BROKEN TOY:

LOCUS: $\langle 0|\varphi|0\rangle = v e^{i\alpha} = v\cos\alpha + i v \sin\alpha$

CHOOSE TO BREAK SYMMETRY BY $\alpha = 0$

$$\left.\begin{array}{l}\langle 0|\varphi_1|0\rangle = v \\ \langle 0|\varphi_2|0\rangle = 0\end{array}\right\} \text{ A } \varphi_2 = 0 \text{ SLICE}$$

$$\langle 0|\begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}|0\rangle = \begin{pmatrix}v\\ 0\end{pmatrix}$$

SHIFT FIELDS USING COMPLEX REPRESENTATION...

$$\varphi = \underbrace{v + \sigma(x)}_{\varphi_1} + \underbrace{i\eta(x)}_{i\varphi_2}$$

TO QUASI PARTICLE SET. $\sigma(x)$ & $\eta(x)$.

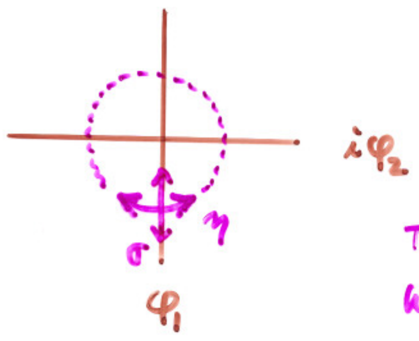
SUBSTITUTE -

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu\eta\,\partial^\mu\eta + \tfrac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma - a|\mu^2|\sigma^2 + \underbrace{\text{cubic}}_{\text{no } \eta^2 \text{ term}} \text{ \& quartic interactions}$$

φ_2 LOST ITS MASS... η IS MASSLESS (THE GOLDSTONE BOSON)

σ IS MASSIVE, $m_\sigma = \sqrt{2a}\,\mu$

LOOK DOWN ON V=0 PLANE...



THE η OSCILLATES WITHIN THE WELL (MASSLESS)... CONNECTING OTHERS OF THE DEGENERATE VACUUMS

Higgs mechanism I:

GOLDSTONE THEOREM IRON-CLAD

- PROVEN BY WEINBERG, SALAM. & GOLDSTONE
  - ⇒ USE BY HEP TO ACCOUNT FOR APPROXIMATE SYMMETRIES WAS DEAD
- EXCEPT.. UNNOTICED BY MANY (EXCLUDING ANDERSON, BY THE WAY...)

THERE IS A LOOPHOLE...

- WE DID GLOBAL $U(1)$ SYMMETRY... WHAT ABOUT LOCAL $U(1)$ SYMMETRY?

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi^+\partial^\mu\varphi - \frac{a\mu^2}{2}\varphi^+\varphi - \frac{\lambda}{4}(\varphi^+\varphi)^2$$

WE KNOW HOW TO MAKE THIS LOCALLY GAUGE INVARIANT...

$\partial^\mu \longrightarrow \partial^\mu + ig\,a^\mu$ SUBSTITUTION + TRANSFORMATIONS:

$\varphi \longrightarrow \varphi' = e^{ig\theta(x)}\varphi(x)$ & $a^\mu \longrightarrow a^{\mu\prime} = a^\mu - \partial_\mu\theta(x)$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \underbrace{\frac{1}{2}(D_\mu\varphi)^+D^\mu\varphi}_{} - \frac{a\mu^2}{2}\varphi^+\varphi - \frac{\lambda}{4}(\varphi^+\varphi)^2$$

encryption of a-φ interaction

$$\frac{1}{2}\partial_\mu\varphi^+\partial^\mu\varphi + \frac{1}{2}g^2a^\mu a_\mu\varphi^+\varphi$$

FORCE $a \rightarrow -|a|$ AND SHIFT FIELDS...

