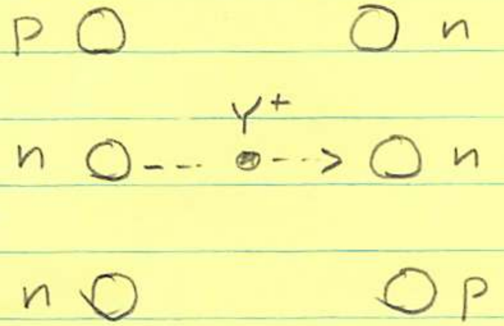
$I = 1/2 \quad I_3 = \pm 1/2$

In that picture, β decay was




• using Dirac's QED
• Pauli's neutrino
• Heisenberg's isodoublet

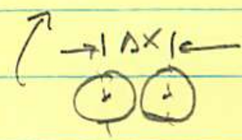
The exchange idea was used by Yukawa to further elucidate β decay




Y – spin 0

$\tau \sim \frac{\hbar}{2\Delta E}$

$mc^2 \sim \frac{\hbar c}{2\Delta x} \sim \frac{197\,\mathrm{MeV\text{-}fm}}{2.1\,\mathrm{fm}}$

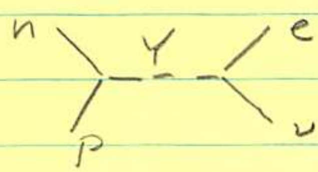



$\Delta x = c\Delta t$

$m_Y \sim 100\ \mathrm{MeV}/c^2$

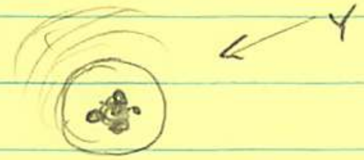
accounting for nuclear binding

& Y caused decay




The race was on to find the Yukawa Particle

The search was for evidence that




would be captured ...
S-state would be inside
a nucleus, so fast
reaction

Instead .. decays were found which were slow ...
1947 eventually sorted out .. They had seen 2
particles

$\pi \to \mu \nu$ ← weakly decaying $\tau \sim 2 \times 10^{-6}$ s

↑ Y .. the pion $\tau \sim 2.6 \times 10^{-8}$ s

$\pi^{\pm}$ and eventually $\pi^0 \to 2\gamma$ were identified.

Then it got confusing .. strange  1947 still

3cm Pb

"VEE" event ... ONE event
something $V \to$ 2 charged particles.

then .. nothing for 2 years

By 1950, a bunch of particles began to emerge

The original VEE : $K^0 \to \pi^+ \pi^-$ $m_K \sim 500\ MeV/c^2$

$K^+ \to \mu^+ \nu$

$\Lambda^0 \to p \pi^-$ $m_\Lambda \sim 1\ GeV/c^2$

$\Sigma^+ \to n \pi^+$ $m_\Sigma \sim 1.2\ GeV$
$\to p \pi^0$

$\Xi^- \to \pi^- \Lambda^0$ $m_\Xi \sim 1.3\ GeV$
$\hookrightarrow p \pi^-$

All of these objects
had very long lifetimes
$\Rightarrow$ weak decays

BUT liked to be produced in nucleon collisions
like $\pi n$ etc.

Further, found to be produced in PAIRS
"associated production"

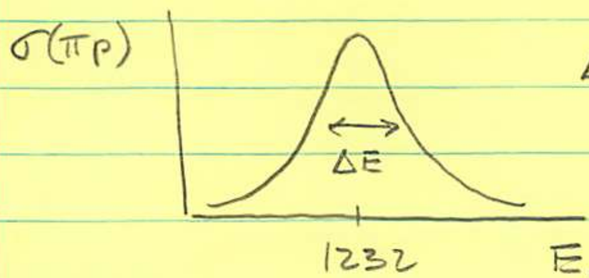
$\pi^+ + n \rightarrow \Lambda^0 + K^+$

$\pi^- + p \rightarrow \Lambda^0 + K^0$

suggests some quantum number conservation

Finally, Fermi produced "artificially" (cyclotron)

$\pi^+ + p \rightarrow \pi^+ + p$ at particular $\pi$ energies -- something




$\Delta E \sim 100$ MeV

like a "resonance" called $\Delta^{++}$ $m_\Delta \sim 1.2$ GeV.

and partners $\Delta^+$ $\Delta^0$ $\Delta^-$

and more - $\pi^- + p \rightarrow n + \rho^0$ $m_\rho \sim 770$ MeV

$\hookrightarrow \pi^+\pi^-$

and partners $\rho^+$ $\rho^0$ $\rho^-$

So, can't be some nucleon-nucleon collective effect –
It's thought to be the production of a new
"resonant" state

