NOW FOR SOMETHING *COMPLETELY* DIFFERENT...

THE BASES ON WHICH THE ARCANE MECHANICS OF THE "STANDARD MODEL" ARE BUILT ARE NOT ALWAYS TAUGHT/WRITTEN ... but they are fun.

- THE DEVELOPMENT OF ESPECIALLY THE ELECTROWEAK MODEL IS FULL OF INTERESTING HISTORY, FALSE STARTS, INTRIGUE, MYSTERY, & SOME PRETTY NON-HEP.
- I PROPOSE TO SCHEMATICALLY.. WITH A MINIMUM OF MATHEMATICS..

TALK AROUND THE EDGES OF

THE STANDARD MODEL OF ELECTRO-WEAK PHYSICS

- QCD has a couple of important uses... one of which is as an important systematics tool in EW physics.
- we wanted to give a flavor ⇒ knowing something about standard model

- We're all high energy physicists here.. so we all share a common characteristic -- we think that we're pretty special!

- What I'd like to do is to chip away a bit at the large measure of hubris which is a necessary personality trait in our species — by appealing to "the guys in the basement"

→ every physics dept. has a set of low temperature physicists --

they explore temperatures which are inherently interesting:

note The universe has presumably never been colder than 3K.. so any exploration of the states of matter below that temperature is NEW.. not been created naturally ever.

Who cares? We do.

R. BROCK
MICHIGAN STATE

GAUGE - THEORIES

- an eccentric introduction

A RECREATION IN THE HISTORICAL AND CROSS-CULTURAL ROOTS OF MODERN GAUGE THEORIES OF THE ELECTROMAGNETIC, WEAK, AND STRONG INTERACTIONS

introduction
uses of symmetry & invariance in physics
gauge principle
weak interactions
critical phenomena
BROKEN symmetry
Higgs, et. al. mechanism
PUTTING IT TOGETHER → WEINBERG & SALAM MODEL

I WANT TO TELL YOU a story...

previous lectures → meat & potatoes.

Desert.

PC

pedagogically complete

A CATALOG WILL SUFFICE...

FREE LAGRANGIANS	EQUATIONS OF MOTION
scalar fields:	
$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2$	$\partial_\mu\partial^\mu\phi + m^2\phi^2 = 0$
spin 1/2 fields:	
$\mathcal{L} = \bar{\psi}(x)[i\gamma^\mu\partial_\mu - m]\psi(x) = 0$	$(i\gamma^\mu\partial_\mu - m)\psi(x) = 0$
spin 1, massless fields:	
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$	$\partial_\mu F^{\mu\nu} = 0$
spin 1, massive fields:	
$\mathcal{L} = -\frac{1}{4}f^{\mu\nu}f_{\mu\nu} + \frac{1}{2}M^2B^\mu B_\mu$, $f^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$	$\partial_\mu f^{\mu\nu} + m^2B^\nu = 0$

INTERACTIONS -

$$\mathcal{L}_{\text{electromagnetic - spin 1/2}} = e_f\bar{f}(x)\gamma^\mu f(x)A_\mu(x)$$

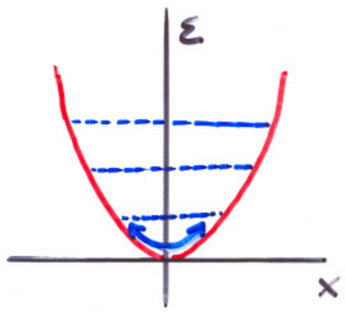
$$\mathcal{L}_{\text{YUKAWA}} = g\,\phi(x)\bar{\psi}(x)\psi(x)$$

PARTICLE SPECTRA -

$$\phi(x) = \int\frac{d^3k}{(2\pi)^3\,2\omega_k}\left[a(k)e^{-ikx} + a^\dagger(k)e^{ikx}\right]$$

creation operator

annihilation operator



... just like quantum oscillator from 1st year quantum mechanics

OUR FAITH HAS COME FULL CIRCLE...

- We are amused at the image of Kepler, among many others, trying to bend the observed universe into an *a priori* notion of how it ought to be — for Kepler, it *ought* to have something to do with the Platonic Solids. For others, a "perfect geometry", circles... then ellipses...

- We are no different now! One of my messages...

- WHAT DID EINSTEIN DO IN SPECIAL RELATIVITY?

  → HE DIDN'T INVENT THE TRANSFORMATIONS NECESSARY
  Lorentz did that earlier

  → HE DIDN'T ESTABLISH THE MATHEMATICAL RIGOR
  Poincaré did that earlier

- WHAT HE DID WAS DERIVE THOSE RESULTS BY ARGUING FROM AN A PRIORI PREDJUDICE REGARDING A PREFERANCE FOR SYMMETRY

THAT WAY OF THINKING CAUGHT ON... SPACETIME SYMMETRIES TOOK ON A FUNDAMENTAL IMPORTANCE IN PHYSICS...

only to be confused & frustrated by:

1. The discovery of non-spacetime symmetries (eg, isospin.. the "INTERNAL" SYMMETRIES)
2. The discovery that Nature is actually RARELY symmetric! .. approximate symmetries!

quick lesson on symmetry in quantum mechanics I:

QUANTUM MECHANICS:

- GROUP OPERATIONS REPRESENTED BY UNITARY OPERATORS, $\mathcal{U}$, IN A LINEAR VECTOR SPACE OF STATE VECTORS, $|\alpha\rangle$

generated by G

vectors transform: $|\alpha\rangle \longrightarrow |\alpha'\rangle = \mathcal{U}|\alpha\rangle$

operators transform: $\Theta \longrightarrow \Theta' = \mathcal{U}\Theta\mathcal{U}^{-1}$

- IF SYSTEM IS SYMMETRIC wrt GROUP, $[\mathcal{H}, G] = 0$

  - An important theorem came out of the incredible mathematics group of F. Klein in Göttingen – written down by Emmy Noether:

SYMMETRY $\Longleftrightarrow$ CONSERVATION LAW

- OF PARTICULAR INTEREST ARE SYMMETRY GROUPS WITH REPRESENTATIONS LIKE $\mathcal{U}(\varepsilon) = e^{-i\sum_j \varepsilon^j Q^j}$

"GENERATORS" OF THE GROUP & OPERATORS HAVING QUANTUM #'s AS EIGENVALUES

INFINITESIMAL PARAMETERS

- CONNECTION THROUGH "CHARGE" & A CONSERVED "CURRENT" –

$$Q \equiv \int d^3x \, j^0(x)$$

where $\quad \partial_\mu j^\mu(x) = 0 \quad$ signifies a conservation law quick lesson on symmetry in quantum mechanics?:

QUANTUM FIELD THEORY:

- $\phi(x)$ IS AN OPERATOR $\quad \phi \to \phi' = U\phi U^{-1}$

$$= (1 - i\sum_j \varepsilon^j Q^j)\phi(1 + i\sum_j \varepsilon^j Q^j)$$
$$\vdots$$
$$= \phi + i\sum_j \varepsilon^j [Q^j, \phi(x)]$$

SO $[Q^j, \phi(x)] = \phi(x) \Rightarrow$

(note: often $U\phi U^{-1} = \exp(i\sum_j \varepsilon^j q^j)\,\phi(x)$ ... a phase)

eigenvalues of Q^j

- SUPPOSE $[H, Q] = 0 \quad \Rightarrow \partial_0 Q = 0$

LET $H|\vec{p}_n\rangle = E_n|\vec{p}_n\rangle$

THEN $QH|\vec{p}_n\rangle = E_n Q|\vec{p}_n\rangle$
$\quad \shortparallel$
$HQ|\vec{p}_n\rangle = E_n Q|\vec{p}_n\rangle$

$|\vec{p}_n\rangle$ & $Q|\vec{p}_n\rangle$ ARE BOTH EIGENSTATES OF H WITH SAME E_n – degenerate

→ MAY REPRESENT ORTHOGONAL STATES WITH DISTINCT QUANTUM NUMBERS...

- THERE IS A SPECIAL EIGENSTATE OF H... THE VACUUM.

$H|0\rangle = 0$ IS ALWAYS TRUE FOR VACUUM STATE

USUALLY, IT IS ASSUMED THAT, FOR $U = e^{iQ\alpha}$

$U|0\rangle = |0\rangle$ FOR ALL SYMMETRIES

$\Rightarrow Q|0\rangle = 0$

IF $Q|0\rangle \neq 0$, THEN THERE MUST BE DEGENERATE VACUA IF ALSO $[H, Q] = 0$. stay tuned!

HISTORICALLY...

- SOON AFTER GENERAL RELATIVITY WAS WRITTEN BY EINSTEIN, H. WEYL PROPOSED A MODIFICATION...

HE ADDED INVARIANCE WITH RESPECT TO

a. $g'_{\mu\nu} = \lambda(x) g_{\mu\nu}$

b. $A'_{\mu} = A_{\mu} - \frac{\partial \lambda(x)}{\partial x^{\mu}}$

same $\lambda(x)$ phase

b. is the regular ambiguity required of electromagnetic potentials.

a. is weird. $\Rightarrow$ $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \longrightarrow \lambda ds^2$: LENGTHS ARE RE-"GAUGED"

- suggests an invariance even though space & time can change <u>over</u> all space and time.
- the mediator which holds the spacetime structure together would be the electromagnetic field.

→ ALL CALLED A "GAUGE TRANSFORMATION"

"Your ideas show a wonderful cohesion. Apart from the agreement with reality, it is at any rate a grandiose achievement of mind." A. Einstein to H. Weyl 1919.

