Worksheet #2 - PHY102 (Spr. 2009)

Formats and List operations (Vectors)

Formating

From the toolbar do the following: "format" followed by "style", the dropdown menu offers you lots of options. "input" is the default, and is the "format" in which you do calculations. However *Mathematica* also allows various fonts and styles of text input. In this problem set you should include a text "cell" before each problem which identifies the problem (e.g. Problem 1), and a text cell at the top of the page with your name and the number of the worksheet (e.g. "worksheet 2").

Last week we did derivatives and integrals using the full *Mathematica* commands. However many of these commands may be entered from "palettes". To activate a palette, from the toolbar do the following: "file" followed by "palettes". You have several options: "basic input" is one I like.

When you get into trouble

Sometimes you will try to use a variable in more than one way. This can confuse Mathematica. There are several ways to clear a variable a, for example

$a = .$ which clears a numerical assignment to a, and

$Clear[a[x]]$ which clears a function assignment to $a[x]$.

If you want to remove all of your prior definitions you can use,

$Remove["Global` *"]$

At some point Mathematica will get really unhappy and start doing a really long winded calculation which you did not think you asked it to do. In that case you can go to the "kernel" tab, and click on "abort evaluation". Sometimes that does not work, in which case you can click on "quit kernel". This stops the mathematica kernel and you loose the evaluations you have already carried out. If neither of these works, scream and rant about stupid computer programs and computers which will probably get my attention.

Lists and Vectors

By now you must have read about vectors. A vector is a quantity which, unlike a scalar, can have many components. For example in Newton's second law of motion

$$\vec{F} = m\frac{d^2\vec{r}}{dt^2} \tag{1}$$

the quantity m (mass) is a scalar. But the force $\vec{F}$ and the acceleration $\vec{a} = \frac{d^2\vec{r}}{dt^2}$ are vectors. As you can see in Eq. (1), and which is true in general, multiplying a vector $\vec{a}$ with a scalar m, gives a vector $\vec{F}$. A vector is described by its components in a chosen co-ordinate system. For example a vector $\vec{A}$ in cartesian co-ordinates is given by, $\vec{A} = (A_x, A_y, A_z)$.

In Mathematica vectors are represented in the same way. In Mathematica this object is called a *list*, because it can be used for more general objects such as matrices and tensors. In this worksheet we just work with vectors.

Type "A = {Ax, Ay, Az}". This means Mathematica associates the object A with the list {Ax, Ay, Az}. Now type "B = {Bx, By, Bz}". Type "Dot[A,B]". This will give the dot product $\vec{A} \cdot \vec{B}$=AxBx+AyBy+AzBz which is the same as $|A||B|cos(\theta)$, where θ is the angle between the vectors $\vec{A}$ and $\vec{B}$.

Likewise, the cross product of two vectors $(\vec{A} \times \vec{B})$ yields another vector $\vec{C}$ = {AyBz-AzBy, AzBx-AxBz, AxBy-AyBx}. "Type Cross[A,B]" and verify that you indeed get the above expression in terms of the components of $\vec{A}$ and $\vec{B}$. Unit vectors can be easily written with lists as: $\hat{x}$ = {1,0,0}, $\hat{y}$ = {0,1,0}, $\hat{z}$ = {0,0,1}. Check with Mathematica that $\hat{x}\cdot\hat{y} = \hat{y}\cdot\hat{z} = \hat{z}\cdot\hat{x} = 0$.

You can see that the elements in the list {Ax, Ay, Az} of the vector $\vec{A} = \hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ are its x, y, and z components. How do we access the individual components from A?.

Type "A[[2]]" and check that this gives Ay. How would you get *Mathematica* to print out the second element of the cross product "Cross[A,B]"?

Assignment 2. - Hand in by Friday Jan. 30th

Problem 1. Consider two vectors $\vec{A} = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 0)$, and $\vec{B} = (\frac{1}{2}, \frac{\sqrt{3}}{2}, 0)$. Using *Mathematica*:
(i) Check that they are both of unit magnitude.
(ii) Find $\vec{A} \cdot \vec{B}$.

(iii) Find the angle between these two vectors.

(iv) Find the cross pruduct of these two vectors.

Problem 2. Consider the unit vectors along x, y, and z directions: $\hat{x} = \{1,0,0\}$ $\hat{y} = \{0,1,0\}$ $\hat{z} = \{0,0,1\}$. Verify: $\hat{x} \times \hat{y} = \hat{z}$, $\hat{y} \times \hat{z} = \hat{x}$, $\hat{z} \times \hat{x} = \hat{y}$.

Problem 3. Verify that for any three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ that $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$.

Problem 4. The motion of a particle is given by $\vec{r}(t) = a(\hat{x}\cos(\omega t) + \hat{y}\sin(\omega t))$ Find its velocity $\vec{v}$. Calculate $\vec{\Omega} \times \vec{r}$, where $\vec{\Omega} = (0,0,\omega)$, and verify that $\vec{v} = \vec{\Omega} \times \vec{r}$. Do you recognise this motion? Plot the motion to confirm your intuition (use the help menu to look up how to use the command "ParametricPlot" for this problem - you will need to choose values for a and ω).