Sample One Hour Final Exam: PHY102

Due Friday April 18th, 6pm

Use Mathematica to solve all parts of the three problems

Vectors, Lists and Matrices

Problem 1. Consider the circuit shown in the figure. The equations for the currents I_1, I_2, I_3 in a circuit are given by,

$$V_1 - I_1 R_1 - I_2 R_2 = 0$$

$$V_1 - V_2 - I_1 R_1 - I_3 R_3 = 0$$

$$I_1 - I_2 - I_3 = 0$$

where V_1 and V_2 are applied voltages and R_1, R_2, R_3 are resistances. Write these three equations as a matrix equation and use Mathematica to find the currents I_1, I_2, I_3 as a function of the applied voltages and the resistances. Check that your code is correct by setting $V_2 = 0$, $R_1 = 0$, $R_2 = R_3$ and solving the equations by hand. What are the currents for the particular case $V_1 = 1$, $V_2 = 4$, $R_1 = 1, R_2 = 10$, $R_3 = 4$?

Ordinary differential equations and plotting

Problem 2. A mass($m = 1kg$), spring($k = 1N/m$) system hangs vertically at equilibrium in Earth's gravity. It is driven vertically by a periodic force $acos(\omega t)$ where $a = .25$ and it experiences a damping of $-b\vec{v}$, where $b = 0.1$. Consider a small amplitude initial displacement of 0.25 and initial velocity of zero. Solve the linear differential equation for this problem. $(x''(t) + bx'(t) + x(t) = asin(\omega t))$. Plot $x(t)$ for $\omega = 0.1, \omega = 1, \omega = 10$. This is a damped driven oscillator which has a surprisingly rich behavior and is used to model many dynamical systems.

Partial differential equations and Plotting

Problem 3. Confirm that the travelling soliton

$$\rho(x,t) = 3cSech^2[c^{1/2}(x - ct)/2]$$

solves the kdV equation given by,

$$\frac{\partial \rho(x,t)}{\partial t} + \frac{\partial^3 \rho(x,t)}{\partial x^3} + \rho(x,t)\frac{\partial \rho(x,t)}{\partial x} = 0,$$

provided that $c > 0$. For $c = 1$, $t = 0$ plot $\rho(x,t)$ as a function of x for the x-range $-10 < x < 10$. What is the velocity of the soliton? Solitons play an important role in studies of non-linear process in physics, engineering and biology. For example solitons in the kdV equation provide a simple model for a Tsunami.