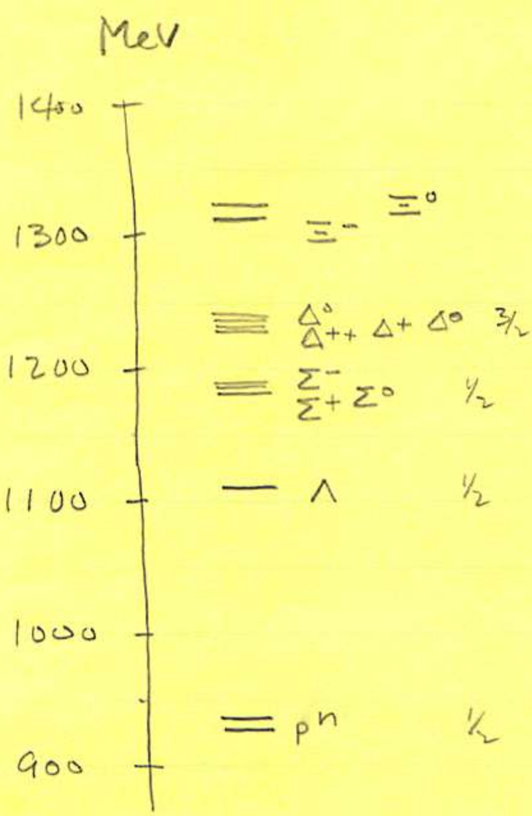
$\Delta^{++}$ $\quad m_{\Delta^{++}} \sim 1232$ MeV

partners: $\quad \Delta^+, \Delta^0, \Delta^-$

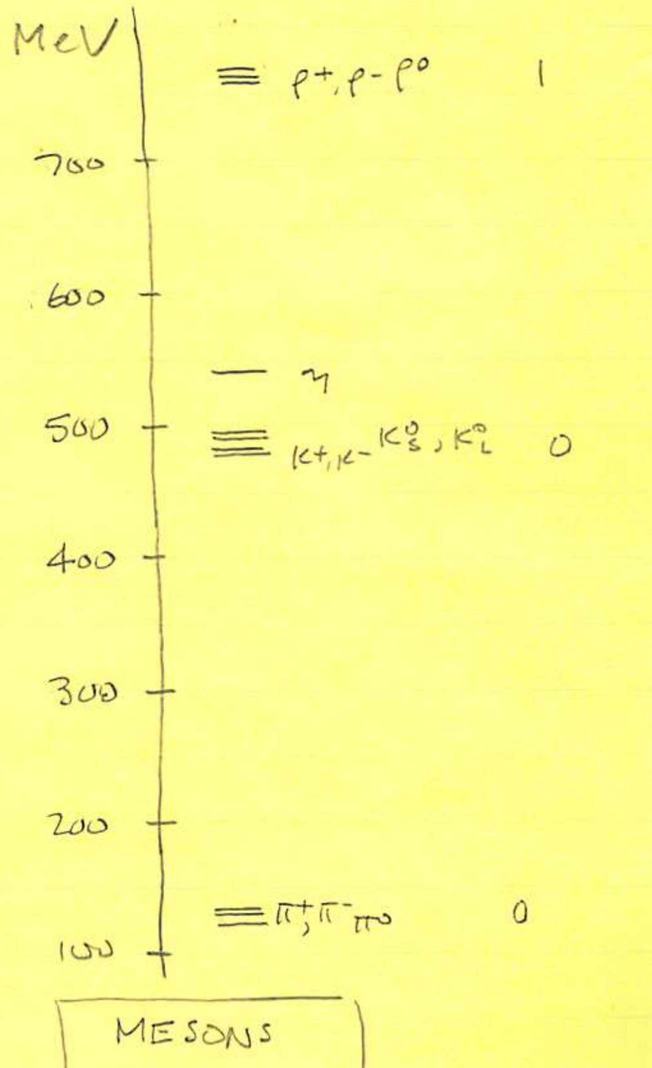
More.. $\quad \pi^- + p \to n + \rho^0$ ← "rho"

$\quad \hookrightarrow \pi^+\pi^-$

and it comes with partners. $\quad \rho^+, \rho^0, \rho^- \quad m_\rho \sim 770$




BARYONS
spin = 1/2, 3/2

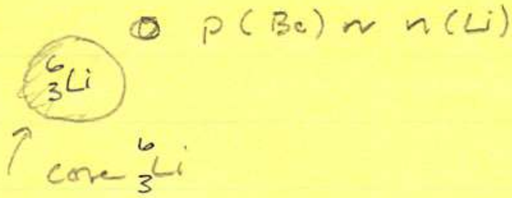



MESONS
spin = 0, 1

ISOSPIN & CHARGE INDEPENCE

Very early, it was apparent that mirror nuclei like $^7_3Li$ and $^7_4Be$ were very similar in their nuclear levels - the ground states of these nuclei differed by only ~1 MeV

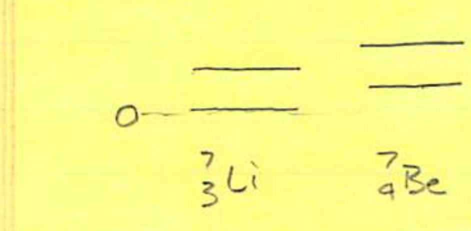
Each nucleus:




$^7_3Li$ has 3 p-p, 6 n-n } pairings

$^7_4B$ has 3 n-n, 6 p-p } pairings

$\Rightarrow$ the n-n and p-p attractions are the same.