$\langle 0|\varphi_1|0\rangle = v \equiv \frac{a\mu^2}{\lambda}$ $\qquad \langle 0|\varphi_2|0\rangle = 0$

$\varphi = v + \sigma + i\eta$ AGAIN

Higgs mechanism 2:

- SUBSTITUTE

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \underline{\frac{1}{2}\partial_\mu\eta\partial^\mu\eta} - 1/4\, F_{\mu\nu}F^{\mu\nu}$$

$$\underline{-\frac{1}{2}\,2gv\,\partial_\mu\eta\, a^\mu} + g^2 v\sigma a^2 + \underline{\frac{1}{2}g^2v^2a^2} - a\mu^2\sigma + \text{cubic \& quart. interactions}$$

LOOK AT ▬▬ TERMS...

$$\frac{1}{2}\left(\partial_\mu\eta\,\partial^\mu\eta - 2gv\,\partial_\mu\eta\, a^\mu + g^2v^2a^2\right)$$

$$= \frac{1}{2}\left(gva_\mu - \partial_\mu\eta\right)^2 = \frac{1}{2}g^2v^2\left(a_\mu - \frac{1}{gv}\partial_\mu\eta\right)^2$$

(RE)DEFINE $\alpha_\mu \equiv a_\mu - \frac{1}{gv}\partial_\mu\eta$ (looks like gauge transformation. -- doesn't affect F's)

so, $\Phi_{\mu\nu} \equiv \partial_\mu\alpha_\nu - \partial_\nu\alpha_\mu$ -- or Φ's...

$$\mathcal{L} = \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}\Phi_{\mu\nu}\Phi^{\mu\nu} + \frac{1}{2}g^2v^2\alpha^2 - a\mu^2\sigma^2 + \text{int terms}$$

LOTS OF MAGIC HERE

- η HAS DISAPPEARED! THERE ARE NO MASSLESS BOSONS!
- σ HAS A MASS! $m_\sigma = \sqrt{2a\mu^2}$
- a_μ HAS DISAPPEARED... AND BEEN REPLACED BY α_μ WHICH HAS GAINED A MASS!! $m_\alpha = \frac{gv}{\sqrt{2}}$
- THE GRADIENT OF THE GOLDSTONE BOSON COMBINED WITH THE MASSLESS a_μ ... IN MOM. SPACE $\underbrace{k_\mu\eta}_{\text{behavior of longitudinal dof for SPIN 1.}}$

a_μ ATE THE η FIELD...

ACTUALLY... IT WAS "GAUGED AWAY".

THIS WAS DISCOVERED BY...

Anderson, Nambu, Englert, Brout, Gilbert, Guralnik, Higgs, Hagen, & Kibble

SO IT IS NATURALLY CALLED THE HIGGS MECHANISM.

σ IS THE HIGGS BOSON... A NECESSARY RELIC OF THIS APPROACH

- START OUT WITH:
  - 2 COMPONENT, DEGENERATE BOSON PAIR
  - MASSLESS SPIN 1 BOSON which insures local U(1) symmetry
- EMPLOY THE LANDAU MECHANISM...
- END UP WITH:
  - 1 MASSIVE SPIN 0 BOSON
  - 1 MASSIVE SPIN 1 BOSON

MAGIC? nope... that's superconductivity.

- THE CURRENT DENSITY IN SUPERCONDUCTIVITY IS

$$\vec{j} = \frac{-ie\hbar}{2m}\left(\psi^* \vec{\nabla}\psi - \psi\vec{\nabla}\psi^*\right) - \frac{2e^2}{mc}|\psi|^2\vec{A}$$

ψ – WAVEFUNCTION FOR COOPER PAIRS .. THE ORDER PARAMETER

$\vec{A}$ – ELECTROMAGNETIC FIELD, INTRODUCED BY DEMANDING LOCAL GAUGE INVARIANCE IN H.

$$\psi \to \psi' = \sqrt{\frac{n_s}{2}}\, e^{-2ie\phi(x)/\hbar c}$$

SO

$$\vec{j} = \frac{n_s e^2}{mc}\left(\vec{\nabla}\phi - \vec{A}\right)$$

$$\vec{\nabla}\cdot\vec{j} = 0 = \vec{\nabla}\cdot\vec{\nabla}\varphi - \underbrace{\vec{\nabla}\cdot\vec{A}}_{= 0 \text{ in Coulomb gauge}}$$