THE THEORY -- AN EARLY ATTEMPT TO UNIFY GRAVITATION WITH ELECTROMAGNETISM -- DIDN'T WORK.

... but, the name stuck.

IN 1927 London revived the idea... but the symmetry isn't the scale of spacetime, rather the phase of the wave function.

GLOBAL, U(1) SYMMETRIES:

$$U(\theta) = e^{i\theta Q}$$

"GLOBAL" ⇒ SAME PHASE, INDEPENDENT OF SPACETIME $\theta \neq \theta(x)$

"U(1)" ⇒ 1 PARAMETER LIE GROUP HAVING Q AS GENERATOR

$$\psi(x) \longrightarrow \psi'(x) = U\psi(x)U^{-1}$$
$$= e^{i\theta q}\psi(x)$$

SIMPLE EXERCISE 1: For the Dirac free field, show that a local U(1) transformation leads to an invariance, and hence conserved quantum numbers, q.
ie. show $\delta\mathcal{L} = \mathcal{L}(\psi) - \mathcal{L}(\psi') = 0$.

GLOBAL SYMMETRIES NOT VERY RESTRICTIVE & NOT REALLY CONSISTENT WITH RELATIVITY & LOCAL FIELD THEORY...

LOCAL, U(1) SYMMETRIES:

$$U(\theta) = e^{i\theta(x)Q}$$

"LOCAL" ⇒ POTENTIALLY DIFFERENT PHASE AT ALL SPACETIME POINTS $\theta = \theta(x)$

$$\psi(x) \longrightarrow \psi'(x) = e^{i\theta(x)q}\psi(x)$$

NOT SO SIMPLE...

$$\mathcal{L}(\psi) \longrightarrow \mathcal{L}(\psi') = e^{-i\theta(x)q}\,\bar{\psi}(x)\left[\underline{i\gamma^\mu\partial_\mu} - m\right]e^{i\theta(x)q}\psi(x)$$

$$= \bar{\psi}(x)\left[i\gamma^\mu\partial_\mu - m\right]\psi(x) - \underline{q\,\partial_\mu\theta(x)\,\bar{\psi}(x)\gamma^\mu\psi(x)} \neq \mathcal{L}(\psi)$$

SIMPLE EXERCISE 2: For $\psi = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}$ and $U = e^{i\theta Q}$ ... what is the physical symmetry for Global U(1)?

Derivative term causes trouble ... define a new divergence operator to cancel the unwanted term!

$$D_\mu \equiv \partial_\mu + X_\mu$$

as-yet unnamed vector operator

goal is to get the gradient term to transform simply...

$$(D_\mu \psi) \longrightarrow (D_\mu \psi)' = e^{iq\theta(x)} (D_\mu \psi)$$

- START OUT WITH $\mathcal{L} = \bar{\psi}(x)\left[i\gamma^\mu D_\mu - m\right]\psi(x)$
$= \bar{\psi}(x)\left[i\gamma^\mu \partial_\mu + i\gamma^\mu X_\mu - m\right]\psi(x)$

transform $\psi \to \psi'$

$$\mathcal{L}(\psi) \to \mathcal{L}(\psi') = \bar{\psi}'(x)\left\{ i\gamma^\mu\left[\partial_\mu + \underline{X_\mu - iq\,\partial_\mu\theta(x)}\right] - m\right\}\psi'(x)$$

STILL NOT RIGHT!

must simultaneously transform $X_\mu \longrightarrow X_\mu' = X_\mu - iq\,\partial_\mu\theta(x)$

aha! Denote $X_\mu \equiv iq A_\mu(x)$ so the gradient looks like

$$D_\mu \equiv \partial_\mu + iq A_\mu$$

& TOTAL TRANSFORMATION NECESSARY TO LEAVE $\mathcal{L}$ ALONE IS:

$$\psi(x) \to \psi'(x) = e^{iQ\theta(x)}\psi(x)$$

$$A_\mu(x) \to A_\mu'(x) = A_\mu(x) - \partial_\mu\theta(x)$$

GAUGE INVARIANCE OF 2nd KIND

$$\mathcal{L} = \underbrace{\bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi}_{\text{free }\psi} - \underbrace{qA_\mu\bar{\psi}\gamma^\mu\psi}_{\text{"interaction"}}\ \underbrace{\left(-\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\right)}_{\text{added free }A_\mu}$$

$\mathcal{L}$ IS GAUGE INVARIANT

TURNING THE UTILITY OF SYMMETRY

upside-down...

IF INVARIANCE WITH RESPECT TO LOCAL, U(1) SYMMETRY

IS, a priori, OF PARAMOUNT IMPORTANCE...

one is forced to invent the photon.

DEMAND OF A SYMMETRY... GET NEW FIELDS AND DYNAMICS !!

OTHER SYMMETRIES $\longrightarrow$ NEW SPIN 1, 2... FIELDS ?

THE INTRIGUING RESEARCH PROJECT IN 1954 OF
YANG & MILLS.. AND INDEPENDENTLY BY SHAW

a) LOCAL SU(2) SYMMETRY $\longrightarrow$ ISOTRIPLET OF SPIN 1 FIELDS

b) GRAVITON?

DEMANDING $U = e^{i \sum_a \vec{\theta}(x) \cdot \vec{\tau}/2}$ $\longrightarrow$ $\vec{b}_\mu(x) \left\langle \begin{array}{l} \text{2 charged} \\ \text{1 neutral} \end{array} \right.$

isovector &
Lorentz vector

Yang Mills I:

AGAIN: $\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$

now $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ as bases for SU(2) operators

DEFINE A NEW COVARIANT DERIVATIVE...

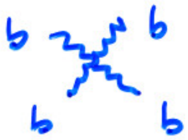
$$\mathcal{D}_\mu \equiv \partial_\mu + ig\,\vec{b}_\mu \cdot \vec{\tau}/_2$$

& substitute & lots of algebra–

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - \frac{g}{2}\bar{\psi}\gamma^\mu \vec{\tau}\psi \cdot \vec{b}_\mu - \underbrace{\tfrac{1}{4}\vec{f}_{\mu\nu}\cdot\vec{f}^{\mu\nu}}_{\text{complicated}}$$

$$\begin{aligned} -\tfrac{1}{4}\vec{f}^{\mu\nu}\cdot\vec{f}_{\mu\nu} = &-\tfrac{1}{2}(\partial_\nu \vec{b}_\mu - \partial_\mu \vec{b}_\nu)\cdot\partial^\nu \vec{b}^\mu \\ &+ g\,\vec{b}_\nu \times \vec{b}_\mu \cdot \partial^\nu \vec{b}^\mu \\ &- \tfrac{1}{4}g^2\left[(\vec{b}_\nu\cdot\vec{b}^\nu)^2 - (\vec{b}_\nu\cdot\vec{b}_\mu)(\vec{b}^\mu\cdot\vec{b}^\nu)\right] \end{aligned}$$

– get self-couplings for $\vec{b}$'s.

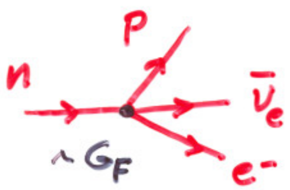


$\vec{b}_\mu$ FIELD IS STILL MASSLESS

ONE MIGHT HAVE HOPED THAT THE $\vec{b}_\mu$ WOULD HAVE FOUND WORK AS $\vec{W}_\mu$ ... but masslessness is a fatal flaw.

weak interactions, circa 1960 :

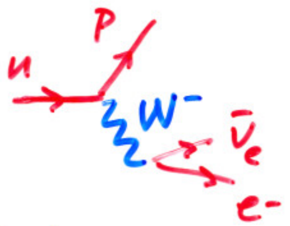
SINCE PAULI & FERMI IN 1930's...



$G_F \sim 10^{-5}/M_p^2$

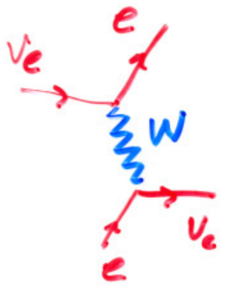
20 YEARS OF CONTRADICTORY EXPERIMENTAL RESULTS, SURPRISES, BEAUTIFUL THEORY (1958 Feynman & Gell-Mann)... A RAG-TAG BUNDLE OF DECAYS WERE FINALLY ALL RECOGNIZED TO BE "WEAK" & PARITY-VIOLATING

...HURISTICALLY DESCRIBED BY:

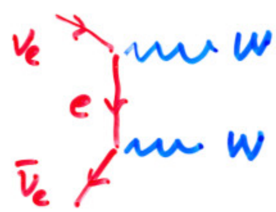


$W^\pm$: charged
isospin raising/lowering
massive

THERE WERE WELL-KNOWN PROBLEMS



violates Unitarity

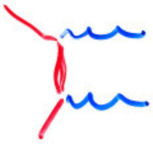


σ unbounded

2W Production

Contains an important hint...

hint 1:

THE PROBLEM WITH  LIES WITH THE

LONGITUDINAL DEGREE OF FREEDOM

- MASSLESS SPIN 1 FIELDS HAVE 2 dof --- polarizations, L, R (Gauge invariance)

- MASSIVE SPIN 1 FIELDS HAVE 3 dof... USUALLY TAKEN AS L, R, & LONGITUDINAL

$\varepsilon^{\mu}(\lambda=0) \sim \frac{k^{\mu}}{M}$ at high energy

HINT IN ELECTROMAGNETISM... 2γ PRODUCTION

BOTH GRAPHS REQUIRED BECAUSE REQUIRE GAUGE INVARIANCE...