Other nuclear effects among mirror nuclei suggest the same thing n-n, p-p, n-p forces of attraction are all similar - different by small Coulomb eff as can be seen from the plots, $p$-$p$, $p$-$n$, $\bar{p}$-$p$ are all about the same

Heisenberg's idea that $n$ and $p$ are just different charge states of the Nucleon led to the notion of an SU(2) symmetry – identical mathematically to spin angular momentum – which is an "internal symmetry.

Imagine an "isospin space" in which the isospin of a state is represented by a vector $\vec{I}$ in that space → preserving the "length" of I is tantamount to conserving isospin.

So, $\quad N$ has $I = 1/2$ with $I_3 = +1/2 \equiv p$

$$= -1/2 \equiv n$$

* Strong interactions don't distinguish the I projections

Nomenclature:

spin = 1/2, 3/2 .. "FERMIONS" < BARYONS ($p, n, \Lambda, \Sigma$ ...
LEPTONS ($e, \nu_e, \mu, \nu_e, \tau, \nu_\tau$)

spin = 0, 1, 2 ... "BOSONS" < VECTOR BOSONS ($\gamma, W, Z, g$)
MESONS ($\pi, K, B, D$ ... )

And the empirical observation that various conservation rules seem to hold:

* BARYON NUMBER, B    baryons, $B=1$    antibaryons, $B=-1$
* LEPTON NUMBER, L    leptons, $L=1$    antileptons, $L=-1$
$= L_e$ or $L_\mu$ or $L_\tau$ and $L = L_e + L_\mu + L_\tau$

| | $\pi^+$ | $\rightarrow$ | $\mu^+$ | + | $\nu_\mu$ |
|---|---|---|---|---|---|
| $L (\doteq L_\mu)$ | 0 | | $-1$ | | $+1$ |

| | $\nu_e$ | + | $n$ | $\rightarrow$ | $e^-$ | + | $p$ |
|---|---|---|---|---|---|---|---|
| $L (= L_e)$ | 1 | | 0 | | 1 | | 0 |
| B | 0 | | 1 | | 0 | | 1 |

| | $p$ | + | $n$ | $\rightarrow$ | $\bar{p}$ | + | $n$ | + | $p$ | + | $p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| B | 1 | | 1 | | $-1$ | | 1 | | 1 | | 1 |

| | $\mu^+$ | $\rightarrow$ | $\bar{\nu}_\mu$ | + | $e^+$ | + | $\nu_e$ |
|---|---|---|---|---|---|---|---|
| $L(L_\mu)$ | $-1$ | | $-1$ | | 0 | | 0 |
| $L(L_e)$ | 0 | | 0 | | $-1$ | | 1 |
| L | $-1$ | | $-1$ | | $-1$ | | 1 |

"HADRONS" = BARYONS & MESONS which undergo the strong interaction

(charged Baryons & Mesons undergo the electromagnetic interaction)

Remember that Strange Particles seem to be produced in pairs.

$$\pi^- p \to \Lambda^0 + K^0$$

* STRANGENESS, S

| | |
|---|---|
| $S(K^0) \equiv 1$ | $S(p) = 0$ |
| $S(\Lambda) \equiv -1$ | $S(n) = 0$ |
| $S(K^-) \equiv -1$ | $S(\pi) = 0$ |
| $S(K^+) \equiv +1$ | |
| and others | etc for "non-strange" particles. |

* HYPERCHARGE, Y is used

$$Y = B + S$$

So... back...

| | Isospin | Y | Q |
|---|---|---|---|
| Strong | ✓ | ✓ | ✓ |
| EM | × | ✓ | ✓ |
| Weak | × | × | ✓ |

There are other symmetries .. C, P, CP ..

For strong interactions: B, Q absolutely conserved.

and

$$\frac{Q}{e} = \frac{B}{2} + I_3$$

holds .. absolutely for strong.
$I_3$ is absolutely conserved for strong — not weak.

The pion comes in 3 charge states $\pi^+, \pi^0, \pi^-$
and if the $\pi$ is considered an $I=1$, isotriplet,
then

$$\begin{array}{ll} I_3 = 1 & \pi^+ \\ 0 & \pi^0 \\ -1 & \pi^- \end{array}$$

works.