$\Rightarrow \vec{\nabla}\varphi$ CONSTANT... a number

LONDON EQUATION

$$\vec{j} = -\frac{n_s e^2}{mc}\vec{A}$$

MANIPULATE:

$$\vec{\nabla}\times\vec{j} = -\frac{e^2 n_s}{mc}\vec{B}$$

WITH Ampere's law $\quad \vec{\nabla}\times\vec{B} = \vec{j}$

$$\nabla^2\vec{B} = -\frac{e^2 n_s}{mc}\vec{B} \qquad *$$

WITH SOLUTION $\quad \vec{B} = \vec{B}_0\, e^{-x/\lambda} \qquad \lambda = \frac{mc}{e^2 n_s}$

A SHORT PENETRATION INTO SUPERCONDUCTOR OF MAGNETIC FIELD .. THE MEISNER EFFECT

ANOTHER INTERPRETATION...

superconductivity 2:

* LOOKS LIKE $\nabla^2 \vec{B} + \frac{1}{\lambda} \vec{B} = 0$

WHICH LOOKS LIKE THE KLEIN GORDON EQUATION FOR A PHOTON OF MASS $\left(1/\lambda \right)$ ...

EXPULSION OF $\vec{B}$ FROM GROUND STATE (actually arranged by collective "super currents" of Cooper pair electrons) MAKES IT APPEAR TO BE "HEAVY".

- THE COOPER PAIRS ARE ELECTRONS PAIRED WITH SPINS $\downarrow\uparrow$ ... $J=0$ $\Rightarrow$ LOOK LIKE BOSONS $\rightarrow$ THEY ARE THE HIGGS FIELDS

  - composite
  - macroscopic $\rightarrow$ yet quantum mechanical
  - screen out em fields $\rightarrow$ making γ massive

ANDERSON KNEW THIS... but nobody asked!

standard model !!

ALL TOOLS IN PLACE...

WEAK INTERACTIONS
NEED SPIN 1 $W^\pm$
PROPAGATOR

↓ YANG-MILLS, 1954

DEMAND LOCAL SU(2) GAUGE INVARIANCE —RESULT→ GET MASSLESS SPIN 1 TRIPLET

↓ GINSBURG/LANDAU, 1950

PLUS... ISODOUBLET
SCALAR FIELDS
LIKE BOSE-GAS

↓ HIGGS & friends, 1964

SPONTANEOUSLY BREAK GAUGE SYMMETRY —RESULT→ GET MASSIVE SPIN 1 TRIPLET & MASSIVE SPIN 0 SINGLET
NO PHOTON

↓ WEINBERG, 1967

SPONTANEOUSY BREAK PRODUCT GAUGE SYMMETRY.. $SU(2) \otimes U(1)$ —RESULT→ GET MASSIVE SPIN 1 TRIPLET & MASSLESS SPIN 1 SINGLET & MASSIVE SPIN ϕ SINGLET

Standard model 2:

BUILD A MODEL: OF LEPTONS.

WEINBERG PRL 19, 1264, 1967.

– JUST DETAILS FROM THIS POINT.

- $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\phi + \mathcal{L}_{\phi \ell}$

- $\mathcal{L}_0 = \bar{L}\, i\gamma^\mu \left(\partial_\mu + \frac{i}{2} g' a_\mu - i g \frac{\vec{\tau}\cdot\vec{b}_\mu}{2}\right) L$

$$+ \bar{R}\, i\gamma^\mu (\partial_\mu + i g' a_\mu) R$$

$$- \frac{1}{4} f^i_{\mu\nu} f^{\mu\nu\, i} - \frac{1}{4} \Phi_{\mu\nu} \Phi^{\mu\nu}$$

WHERE $L \equiv \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}$ $\quad R \equiv \ell_R$ & $\ell_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\ell$ $\quad \nu_L =$ ditto

$$f^i_{\mu\nu} \equiv \partial_\mu b^i_\nu - \partial_\nu b^i_\mu + g\, \varepsilon^{ijk} b^j_\mu b^k_\nu$$

$$\Phi_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$$

- $\mathcal{L}_\phi = (\Delta_\mu \phi^\dagger)(\Delta^\mu \phi) - V(\phi)$

WHERE $\Delta_\mu \phi = \left(\partial_\mu - i g \frac{\vec{\tau}\cdot\vec{b}_\mu}{2} - \frac{i g' a_\mu}{2}\right)\phi$