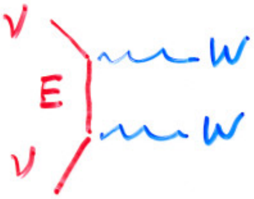
PRETEND THAT γ HAD A MASS... & THEREFORE A LONGITUDINAL dof.

THIS BADLY-BEHAVED POLARIZATION TERM CANCELS BETWEEN THE GRAPHS...

IN HINDSIGHT, CANCELATION CAN BE ARRANGED FOR W.I.

either, require a new, heavy electron

$\begin{pmatrix} E^+ \\ \nu_e \\ e^- \end{pmatrix}$



or, require a new, heavy spin 1 field

stay tuned.

hint 2:

ENCOURAGEMENT (!)...

ELECTROMAGNETISM EXHIBITS A MAGICAL BEHAVIOR...

(messy hadron)

(clean lepton)

SAME COUPLING (in limit)

SO DO WEAK INTERACTIONS...

(messy hadrons)

(clean leptons)

SAME COUPLING ! (vector weak charge)

↳ conservation of something

↳ a gauge symmetry?

spin 1 propagator...

CVC...

COULD THE WELL-BEHAVED ELECTROMAGNETIC INTERACTION BE RELATED TO THE ILL-BEHAVED, BADLY-BRED WEAK ?

Schwinger, Salam, Ward, Glashow, Weinberg... — using Yang-Mills ideas...!

$\begin{pmatrix} W^+ \\ \gamma \\ W^- \end{pmatrix}$? $\quad \begin{pmatrix} W^+ \\ Z^0 \\ W^- \end{pmatrix}$ & γ

BUT... YANG-MILLS FIELDS MUST BE MASSLESS... *sigh*

... AN INTERLUDE ...

MEANWHILE – CONDENSED MATTER PHYSICS WAS HAVING GREAT CONCEPTUAL & EXPERIMENTAL SUCCESS WITH 2nd ORDER PHASE TRANSITIONS

... cooperative phenomena in many-body physics

~ MINI-AGENDA ~

- LIGHT-SPEED REVIEWS OF
  - the thermodynamics of phase transitions
  - Mean Field Theory & the Ginsburg-Landau phenomenology
- FERROMAGNETISM AS AN EXAMPLE OF A "BROKEN SYMMETRY"

  ... AN INTERLUDE WITHIN AN INTERLUDE ...
- GOLDSTONE THEOREM
- DILUTE BOSE GAS AS AN EXAMPLE OF THE GOLDSTONE THEOREM
- GOLDSTONE – not!
  - superconductivity

BACK TO PARTICLE PHYSICS WITH THE SOLUTION — 1967

WHAT IS A PHASE ?

FORMALLY... A REGION OF ANALYTICITY OF THE FREE ENERGY...

$$f = -k_B T \ln Z \qquad \text{from statistical mechanics}$$

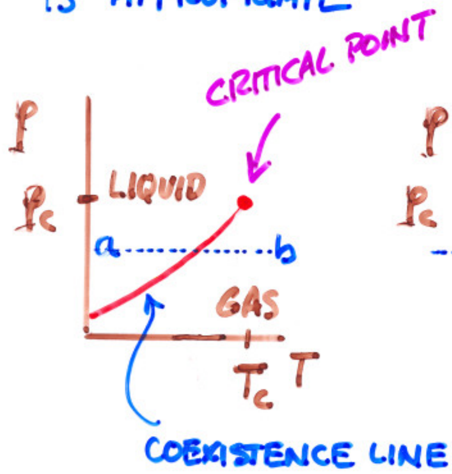
$\hookrightarrow \mathrm{Tr}\, e^{-H/k_BT}$

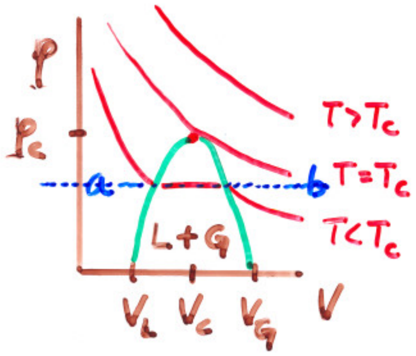
thermodynamics comes from derivatives of f

f: $F = U - TS$ (Helmholtz), $G = F + pV$ (Gibbs) — from thermodynamics

$$S = \left(-\frac{\partial G}{\partial T}\right)_{p,N} = \left(-\frac{\partial F}{\partial T}\right)_{V,N}$$

- A PARTICULAR PHASE MIGHT BE REALIZED WITH MINIMUM G...
- MORE THAN 1 PHASE MIGHT BE POSSIBLE (WITH SAME H), SUGGESTING THAT ANALYSIS OF f FOR NON-ANALYTIC BEHAVIOR IS APPROPRIATE

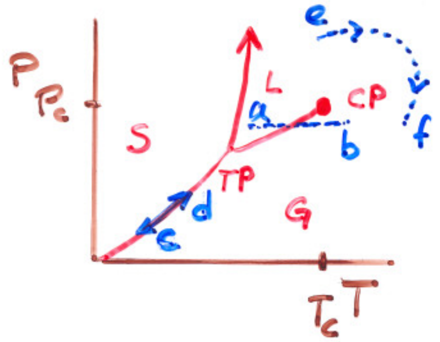




HEAT AT CONSTANT P & DENSITY, $a \to b$

$\Rightarrow dG_L = dG_G$ ACROSS COEXISTENCE LINE

thermodynamics of phase transitions 2:



IMAGINE HEATING, WHILE MAINTAING EQUILIBRIUM BETWEEN S & G, c→d

$dG_S = dG_G$ where $dG_i = V_i dP - S_i dT$

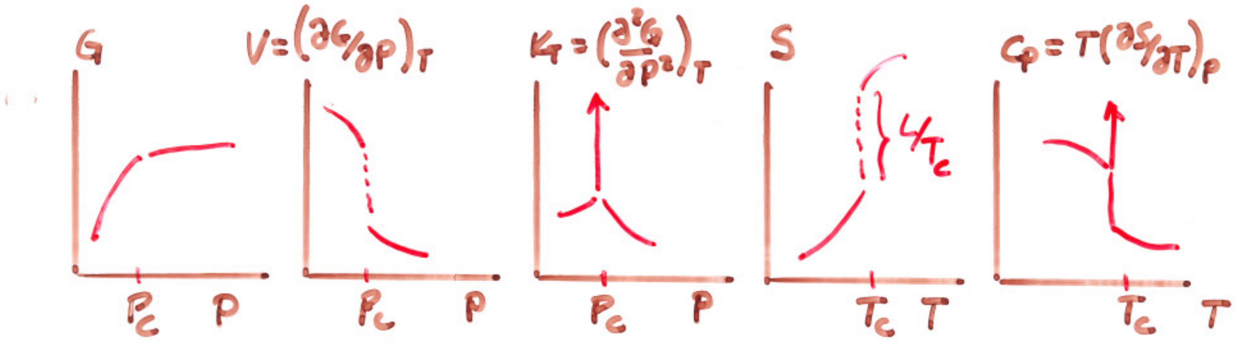
$\Downarrow$

$$\frac{dP}{dT} = \frac{S_S - S_G}{V_S - V_G} = \frac{\Delta S}{\Delta V} = \frac{L}{T\Delta V}$$

entropy change ⇒ heat absorbed in "crossing the line"

(Clausius-Clapeyron)

→ latent heat



FIRST DERIVATIVE OF G IS DISCONTINUOS ⇒ "1st ORDER P.T."

TAKES PLACE ACROSS COEXISTENCE CURVE

CRUCIAL CONCEPT IS THE SYMMETRY OF THE PHASES...

- A SYSTEM EITHER HAS A SYMMETRY... OR IT DOESN'T
- IF THERE IS A SYMMETRY CHANGE → P.T. HAS TAKEN PLACE

HIGH DEGREE OF SYMMETRY ⇒ LACK OF ORDER

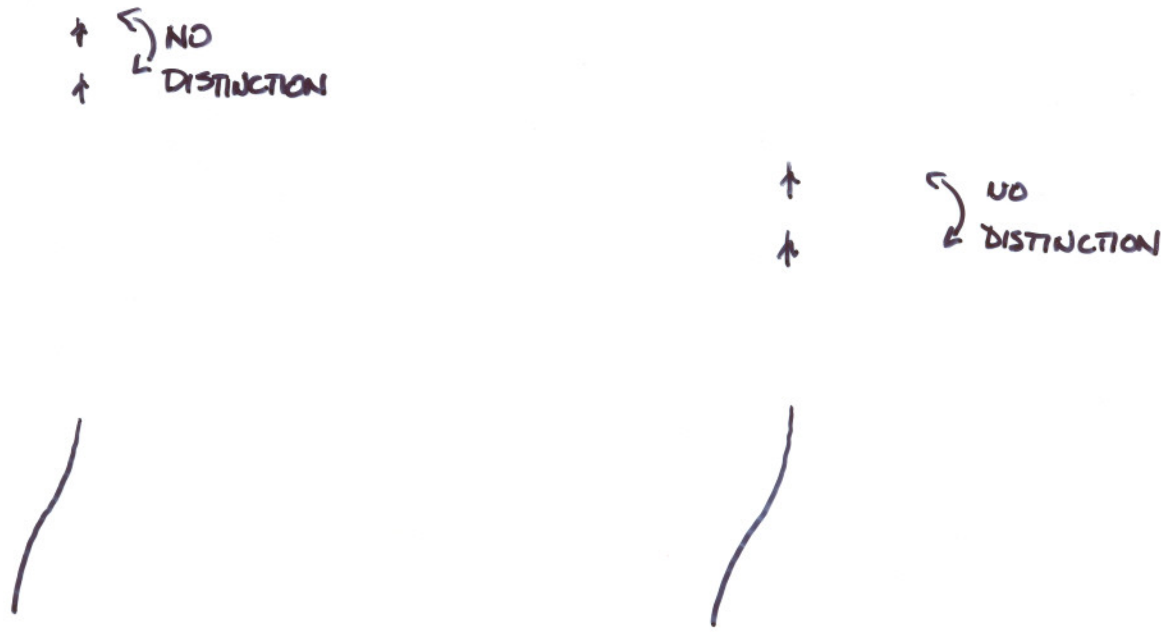
$\Downarrow$ $\Downarrow$

MORE SYMMETRY OPERATIONS ⇒ HIGH ENTROPY

@ HIGH TEMPERATURE

NOTE: e → f DOESN'T INVOLVE A SYMMETRY CHANGE.