So, things like

$$\begin{array}{lcccc} & \pi^+ & p \rightarrow & \pi^+ & p \\ I & 1 & 1/2 & 1 & 1/2 \\ I_3 & 1/2 & 1/2 & 1/2 & 1/2 \end{array}$$

$$\begin{array}{lcccc} & \pi^- & p \rightarrow & \pi^0 & n \\ I & 1 & 1/2 & 1 & 1/2 \\ I_3 & -1 & +1/2 & 0 & -1/2 \end{array}$$

are characteristic

Also,

| | $\pi^\pm$ | $p$ | $\rightarrow$ | $\Sigma^\pm$ | $K^+$ |
|---|---|---|---|---|---|
| $I$ | $1$ | $1/2$ | | $1$ | $1/2$ |
| $I_3$ | $\pm 1$ | $1/2$ | | $\pm 1$ | $1/2$ |
| $S$ | $0$ | $0$ | | $-1$ | $1$ |

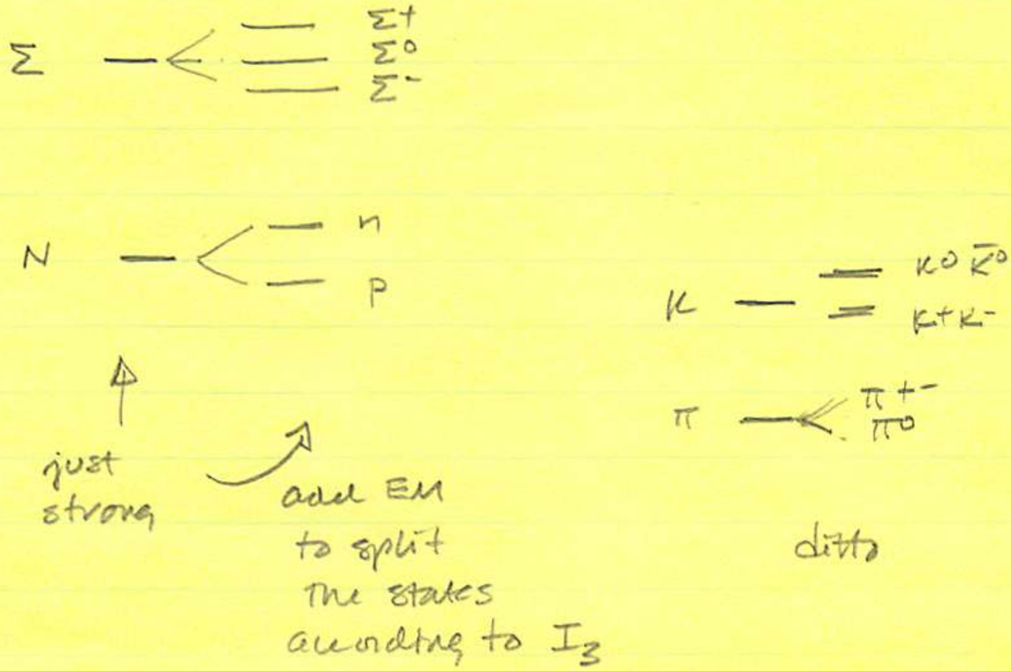
| | $\Sigma^\pm$ | $\rightarrow$ | $n$ | $\pi^+$ | | |
|---|---|---|---|---|---|---|
| $I$ | $1$ | | $1/2$ | $1$ | $\times$ | because weak |
| $I_3$ | $\pm 1$ | | $-1/2$ | $1$ | | OR $\Rightarrow$ weak |
| $S$ | $-1$ | | $0$ | $0$ | $\times$ | weak |

There is also

| | $\Sigma^0$ | $\rightarrow$ | $\Lambda$ | $\gamma$ | | |
|---|---|---|---|---|---|---|
| $I$ | $1$ | | $0$ | $0$ | $\times$ | EM |
| $I_3$ | $0$ | | $0$ | $0$ | | |
| $S$ | $-1$ | | $-1$ | $0$ | $\checkmark$ | |