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- $\mathcal{L}_{\phi\ell} = -G_\ell (\bar{R}\phi^\dagger L + \bar{L}\phi R)$

A FANCY WAY TO IMPLEMENT SYMMETRY BREAKING IS TO TAKE ADVANTAGE OF A DIFFERENT GAUGE.. U-gauge

$$\langle 0|\varphi|0\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad v = \sqrt{\mu^2/\lambda}$$

$$\phi(x) = \exp(-i\, \vec{\xi}(x)\cdot\vec{\tau}/2v) \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$ like polar coordinates...

standard model 3:

$$\varphi \to \varphi' = \underbrace{\exp\left(-i\,\frac{\vec{\xi}\cdot\vec{\tau}}{2v}\right)}_{u(\vec{\xi})}\varphi = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix} = \frac{v+\eta(x)}{\sqrt{2}}\,\chi$$

where $\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

OTHER OBJECTS TRANSFORM...

$$L' = u\,L$$

$$\frac{\vec{\tau}\cdot\vec{b}'_\mu}{2} = u(\xi)\,\frac{\vec{\tau}\cdot\vec{b}_\mu}{2}\,u^{-1}(\xi) - \frac{i}{g}\left[\partial_\mu u(\xi)\right]u^{-1}(\xi)$$

$$a'_\mu = a_\mu$$

$$R' = R$$

$$f' = f \qquad \Phi' = \Phi$$

- ALL OF THE ACTION IS IN THE $\mathcal{L}_\phi$ TERM...

$$\mathcal{L}_\phi \to \mathcal{L}'_\phi = \left(\partial_\mu - ig\,\frac{\vec{\tau}\cdot\vec{b}'_\mu}{2} - ig'\,\frac{a_\mu}{2}\right)(\text{ditto}^\mu)\underbrace{\left(\frac{v+\eta(x)}{\sqrt{2}}\right)\chi}_{\text{leads to terms quadratic in spin 1 fields}}$$

just that piece:

$$= \frac{v^2}{8}\left\{ g^2\left[(b'^1_\mu)^2 + (b'^2_\mu)^2\right] + (\underbrace{g'a'_\mu - g\,b'^3_\mu}_{\text{2 "neutral" fields mixed up...}})^2 \right\}$$

DEFINE

$$W^\pm_\mu \equiv \sqrt{\tfrac{1}{2}}\left(b'^1_\mu \mp b'^2_\mu\right)$$

$$Z_\mu \equiv \frac{-g'a_\mu + g\,b'^3_\mu}{\sqrt{g^2+g'^2}}$$

$$A_\mu \equiv \frac{g\,a_\mu + g'\,b'^3_\mu}{\sqrt{g^2+g'^2}}$$

DIAGONALIZATION TO FORCE A_μ TO BE MASSLESS...

standard model 4:

this becomes $= M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \; (+ \; 0 \cdot A_\mu A^\mu)$

where $M_W^2 = (\tfrac{1}{2} g v)^2$

$$M_Z^2 = \frac{v^2}{4}\sqrt{g^2+g'^2}$$

for convenience $\tan\theta_W \equiv g'/g$

$$M_Z = \frac{1}{2}\frac{gv}{\cos\theta_W} = \frac{M_W}{\cos\theta_W}$$

θ_W = "WEINBERG ANGLE" = "WEAK ANGLE"

AFTER MUCH ALGEBRA, THE REST OF $\mathcal{L}_\phi$ IS..

$$\mathcal{L}_\phi = \underline{\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2 \eta^2} - \lambda v \eta^3 - \frac{\lambda}{4}\eta^4$$
$$+ M_W^2 W_\mu^- W^{\mu +} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu$$
$$+ \frac{g^2}{8}(\eta^2 + 2v\eta)\left[\frac{1}{\cos^2\theta_W} Z_\mu Z^\mu + 2 W_\mu^+ W^{\mu -}\right]$$