COVER

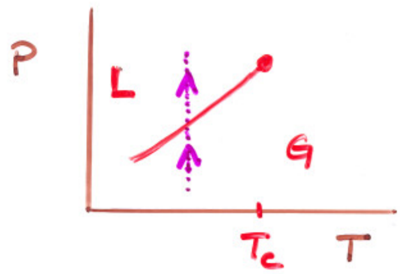


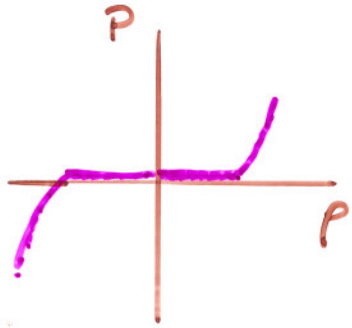
WHILE VERY DIFFERENT, THERE IS CLEARLY SOMETHING <u>THE</u> <u>SAME</u> ABOUT DENSITY IN A FLUID & MAGNETIZATION IN A FERROMAGNET.

... *a universality.*

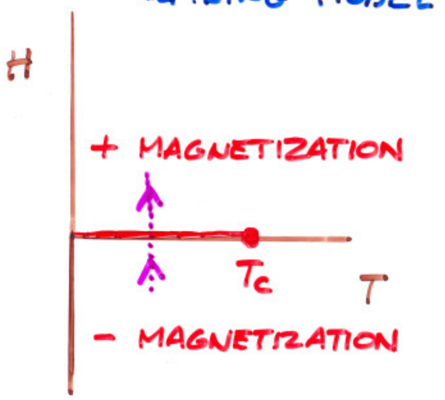
thermodynamics of phase transitions 2 3/4

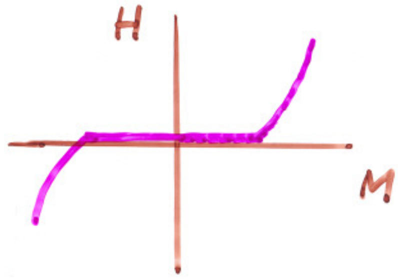
LIQUID-GAS



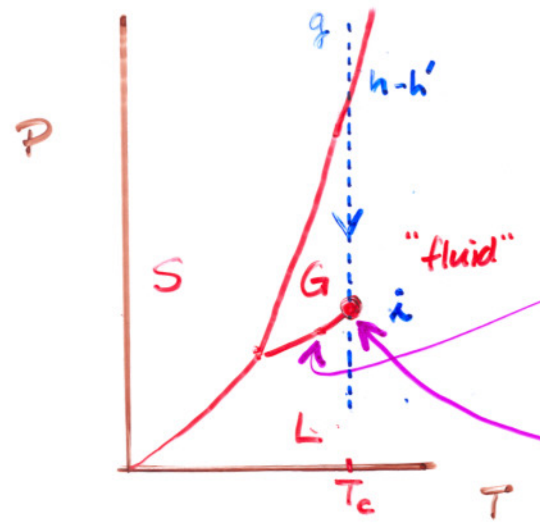


2d ISING MODEL



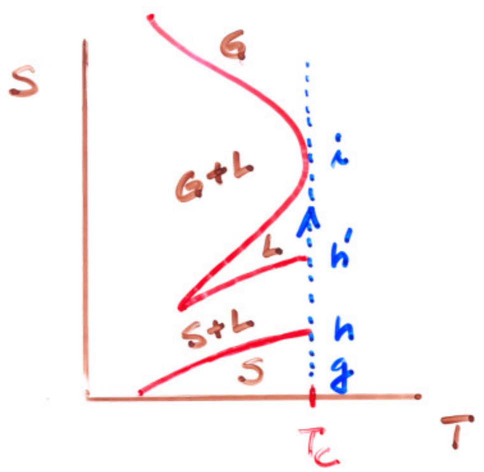


thermodynamics of phase transitions 2½:



PHASE BOUNDARY IS A LINE SEPARATING TWO DISTINCT PHASES OF A SYSTEM... what distinguishes them here? DENSITY

@ T_c, THAT DISTINCTION VANISHES



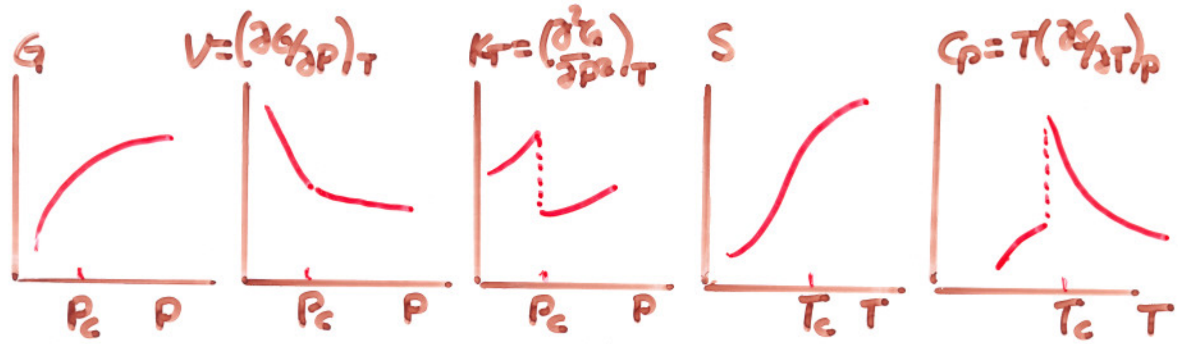
} G & L ARE EQUALLY ORDERED AT T_c

} PASSING THROUGH COEXISTENCE CURVE → LATENT HEAT

TRANSITIONS WHICH DON'T DISPLAY A SUDDEN STATE CHANGE & HAVE A CONTINUOUS ENTROPY CHANGE --- CALLED "ORDER-DISORDER" TRANSITIONS ... DERIVATIVES ARE DISCONTINUOUS .. "LAMBDA" TR.

thermodynamics of phase transitions 3:

THERE ARE P.T. WHICH ARE CONTINUOUS AT 1^{st} ORDER



2^{nd} DERIVATIVE ⇒ "2^{nd} ORDER P.T."

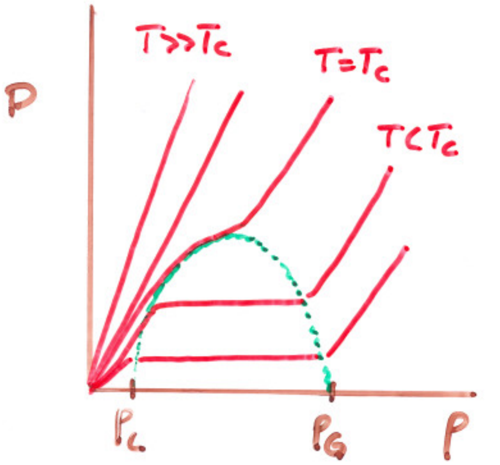
- FOLLOWING ON SYMMETRY FOCUS.. LANDAU & GINSBURG INVENTED A DEGREE OF FREEDOM TO MEASURE THE ORDER IN A SYSTEM : THE ORDER PARAMETER, $\eta(T)$.
  ... and in so doing, universalized the study of phase transitions

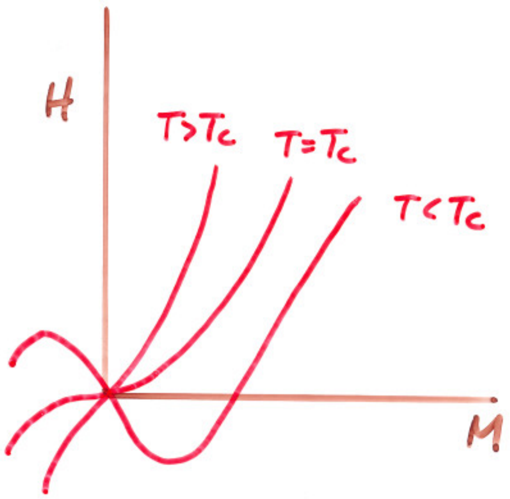
- IF $\eta = 0$ THEN SYSTEM IS IN ORDERED PHASE
  $|\eta| \neq 0$ THEN SYSTEM IS IN DISORDERED PHASE