In this way of looking

Mass



Gell-Mann and Nishijima suggested that

$$\frac{Q}{e} = \frac{B}{2} + \frac{S}{2} + I_3$$

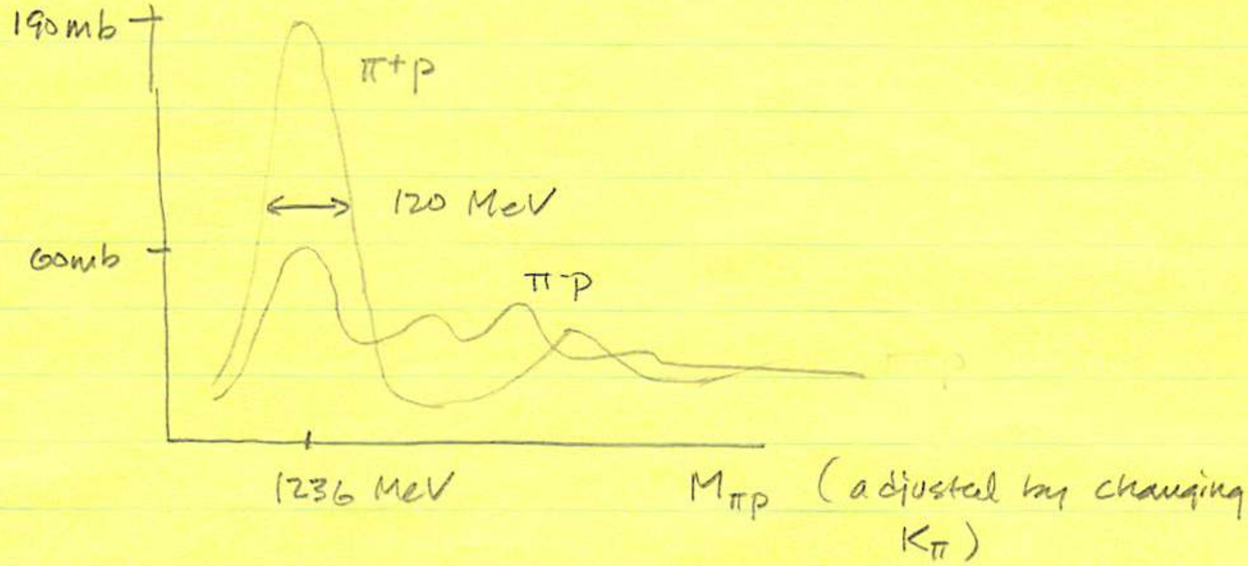
$$\frac{Q}{e} = \tfrac{1}{2}Y + I_3$$

"hypercharge"

holds for strong interactions.

Consequences of charge independence are easy to predict.

Compare $\pi^+p$ to $\pi^-p$




affinity for $\pi p$ at $M_{\pi p} = 1236$ MeV.

$$\frac{\sigma_{\pi^+p}}{\sigma_{\pi^-p}} \sim 3$$

From Isospin analysis —

$$\frac{\sigma_{\pi^+p}}{\sigma_{\pi^-p}} = \frac{\sigma_a}{\sigma_b+\sigma_c} = \frac{|M_3|^2}{\left|\frac{1}{3}M_3 + \frac{2}{3}M_1\right|^2 + \left|\frac{\sqrt{2}}{3}M_3 - \frac{\sqrt{2}}{3}M_1\right|^2}$$

Suppose the $\frac{1}{2}\to\frac{1}{2}$ channel dominated

$$\frac{\sigma_a}{\sigma_b+\sigma_c} \sim 0$$

Suppose the $\frac{3}{2}\to\frac{3}{2}$ channel dominated

$$\frac{\sigma_a}{\sigma_b+\sigma_c} \sim \frac{M_3^2}{\frac{3}{9}M_3^2} = 3$$

So, this "affinity" happens for the $I = 3/2$ channel

It's the production of a short-lived $I = 3/2$ state:

in $\pi^+ p$ scattering: $I = 3/2$, $I_3 = 3/2$ $Q = 2e$

$\pi^- p$ scattering: $I = 3/2$, $I_3 = -1/2$ $Q = 0e$

$I_3 = 1/2$ $= 1e$

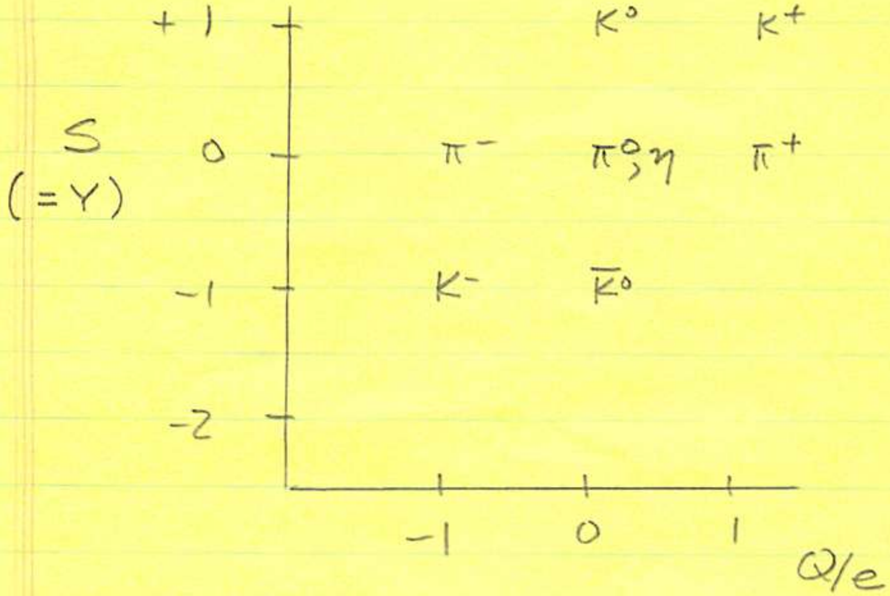
$I_3 = -3/2$ $= -1e$

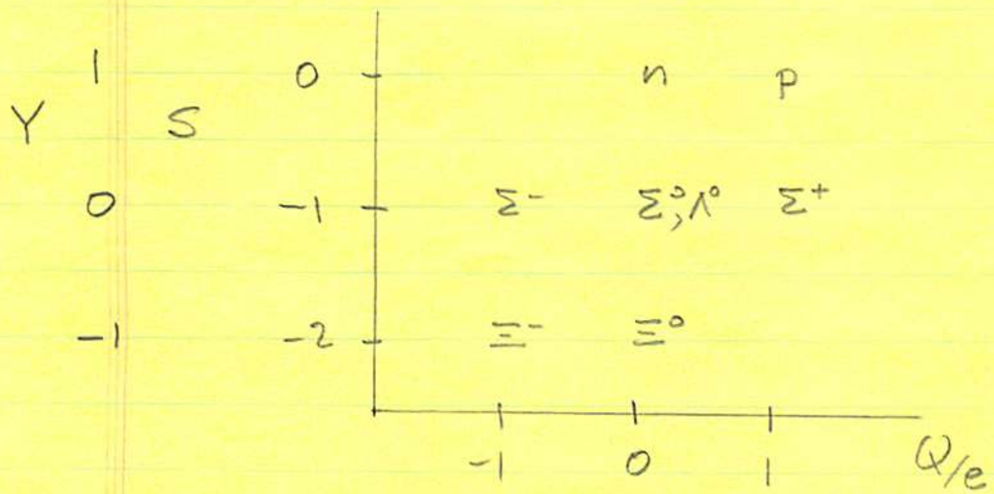
It's actually a quartet of states.. an isomultiplet

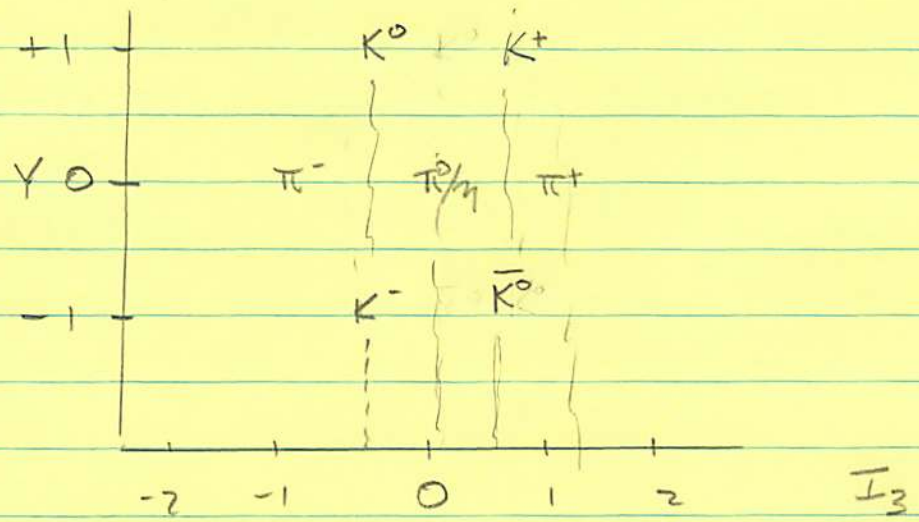
$\Delta^{++}$, $\Delta^{+}$, $\Delta^{0}$ and $\Delta^{-}$

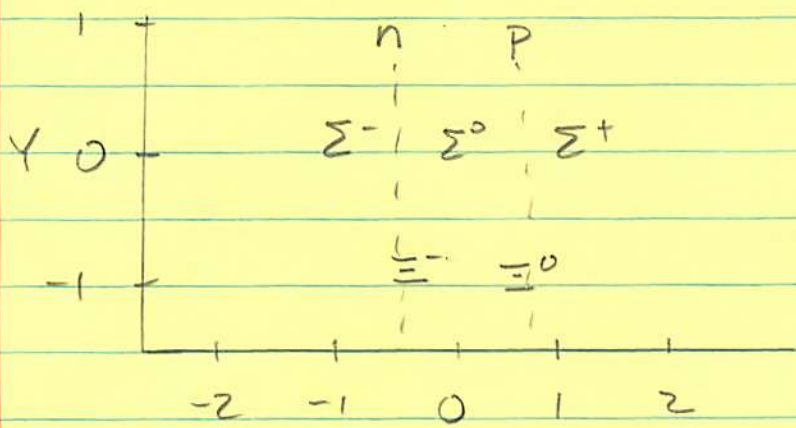
The $\Gamma \approx 120$ MeV $\Rightarrow \tau = 5 \times 10^{-24}$ s as discussed.

These known multiplets seemed to form patterns:



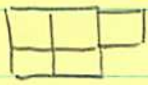
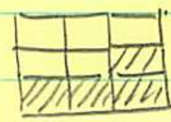


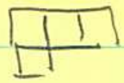




And this pattern was becoming clear about 1960.