— INDICATES HIGGS FREE PORTION $\Rightarrow m_{HIGGS} = \sqrt{2}\,\mu$

- THE REGULAR WEAK INTERACTIONS LIVE IN THE $\mathcal{L}_0$

$$\mathcal{L}_0 = \ldots \bar{R}' i\gamma^\mu \partial_\mu R' + \bar{L}' i\gamma^\mu \partial_\mu L + \underbrace{g\, \vec{b}_\mu' \cdot \bar{L} \frac{\vec{\tau}}{2}\gamma^\mu L}_{\text{CONTAINS REGULAR W.I.}} + \underbrace{\frac{g'}{2} a_\mu \left[2\bar{R}\gamma^\mu R + \bar{L}\gamma^\mu L\right]}_{\text{CONTAINS REGULAR EM}}$$

standard model 5:

AFTER WRITING IN TERMS OF W's, Z's, and A ... THIS BREAKS DOWN INTO

$$\mathcal{L}_c = \frac{g}{\sqrt{2}}\left(j^+_\mu W^{+\mu} + \text{H.C.}\right)$$

where $j^+_\mu = \bar{\nu}_L \gamma_\mu l = \frac{1}{2}\bar{\nu}_l \gamma_\mu (1-\gamma_5) l$

IDENTIFY $\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \Rightarrow v \simeq 260 \text{ GeV}$

$$\mathcal{L}_N = g\, j_\mu^3\, b'^{3\mu} + \frac{g'}{2} a_\mu j_Y^\mu$$

$$\vdots$$

$$= g\sin\theta_W A_\mu J^\mu + \frac{g Z^\mu}{\cos\theta_W}\left[j_\mu^3 - \sin^2\theta_W J_\mu\right]$$

IDENTIFY $e = g\sin\theta_W$

WHERE $j_{\mu Y} \equiv 2\bar{R}\gamma_\mu R + \bar{L}\gamma_\mu L$ & $j_{\mu 3} = \bar{L}\frac{\tau_3}{2}\gamma_\mu L$

$J_\mu \equiv j_\mu^3 + \frac{1}{2} j_{\mu Y}$

• FROM $\mathcal{L}_{\phi l}$ COME...

$$\mathcal{L}_{\phi l} = -G_l\left[\frac{v}{\sqrt{2}}\left(\bar{l}'_R l'_L + \bar{l}'_L l_R\right) + \frac{y}{2}\left(\bar{l}_R l_L + \bar{l}' l_R\right)\right]$$

... the charged lepton mass comes from the primordial Yukawa coupling, G_l, & VEV. SPONTANEOUSLY GENERATED AS WELL.

IMMEDIATE CONCLUSIONS:

- BEAT THE SPIN 1 MASS PROBLEM .. Higgs Mech.
- GAIN NEW WEAK INTERACTION MEDIATED BY Z_μ, A NEW SPIN 1 FIELD.
- GAIN PREDICTION FOR M_Z, IN TERMS OF MIXING PARAMETER, θ_W & M_W.
- GAIN A NEW WAY OF LOOKING AT THE WORLD...!
  DEMAND OF SYMMETRY → DYNAMICS

IMMEDIATE IMPACT IN 1967:

ZERO

- WEINBERG'S PHYSICAL REVIEW LETTER WAS 3 PAGES LONG
  PRL 19, 1264, 1967. still as readable today as then...

NUMBER OF CITATIONS THROUGH 1969:

one (Salam)

3 SIGNIFICANT OCCASSIONS IN 1970's:

1. 1971 't Hooft shows model to be renormalizable
2. 1973 weak neutral currents discovered at CERN
3. 1973 Politzer & Gross & Wilczek "discover" asymptotic freedom... and learn to leave scalars out of SU(3) gauge invariant theory → QCD.

SO... WHERE ARE WE ?

WEINBERG'S WONDERFUL ANALOGY...

SUPPOSE YOU LIVED INSIDE A FERROMAGNET BELOW T_c ... WHAT EXPERIMENTS COULD YOU PERFORM WHICH WOULD TELL YOU THAT THE SYMMETRY OF THE HAMILTONIAN OF YOUR SO(2)-APPEARING UNIVERSE IS SO(3) ?

IF THE ANALOGIES THAT I'VE TALKED ABOUT ARE REASONABLE ... *then, that's our situation!*

HOWEVER, WE HAVE A CLUE ... θ_W & PREDICTIONS

→ THE ELECTROWEAK PROGRAM WORLDWIDE.