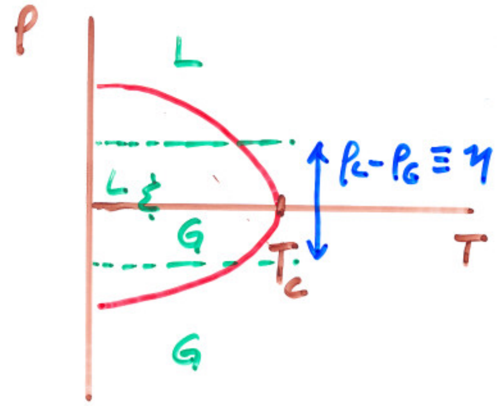
- IF $\eta(T) \rightarrow 0$ CONTINUOUSLY THEN P.T. IS 2^{nd} ORDER

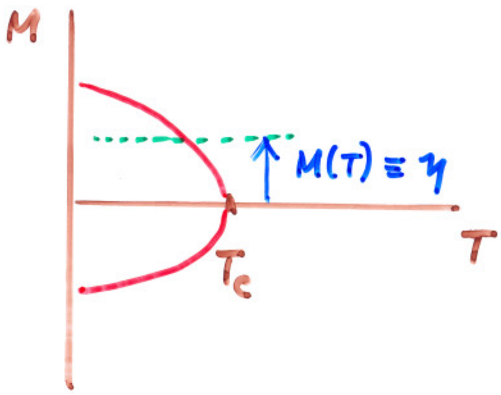
SYSTEM	η	EXAMPLE	T_c (K)
liquid-gas	$\rho_L - \rho_G$	H_2O	647
ferromagnet	M	Fe	1044
superfluid	$\psi_{\text{ground state}}$	^4He	2
superconductivity	$\psi_{\text{cooper pairs}}$	Pb	7
ferroelectrics	P	truglycerine sulfate	323
binary alloys	concentration	Cu-Zn	739

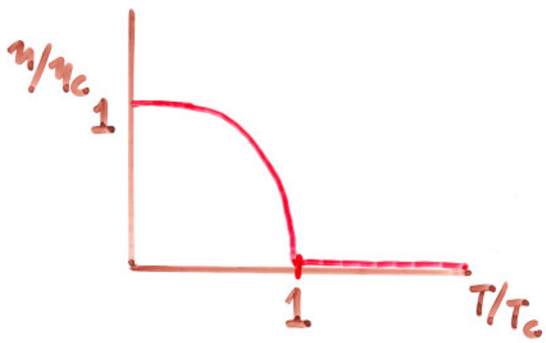
thermodynamics of phase transitions 4:











thermodynamics of phase transitions 5:

NEAR T_c, LANDAU POSTULATED THAT WE CAN WRITE A FUNCTION, L (Landau free energy) ... RELATED TO G. $\propto V$

$$L(P,T,\eta) = L_0 + \beta(P,T)\eta^2 + \delta(P,T)\eta^4 \ldots$$

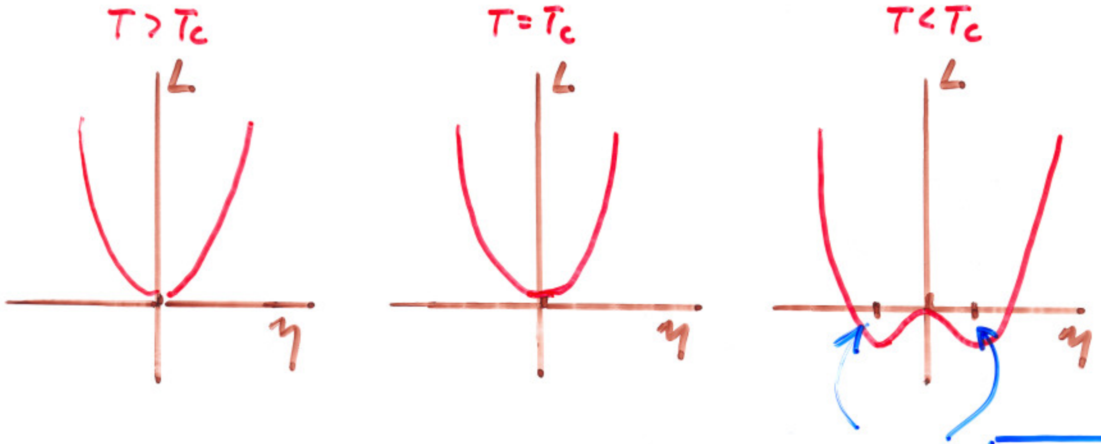
$$\left.\begin{array}{lll} \beta > 0 & \Rightarrow & T \geq T_c \\ \beta < 0 & \Rightarrow & T < T_c \end{array}\right\} \text{FLIP, ABOVE \& BELOW PHASE TRANSITION}$$

$\delta > 0$ ALWAYS

- MINIMIZATION OF L GUARANTEES A STABLE GROUND STATE

... PRESUME GOOD BEHAVIOR NEAR T_c: $\beta(P,T) = b(P)(T-T_c) + \ldots$

$$L = L_0 + b(T-T_c)\eta^2 + \delta\eta^4$$



SIMPLE EXERCISE 3: SHOW

$$\left\{\begin{array}{l} \eta = \pm\sqrt{\dfrac{b}{2\delta}|T-T_c|} \\ L = -\dfrac{b^2}{2\delta}(T-T_c)^2 \end{array}\right.$$

TWO CHARACTERISTICS FOR $T < T_c$:

1. GROUND STATE ENERGY <u>LOWERED</u>
2. MULTIPLE GROUND STATE CONFIGURATIONS POSSIBLE

ferromagnetism I:

SYMMETRY IN PHASE TRANSITIONS

... FERROMAGNET

$T > T_c$

$\langle M \rangle = 0$

- ALL DIRECTIONS EQUALLY PROBABLE... GROUND STATE IS INVARIANT wrt $SO(3)$, U_3
- $[H, G] = 0$

  $\Rightarrow$ HAMILTONIAN INVARIANT wrt $SO(3)$

$T < T_c$

$\langle M \rangle \neq 0$

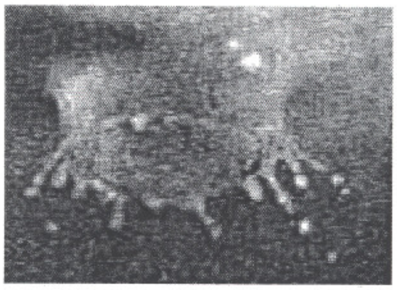
- A SINGLE, RANDOM DIRECTION IS SINGLED OUT
- SYMMETRY OF GROUND STATE IS LOWERED $SO(3) \rightarrow SO(2)$
- $[H, G] = 0$ still

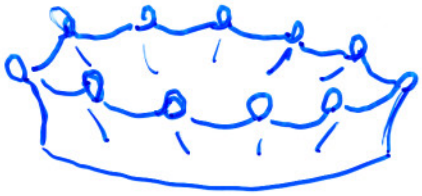
- SPECIAL STATE OF AFFAIRS.. common to 2^{nd} order p.t. ... SYMMETRY OF GROUND STATE IS LOWERED FROM THAT OF THE HAMILTONIAN

- SYMMETRY IS SAID TO BE "SPONTANEOUSLY BROKEN". (lousy phrase.. better is "HIDDEN SYMMETRY")

SYSTEMS WITH SYMMETRIES WHICH ARE NOT BROKEN ARE RARE! classically & quantum mechanically

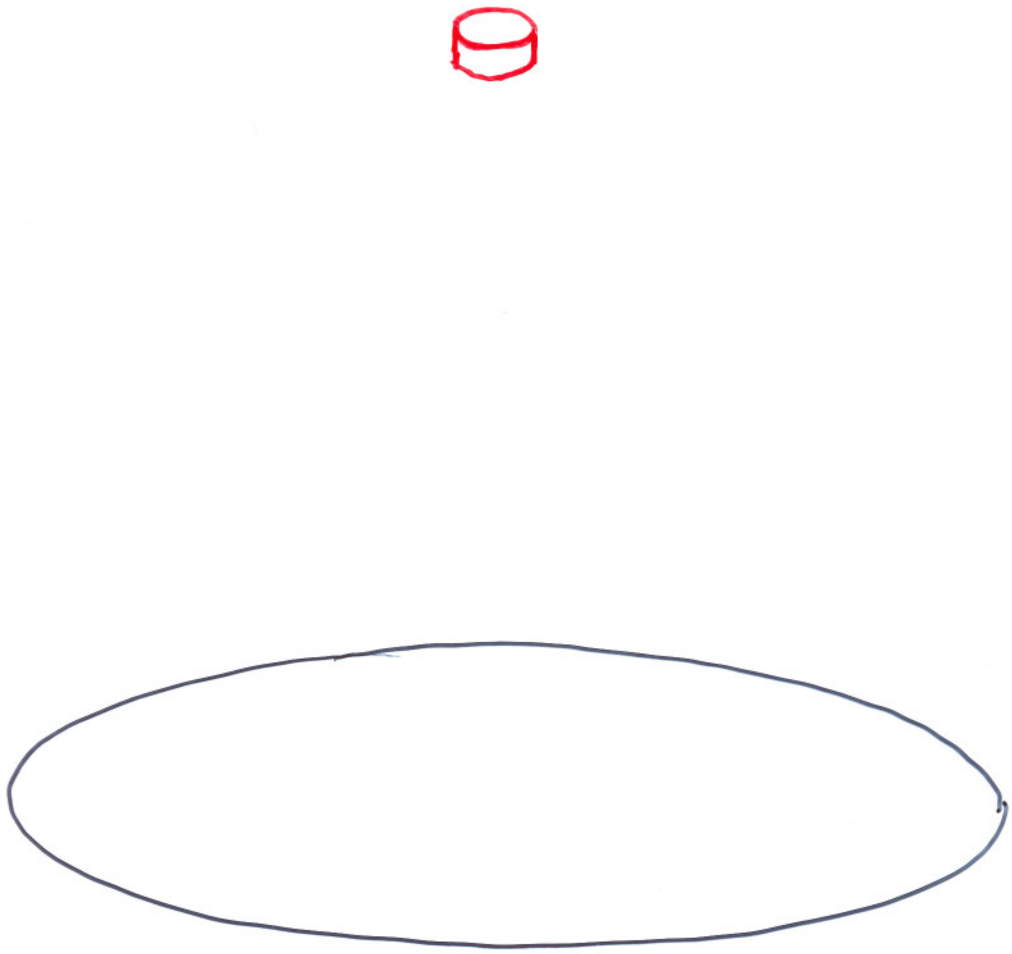
WORTHINGTON – turn of the century





WHERE DOES THE SYMMETRY "GO"?