Continuing with the notion of the conjugate state...
it's basically defined by that state, which when
multiplied by the "un-conjugate" state gives "1".
That's the singlet. For SU(n), that's whatever is
required to make columns n-boxes tall.  $\}\, n = \bullet$

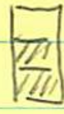
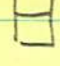
So, for SU(2),

 from 

$\Rightarrow$  is conjugate

How about the fundamental states?

SU(2): □ is the fundamental $\underline{2}$
the conjugate comes from forming 
so, □ is also the $\underline{2}^*$
it's self-adjoint.

For SU(3) □ is the fundamental $\underline{3}$
the conjugate:  $\Rightarrow$  is $\underline{3}^*$

For SU(4) □
so  $\Rightarrow$  is the $\underline{4}^*$

SU(2) is more than spin... in particular, isospin.

$\begin{pmatrix} p \\ n \end{pmatrix}$ is an old-time SU(2) doublet, which you could think of as the fundamental basis state of the Nucleon, $I = 1/2$

$I_3 = 1/2$ p

$-1/2$ n

The fundamental spinor is

$$\xi = \begin{pmatrix} \xi^1 \\ \xi^2 \end{pmatrix} \longrightarrow \xi'^i = U^i{}_j \, \xi^j$$

where $U = e^{i \vec{\theta} \cdot \vec{\sigma}/2}$ or for isospin we use "$\vec{\tau}$" ($\equiv$ "$\vec{\sigma}$")

The conjugate is

$$\xi^* = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

For spin, this doesn't matter. For Nucleons, it does since we have antiprotons and antineutrons ($\bar{p}$ and $\bar{n}$) or (P and N)

Define a combination $S\xi^*$ that would have the same transformation properties as $\xi$

$$(S\xi'^*) = u(S\xi^*)$$

since $\quad \xi'^* = u^*\xi^*$

$$S\xi'^* = Su^*\xi^*$$

so

$$Su^* = uS \quad \Rightarrow \quad u^* = S^{-1}uS$$

In $SU(2)$

$$-\vec{\sigma}^* = S^{-1}\vec{\sigma}S \qquad \left(\text{since } u = e^{i\theta\sigma/2} \text{ so } u^* = e^{-i\theta\sigma^*/2}\right)$$

The only imaginary generator is $\sigma_2$ so $\quad S \sim \sigma_2$ and chosen by convention to be

$$S = i\sigma_2$$

(related to charge conjugation operation in field theory)

Then

$$S\xi^* = i\sigma_2\xi^* = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix}$$

$$= \begin{pmatrix} -\xi_2 \\ \xi_1 \end{pmatrix}$$

This is given a new name

$$\eta^i \equiv \begin{pmatrix} -\xi_2 \\ \xi_1 \end{pmatrix} = \begin{pmatrix} -\xi^{2*} \\ \xi^{1*} \end{pmatrix}$$

$$\eta^1 = -\xi_2 \quad \& \quad \eta^2 = \xi_1$$

Now, suppose $\xi^1 = p$ then
$\xi^2 = n$

$$\xi^{1*} = \xi_1 = \bar{p}$$
$$\xi^{2*} = \xi_2 = \bar{n}$$

So, we have

$$\xi^i = \begin{pmatrix} p \\ n \end{pmatrix} \quad \text{and} \quad \eta^i = \begin{pmatrix} -\bar{n} \\ \bar{p} \end{pmatrix}$$

Fermi and Yang used this to make the first effort at building particles out of other particles.

Namely, they tried – in 1949! – to make the pion triplet out of protons & neutrons and their antiparticles.

What they knew was that the pion appeared to be an $I = 1$, isospin triplet. The clever idea was to make it by combining other states.

How about $N$-$N$?

$$\xi^i \phi^j = \tfrac{1}{2}(\xi^i \phi^j + \xi^j \phi^i) + \tfrac{1}{2}(\xi^i \phi^j - \xi^j \phi^i)$$

So, one has a triplet $\quad \square \otimes \square = \square\!\square + \cdot$

$$\xi^1\phi^1 = pp \qquad I_3 = 1 \qquad Q = 2$$
$$\xi^1\phi^2 = pn + np \qquad = 0 \qquad = 1$$
$$\xi^2\phi^2 = nn \qquad = -1 \qquad = 0$$

$$\xi^1\phi^2 = pn - np \qquad I_3 = 0 = I \qquad = 1$$

So, that doesn't work

But, how about $N - \bar{N}$ ?

Same thing

$$\xi^i\eta^j = \tfrac{1}{2}(\xi^i\eta^j + \xi^j\eta^i) + \tfrac{1}{2}(\xi^i\eta^j - \xi^j\eta^i)$$

still $\Box \otimes \Box = \Box\!\Box + \cdot$ for conjugate $SU(2)$

But now–

$$\xi^1\eta^1 = -p\bar{n} \qquad I_3 = 1$$
$$\xi^1\eta^2 = p\bar{p} - n\bar{n} \qquad = 0$$
$$\xi^2\eta^2 = n\bar{p} \qquad = -1$$

$$\xi^1\eta^2 = p\bar{p} + n\bar{n} \qquad = 0$$

The problem with this model? No strange particles.