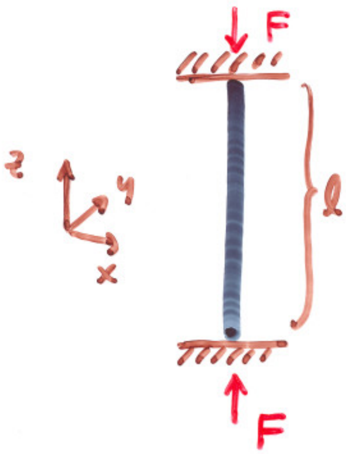
IT'S STILL THERE... INSIDE OF THE ENSEMBLE OF ALL POTENTIAL SPLASHES



Spontaneously broken symmetries !!

HOW DOES SYMMETRY GET LOST? WHERE DOES IT GO?

CLASSIC... CLASSICAL... EXAMPLE (solved by Euler):



$$EI\frac{d^4x}{dz^4} + F\frac{d^2x}{dz^2} = 0$$

$$EI\frac{d^4y}{dz^4} + F\frac{d^2y}{dz^2} = 0$$

} $x = y = 0$ is a solution

BUT, WHEN $F > \frac{4\pi^2 EI}{l^2} \equiv F_c$

$x \text{ (or } y) = C \sin kz \qquad k = \sqrt{|F|/EI}$

SYMMETRY IS LOST... HIDDEN (same equation of motion)

→ ROD COULD HAVE PICKED AN INFINITE NUMBER OF DIRECTIONS TO BULDGE... IN ACCORDANCE WITH ORIGINAL SYMMETRY

...just like ferromagnet

~ INTERLUDE ~ AROUND 1960 HEP THEORISTS WERE STRUGGLING WITH A NUMBER OF BROKEN SYMMETRIES: SU(3), SU(2), PARITY...

★ Weinberg got a whiff of CMP's success & began trying to apply some of these ideas → idea that symmetry isn't gone, but hidden was appealing to him... and wrong.

WRONG BECAUSE ...

GOLDSTONE THEOREM (G.T.): A SYSTEM WHICH HAS A SPONTANEOUSLY BROKEN CONTINUOUS SYMMETRY <u>MUST</u> HAVE MASSLESS, BOSE-EXCITATIONS.

(This spoiled Weinberg's hopes, as there are no massless spin zero particles...)

G.T. WORKS FINE FOR CMP...

eg ferromagnetism

↑ ↑ ↑ ↑ ↑ ↑ ↑ — GROUND STATE

↑ ↑ ↑ ↓ ↑ ↑ ↑ — 1 EXCITED STATE

but that's not what magnets do (<u>large</u> magnets..!)

energetics favor: ↑ ↗ → → ↘ ↓ ↓ ↙ ↙ ↙ ← ← ↖ ↑ ↑

get a long-wavelength MACROSCOPIC, QUANTIZABLE excitation with energy

$$\epsilon = \hbar^2 S \sum_{\vec{a}} (1 - \cos \vec{q}\cdot\vec{a}) \qquad (\text{"dispersion"})$$

AS $\vec{q} \to 0$, $\epsilon \to 0$ $\Rightarrow$ "MASSLESS"

... as if the ground state is full of SPIN WAVE excitations...

IF YOU LIVED INSIDE AT $T < T_c$, HOW WOULD YOU RECOGNIZE THAT THE SYMMETRY OF THE HAMILTONIAN IS SO(3)!?

... that's our situation.

PROOF:

- SUPPOSE WE HAVE A CONSERVED CURRENT, $\partial_\mu j^\mu(x) = 0$ FOR SOME SYSTEM CHARACTERIZED BY FIELDS $\phi(x)$

$$\partial_\mu [j^\mu(x), \phi(x')] = 0$$

$$\partial_0 [j^0(x), \phi(x')] - \vec{\nabla} \cdot [\vec{j}(x), \phi(x')] = 0$$

$$\partial_0 \int d^3x\, [j^0(x), \phi(x')] - \int d\vec{S} \cdot [\vec{j}(x), \phi(x')] = 0$$

(using Divergence theorem)

if $\int d\vec{S} \cdot [\vec{j}(x), \phi(x')] = 0$ over the surface

field operator

then $\partial_0 [Q(t), \phi(x')] = 0 \Rightarrow [Q, \phi(x')] = \text{constant}, C$

- TAKE EXPECTATION VALUE OF THIS QUANTITY IN VACUUM...

$$\langle 0 | [Q, \phi(x')] | 0 \rangle = \langle 0 | C | 0 \rangle$$

without identifying this quantity, yet.

- USE COMPLETENESS TO INSERT THE SPECTRUM OF A COMPLETE SET OF INTERMEDIATE STATES OF THE ϕ's, $|n\rangle$...

$$\sum_n \left[\langle 0 | Q | n \rangle \langle n | \varphi(x') | 0 \rangle - \langle 0 | \varphi(x') | n \rangle \langle n | Q | 0 \rangle \right] = \langle 0 | C | 0 \rangle$$

- WRITE Q IN TERMS OF $j(x)$ & SHIFT SPACETIME ARGUMENT USING

$$j^0(x) = e^{-iPx} j^0(0)\, e^{iPx} \quad \text{with} \quad e^{iPx}|n\rangle = e^{ik_n x}|n\rangle, \quad e^{iPx}|0\rangle = |0\rangle$$

$$\int d^3x \left\{ \sum_n \langle 0 | j^0(0) | n \rangle \langle n | \phi(x') | 0 \rangle e^{ik_n x} - \langle 0 | \varphi(x') | n \rangle \langle n | j^0(0) | 0 \rangle e^{ik_n x} \right\} =$$

- INTEGRATE

exponentials contain only $\vec{x}$ dependence $\longrightarrow \delta(\vec{k}_n)$

Goldstone theorem 3:

only time dependence

$$\sum_n (2\pi)^3 \delta(\vec{k}_n)\Big[\langle 0|j^0(0)|n\rangle\langle n|\varphi(x')|0\rangle e^{-iE_n t} - \langle 0|\varphi(x')|n\rangle\langle n|j^0(0)|0\rangle e^{iE_n t}\Big] = \langle 0|c|0\rangle \equiv RHS$$

GO BACK: $\langle 0|[Q, \varphi(x')]|0\rangle = \langle 0|Q\varphi'(x')|0\rangle - \langle 0|\varphi(x')Q|0\rangle \equiv LHS$

LEAVING 2 CONSEQUENCES, DEPENDING ON VACUUM & GROUP.

a.) $\mathcal{U}(Q)|0\rangle = |0\rangle \quad \Rightarrow \quad Q|0\rangle = 0$ "Weyl Symmetry"

OR

b.) $\mathcal{U}(Q)|0\rangle \neq |0\rangle \quad \Rightarrow \quad Q|0\rangle \neq 0$ "Goldstone Symmetry"

a.) IS THE USUAL SITUATION ... the vacuum "carries the trivial, one dimensional representation of all symmetry groups." Rowan

b.) HAPPENS ALL THE TIME IN CMP ... $|0\rangle \equiv$ GROUND STATE ☆ NAMBU

IF b) IS THE SITUATION...

$LHS = \langle 0|[Q, \varphi(x')]|0\rangle \neq 0$

$RHS =$ independent of time $\Rightarrow$ in $e^{\pm iE_n t}$ terms $E_n \to 0$

HERE WE GO AGAIN: AS $\underline{\vec{k}_n \to 0}$, $\underline{E_n \to 0}$ $\Rightarrow$ MASSLESS $|n\rangle$'s.

NOTE: IF $\phi(x)$ IS NOT A SINGLET UNDER THE GROUP

$[Q, \phi(x')] = \phi'(x')$ ... some ϕ' must exist "vacuum expectation value"

THEN OUR $\langle 0|c|0\rangle \longrightarrow \underbrace{\langle 0|\phi'(x')|0\rangle}$ VEV OF FIELD ITSELF

vacuum • - - - - - - • vacuum
$\phi'(x)$

ϕ CONNECTS VACUUM TO ITSELF...
VACUUM IS FULL OF ϕ's.

OBSERVATION THAT VEV OF FIELD $\neq 0$ IS A TRIGGER FOR GOLDSTONE THEOREM.