Sakata tried something similar, now with p, n, $\Lambda$ as the ingredients.. gave too many, $\bar{p}\ \bar{n}\ \bar{\Lambda}$

weird states, like a B=3 baryon $pn\Lambda$.

The striking thing about what the zoo was saying was that the particles could be arranged in multiplets distinguished by 2 additive quantum numbers, $I_3$ and $Y$ → linked by $Q = I_3 + Y/2$

→ in group-theory-think, there will be one diagonalizable generator for each.

Look at all of the Rank = 2 groups. Well, $SU(2) \otimes U(1)$ is rank 2, but this wouldn't link $I_3$ and $Y$, Patterns had certainly emerged, but one is clear

$\updownarrow Y =$ | $\pi^- \ldots \pi^+ - \pi^0$ for example

$\longleftarrow I_3 \longrightarrow$

a given isospin multiplet has a single $Y$.

Gell-Mann was playing with groups by 1961 and he and Neeman each looked hard at $SU(3)$. Others looked at $C_2$, $B_2$, $G_2$. The best way to look, is with a geometrical pattern.

This classification of Lie algebras and their groups had been done for a long time. Gell-Mann concentrated on SU(3).

The fundamental basis vector is

$$\xi = \begin{pmatrix} \xi^1 \\ \xi^2 \\ \xi^3 \end{pmatrix}$$

Gell-Mann called them "quarks" and adjusted their properties to fit the data

$$q^i = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

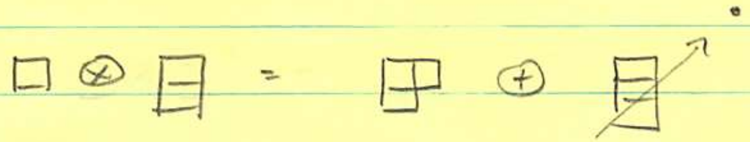
The generators are $3 \times 3$ matrices. The idea is now familiar: make states by putting $q^i$'s into products. Young-Tableaux can help to identify the various multiplets that result.

$$\square \otimes \square = \square\!\square \oplus \begin{array}{c}\square\\[-4pt]\square\end{array}$$

Which for SU(3) corresponds to

$$\underline{3} \otimes \underline{3} = \underline{6} \oplus \underline{3}^*$$

We could also do: $3 \otimes \bar{3}$ or $q \cdot \bar{q}$ states.

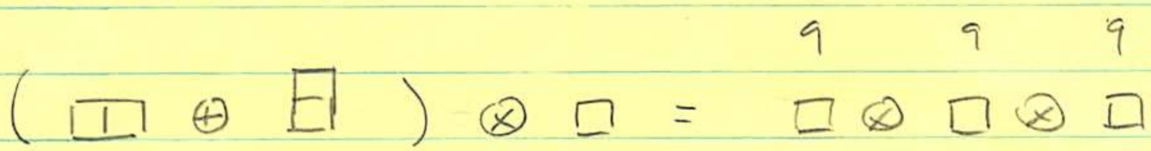


$\underline{3} \otimes \underline{3}^*$ $\qquad$ $\underline{8} \oplus \underline{1}$

So, is there any known particle content matching the $\underline{6}$, $\underline{8}$, $\underline{3}^*$, & $\underline{1}$?

Well, the $\underline{6}$, no. The $\underline{3}^*$, no... so no qq states.

But, if you make 3 states (similar in spirit to the Sakata model)




$\underline{6} \oplus \underline{3}^* \quad \otimes \underline{3}$

$\underline{10}$ $\qquad$ $\underline{8}$ $\qquad$ $\underline{3}^*$ $\qquad$ $\underline{8}^*$

Luckily for G.M. the $\underline{3} \otimes \underline{3}^* = \underline{8} \oplus \underline{1}$ and the $3 \otimes 3 \otimes 3$ had physical content.

Lecture 14 Where we were

A history lesson..

→ the slow appreciation for "charge-independence" of the strong interaction ⇒ Isospin conservation

$$N = \begin{pmatrix} p \\ n \end{pmatrix}$$

→ the steady accumulation of a zoo of hadrons, fermions and bosons

→ the appreciation that they seemed to cluster around mass, isospin, and hyperchange

$Y = B + S$ Baryon # : $p, n$ 1
$\bar{p}, \bar{n}$ −1
etc

and related to

$$Q = \frac{Y}{2} + I_3$$

I showed how to consistently represent a conjugate-antifermion-spinor

$$\xi^i = \begin{pmatrix} p \\ n \end{pmatrix} \qquad \eta^i = \begin{pmatrix} -\bar{n} \\ \bar{p} \end{pmatrix}$$