DILUTE BOSE GAS: STATISTICAL MECHANICS

$n_i > 0 \Rightarrow$
$\varepsilon_i - \mu \geq 0 \Rightarrow$
$\mu \leq 0$

RECALL: # OCCUPATION NUMBER FOR BOSONS $n_i = \dfrac{g_i}{e^{(\varepsilon_i - \mu)/kT} - 1}$ (BE)

TOTAL OCCUPATION: $N = 2\pi V (2m)^{3/2} \displaystyle\int_0^\infty \frac{\varepsilon^{1/2}\, d\varepsilon}{e^{(\varepsilon-\mu)/kT} - 1}$

continuum limit

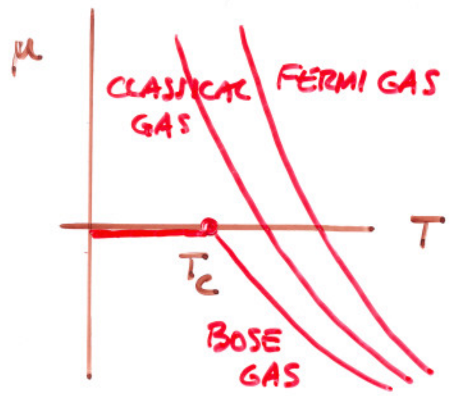
$g(\varepsilon)\,d\varepsilon = 2\pi V (2m)^{3/2} \varepsilon^{1/2} d\varepsilon$
non-relativistic

$\dfrac{1}{e^{-x}-1} = \displaystyle\sum_{j=1}^{\infty} e^{jx}$

$N = V(2m\pi kT)^{3/2} \left[\displaystyle\sum_{j=1}^{\infty} \frac{1}{j^{3/2}} e^{j\mu/kT} \right]$

as $T \to \infty$, $N \to e^{-\mu/kT} \longrightarrow$ M.B. $\quad \mu \to -\infty$, $T \to \infty$
hot $\to$ classical

@ $\mu = 0$. CALL $T \equiv T_c$: $T_c = \dfrac{1}{[\zeta(3/2)]^{2/3}} \left(\dfrac{N}{V}\right)^{2/3} \dfrac{1}{2m\pi k}$



BELOW T_c? SEPARATE GROUND STATE FROM EXCITED STATES...

$N = n_0 + N_\varepsilon$

Ground state

$\varepsilon = 0 \qquad n_0 = \dfrac{g_0}{e^{-\mu/kT} - 1} \qquad \mu = 0^-$ below T_c (to keep n +)

Excited states

$\varepsilon \neq 0 \qquad n_\varepsilon = \dfrac{g_i}{e^{(\varepsilon_i - \mu)/kT} - 1} \qquad \mu = 0$ at $T = T_c$

$N = n_0 + V(2\pi m kT)^{3/2} \sum \dfrac{1}{j^{3/2}} e^{j\mu/kT}$

FOR $T < T_c$ $\mu = 0$ IN SECOND TERM

$N = n_0 + V(2\pi m kT)^{3/2} \zeta(3/2)$

$N = n_0 + N(T/T_c)^{3/2} \Rightarrow \boxed{n_0 = N\left[1 - \left(\frac{T}{T_c}\right)^{3/2}\right]}$

AS $T \to 0$ $n_0 \to N$
CONDENSATE IN G.S.

DILUTE BOSE GAS: QUANTUM MECHANICS

- CONDENSATION INTO GROUND STATE IS A PROBLEM FOR A FIELD THEORY

RECALL: USE WICK'S THEOREM TO ORDER a's AND $a^\dagger$'s IN VEV's ... NEED $a|0\rangle = 0$ & $\langle 0|a^\dagger = 0$ TO BUILD A PERTURBATION THEORY...

need an empty vacuum — Bose-Einstein Condensate is a full vacuum!

$|0\rangle_N \equiv$ VACUUM STATE WITH N PARTICLES

as a Fock state...

$|0\rangle_N = |N, 0, 0 \ldots 0\rangle$

ALL OCCUPY THE $\varepsilon = 0$ STATE AT $T = 0$

- A WAY OUT INVENTED BY BOGOLIUBOV...

$$H = \int d^3x \; \psi^\dagger(x) \left[-\frac{\hbar^2}{2m} \nabla^2 \right] \psi(x) \qquad \text{K.E. term}$$

$$+ \int d^3x \int d^3x' \; \psi^\dagger(x) \psi^\dagger(x') v(x,x') \psi(x) \psi(x') \qquad \text{P.E. term} \quad \psi^4$$

$$+ \mu \int d^3x \; \psi^\dagger(x) \psi(x) \qquad \text{C.P. term} \quad \psi^2$$

$\mu = 0$ IN CONDENSATE:

★ [Reminiscent of Landau free energy with $\psi_{G.S.}$ as order parameter] ★

$$H = \sum_{k} \frac{\hbar^2 k^2}{2m} a_k^\dagger a_k + \sum_{k_1} \sum_{k_2} \sum_{k_3} \sum_{k_4} \underbrace{\delta_{k_1+k_2,\, k_3+k_4}}_{\text{conserves momentum}} f(\vec{k}_2 - \vec{k}_4) a_{k_1}^\dagger a_{k_2}^\dagger a_{k_3} a_{k_4}$$

SINCE ${}_N\langle 0|a^\dagger a|0\rangle_N \neq 0$ & $a|0\rangle_N = N^{\frac{1}{2}}|0\rangle_{N-1} \simeq N^{\frac{1}{2}}|0\rangle_N$ large N

WE HAVE ${}_N\langle 0|\psi|0\rangle_N = {}_N\langle 0|\psi^\dagger|0\rangle_N \neq 0$ FOR FIELDS OF SYSTEM... SHOULD EXPECT TO SEE GOLDSTONE KICKING IN?

Bose gas 3:

$a^\dagger a$, NUMBER OPERATOR

- NEED A BROKEN SYMMETRY... $U = e^{i\lambda N}$ IS TRIVIALLY SATISFIED BY HAMILTONIAN BUT $U|0\rangle_N \neq |0\rangle_N$

SINCE $N|0\rangle_N \neq 0$

YUP.. WE GOT GOLDSTONE.

BUT WE ALSO HAVE THE FIELD THEORY PROBLEM...

- BOGOLIUBOV NOTED $a^\dagger$ AND a ARE ALMOST C-numbers...

SIMPLE EXERCISE 4: SHOW THAT $[a^\dagger, a] \simeq 0$ IN CONDENSATE GROUND STATE AND THAT $\underbrace{a^\dagger \simeq a \simeq \sqrt{n_0}}_{\sim \text{ C-NUMBERS}}$.

BOGOLIUBOV TRANSFORMATION;

two parts -

1. SHIFT AWAY FROM GROUND STATE

$$\psi(x) = e^{i\alpha}\sqrt{n_0} + \chi(x)$$

original field op's... eg ^{4}He atoms

G.S. stuff: number

EXCITED STATE STUFF

$$\chi(x) = \frac{1}{\sqrt{V}} \sum_{\vec{k}\neq 0} a_{\vec{k}}\, e^{i\vec{k}\cdot\vec{x}}$$

IN CONDENSATE: ${}_N\langle 0|\chi(x)|0\rangle_N = 0$

SUBSTITUTE THIS INTO HAMILTONIAN...

$$H = N^2 + \sum_{\vec{k}\neq 0} \omega_k a_k^\dagger a_k + N \sum_{\vec{k}\neq 0} \left(a_k a_{-k} + a_k^\dagger a_{-k}^\dagger\right)$$

where $\omega_k = \frac{\hbar^2 k^2}{2m} + 2N f(\vec{k})$

MESSY EXERCISE 1: SHOW THIS.

NOTE: PRICE IS NON-DIAGONAL INTERACTION --

2. DIAGONALIZE WITH A CANONICAL TRANSFORMATION

$$\left.\begin{aligned} \alpha_{\vec{k}} &\equiv u_k a^\dagger_k + v_k a^\dagger_{-k} \\ \alpha_{-\vec{k}} &\equiv u_{-k} a_{-k} + v_{-k} a^\dagger_k \end{aligned}\right\} \text{ } \alpha\text{'s have same commutation relations as } a\text{'s.}$$

α's CREATE & ANNIHILATE A NEW PARTICLE SPECTRUM ... A "QUASI-PARTICLE" SPECTRUM & $\alpha_k |0\rangle_N = 0 \quad \forall_{k\neq 0}$

$$H = \underbrace{N^2 - \frac{1}{2}\sum_{k\neq 0}(\omega_k - \varepsilon_k)}_{\text{G.S. energy level lowered}} + \underbrace{\frac{1}{2}\sum_{k\neq 0} \varepsilon_k\, \alpha^\dagger_k \alpha_k}_{\text{energy spectrum of quasi particles near ground state}}$$

where $\varepsilon_k = \sqrt{\omega_k^2 - 4N^2 f^2(k)}$

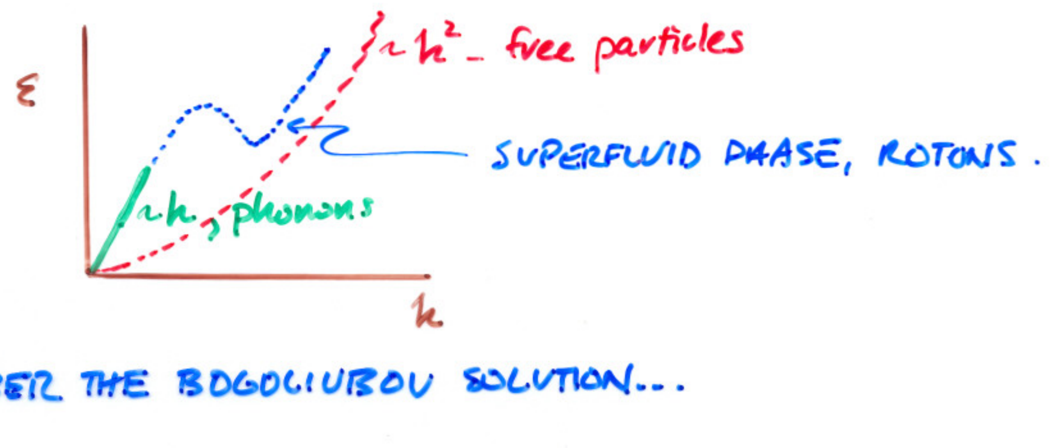
$$\varepsilon_k = \sqrt{\frac{\hbar^4 k^4}{4m^2} + \frac{4\hbar^2 k^2 f(k)}{2m}} \qquad \text{DISPERSION}$$

AS $k \to$ LARGE, $\varepsilon_k \sim k^2$ such as free particles...

AS $k \to$ SMALL, $\varepsilon_k \sim k\sqrt{\frac{4\hbar^2 f(0)}{2m}} \propto k$ LIKE PHONONS...

$\left.\begin{matrix}\to 0 \\ \text{as } k \to 0\end{matrix}\right\}$ massless

* THE MASSLESS QUASI EXCITATIONS ARE PHONONS ... GOLDSTONE BOSONS OF BOSE GAS



REMEMBER THE BOGOLIUBOV SOLUTION...

recap:

MANY PHENOMENA INVOLVE BROKEN SYMMETRIES:
full symmetry is "really" there... natural manifestation hides that fact

$\Rightarrow$

GROUND STATE FULL: $\langle 0|\phi|0\rangle \neq 0$
BROKEN CONTINUOUS SYMMETRY
$\Rightarrow$ MASSLESS GOLDSTONE BOSONS

$\downarrow$

SHIFT FIELD OPERATORS INTO
c-number vacuum term + quasi-particle operator term

$\downarrow$

INSERT INTO MODEL & TRANSFORM INTO QUASIPARTICLE SPECTRUM

GINSBURG-LANDAU PHENOMENOLOGY:
identify order parameter & mechanically induce phase transition

PUT THESE IDEAS TOGETHER WITH THE ASSUMPTION THAT THE MANY-BODY GROUND STATE IS ANALOGOUS TO THE ELEM. PARTICLE VACUUM & $\eta \equiv \phi$

Plunge down along
the coexistence curve.

TOY THEORY: ... mix these ideas up!

- INCORPORATE ALL OF ABOVE NOTIONS INTO A RELATIVISTIC QUANTUM FIELD THEORY... Goldstone 1960
  "Field Theories with 'Superconductor' Solutions"

- FOLLOWING LANDAU FORM... OR EQUIVALENTLY, THE BOSE GAS:

$$\mathcal{L} = \underbrace{\frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi}_{\text{KE term}} - \underbrace{a\frac{|\mu^2|}{2}\varphi^2}_{\text{mass term (like }\mu\text{ in Bose gas)}} - \underbrace{\frac{1}{4}\lambda\varphi^4}_{\text{self interaction}}$$

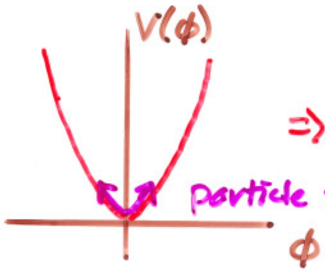
Euler-Lagrange equations of motion: $\partial_\mu\partial^\mu\phi + a\mu^2\phi = \lambda\phi^3$ ✓

ϕ has mass $\sqrt{a}\,\mu$ ...

- INVESTIGATE SYMMETRY...
  REFLECTION SYMMETRY $\phi \rightarrow -\phi$ LEAVES $\mathcal{L}$ ALONE

- IDENTIFY LANDAU FREE ENERGY WITH PE TERM OF $\mathcal{L}$

$$V(\phi) = \frac{a\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$



- MINIMIZE TO FIND GROUND STATE:
  minimum @ $V(\varphi) = 0$

$\Rightarrow \langle 0|\phi|0\rangle = 0$

- BUT, a la LANDAU, ALLOW A 2nd ORDER "PHASE TRANSFORMATION"...
  $a \rightarrow -|a|$

$$V(\phi) = -\frac{a\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

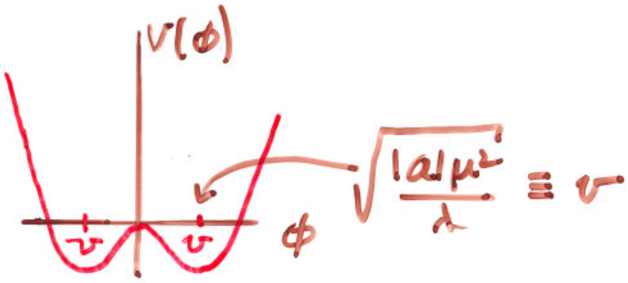
... mass interpretation for $\sqrt{a}\cdot\mu$ is destroyed... now a complicated interaction for massless ϕ particles

$$\partial_\mu\partial^\mu\varphi - a\mu^2\varphi = \lambda\varphi^3$$

wrong for mass term in K.G. equation

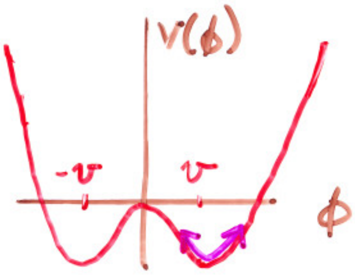
relativistic Goldstone 2:

- MINIMIZE



"VACUUM" OCCURS AT FINITE $\phi \Rightarrow$ "$\langle 0|\varphi|0\rangle \neq 0$"
$= \pm v$

- FULLY REALIZE THE THEORY AS BROKEN, BY CHOOSING ONE OF THE VACUA AS THE VACUUM FROM WHICH THE PARTICLE SPECTRUM IS BUILT..



$\Rightarrow$ A BOGOLIUBOV-LIKE SHIFT $\phi(x) = \langle 0|\varphi|0\rangle + \chi(x)$
$= v + \chi(x)$

& SUBSTITUTE BACK..

$\mathcal{L}(\chi) = \frac{1}{2}\partial_\mu \chi \partial^\mu \chi - |a| \mu^2 \chi^2 +$ quartic & cubic self interactions

Hold the phone... this is the Lagrangian for a χ field of mass $\sqrt{2a}\,\mu$ $\rightarrow$ WHERE HAS GOLDSTONE GONE?

WHAT'S WRONG WITH THE GOLDSTONE THEOREM?
nothing

MESSY EXERCISE 2: SHOW $\mathcal{L}(\chi)$.

HERE, THE SYMMETRY WAS A BROKEN DISCRETE SYMMETRY...
GOLDSTONE THEOREM INVOLVED BROKEN CONTINUOUS SYMMETRIES.

notch it up one step...

NEW TOY:

- FOR A CONTINUOUS SYMMETRY.. NEED MORE THAN 1-COMPONENT OBJECT: φ_1 & φ_2 or φ & $\varphi^+ = \frac{\varphi_1 \pm i\varphi_2}{\sqrt{2}}$

$$\mathcal{L}(\varphi) = \tfrac{1}{2}\partial_\mu \varphi^+ \partial^\mu \varphi - \tfrac{1}{2} a \varphi^+\varphi - \tfrac{1}{4}\lambda(\varphi^+\varphi)^2$$

SYMMETRY: $\varphi \to \varphi' = e^{i\theta}\varphi$ LEAVES $\mathcal{L}$ ALONE...

or $\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix} \to \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}' = \begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$

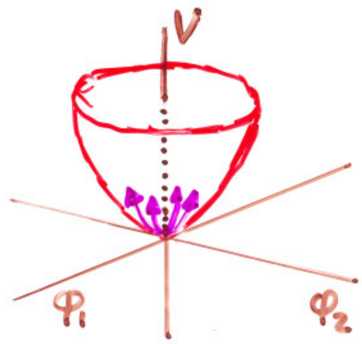
... Global $U(1)$ or $SO(2)$, which are isomorphic.

2 COMPONENT "ISODOUBLET-LIKE" MORE INSTRUCTIVE:

$$\mathcal{L}(\varphi) = \tfrac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \tfrac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 - \frac{a\mu^2}{2}(\varphi_1^2+\varphi_2^2) - \frac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2$$

$$V(\varphi_1,\varphi_2) = \frac{a\mu^2}{2}(\varphi_1^2+\varphi_2^2) + \frac{\lambda}{4}(\varphi_1^2+\varphi_2^2)^2$$

MINIMIZATION LEADS TO:



- NOW, $a \to -|a|$

MINIMIZATION LEADS TO: $\varphi_1^2 + \varphi_2^2 = \frac{|a\mu^2|}{\lambda}$ ... a loci which is a circle @ RADIUS $v = \sqrt{\frac{a\mu^2}{\lambda}}$

number of vacua is now infinite
→ CHOICE OF ONE INVOLVES A SLICE IN $\varphi_1 - \varphi_2$ & BREAKS THE $SO(2)$ SYMMETRY