PHY481 - Lecture 17: Magnets field lines, North and South. Lorentz Force Law
Griffiths: Chapter 5

Magnetic poles and magnetic field lines

There are many analogies between electrostatics and magnetostatics (time independent magnetic fields). Just as there are two types of electric charge, there are two types of magnetic "charge". They are called the North pole "N" and the south pole "S". Magnetic field lines come out of north poles and go into south poles. Two like poles repel and unlike poles attract. No elementary particles with an isolated magnetic charge or Dirac monopole have been discovered. Instead elementary particles often have a property called spin which is quantized (for example the electron has spin half). Associated with spin is a quantum of magnetic dipole moment, not a quantum of magnetic charge. Magnetic dipoles occur as a property of elementary particles, for example a free electron which has spin 1/2 has magnetic dipole moment μ_B which is the Bohr magneton. The magnetic field lines due to a magnetic dipole (e.g.a bar magnet or an electron spin) look the same as the electric field lines due to an electric dipole.

The earth's magnetic field is close to that of a bar magnet, with the poles buried deep inside the earth. The magnetic North pole is actually about 11.3 degrees from the geographic South pole. The magnitude of the earths field at the earths surface is around $30 - 60\mu T$.

In the past couple of years there have been exciting research developments in magnetostatics with the discovery of solid state magnetic materials where unbound magnetic monopoles have been found and magnetic currents have been generated. This means that in those systems $\vec{\nabla} \cdot \vec{B} \neq 0$!. If these materials can be developed further it will be possible to check if our naive expectations about magnetic charge and magnetic currents are true. Here is a preprint of a recent paper describing these discoveries.

DC motor

Most motors are AC motors, but some use DC current. The physics of these motors is $\vec{F} = i\vec{l} \wedge \vec{B}$, as they produce a force on a current carrying wire by placing the wire in a DC magnetic field. A key element of DC motors is that a commutator is needed to change the direction of the current as the motor axle rotates. This ensures that the axle keeps rotating in the same direction. An alternative approach is to turn the DC current into an AC current producing axle rotation at the frequency of the AC current. The latter is also based on $\vec{F} = i\vec{l} \wedge \vec{B}$ but involved time varying currents so we will discuss that in about two weeks.

The magnetic force $\vec{F} = q\vec{v} \wedge \vec{B}$ follows from $\vec{F} = i\vec{l} \wedge \vec{B}$

To show the relation between the magnetic force on a wire and the magnetic force on a moving charge, we need the result $j = nqv$. To see this result, consider a wire of cross-section a, length l, carrier density $n = N/V$, containing carriers that travel in the direction of the wire with average speed v. The current in the wire is the rate at which charge crosses any cross-section of the wire, δQ. In a given time interval δt, the amount of charge that crosses a cross-section is the amount of charge in a segment of length $l = v\delta t$. The amount of charge in this segment is $\delta Q = nqla$ so that,

$$i = \frac{\delta Q}{\delta t} = \frac{nqla}{\delta t} = \frac{nqv\delta ta}{\delta t} \tag{1}$$

The magnitude of the current density $j = i/a = nqv$. The direction of the velocity is the same as that of the current, so that this is also true in vector form, i.e. $\vec{j} = nq\vec{v}$.

Now we can use this result to show that,

$$il = jal = nqval = \frac{N}{V}alqv = Nqv \tag{2}$$

Here N is the number of carriers in the wire, V is the volume of the wire, a is the wire cross-section and l is its length. The force on each charge is then found by setting $N = 1$, so that the force on a charge moving in a magnetic field is the magnetic force $\vec{F} = q\vec{v} \wedge \vec{B}$.

General properties of motion in a magnetic field

A charge q moving in a magnetic field $\vec{B}$ with velocity $\vec{v}$ experiences a force,

$$\vec{F}_B = q\vec{v} \wedge \vec{B} = F_B = qvBsin(\theta) \tag{3}$$

where θ is the angle between the velocity vector and the magnetic field vector. The direction of $\vec{F}_B$ is given by the right hand rule. Note the following:

(i) The force on the charged particle is always *perpendicular* to both the velocity vector and to the magnetic field vector.

(ii) If the particle moves in the direction of the magnetic field, it experiences no magnetic force.
(iii) If the particle moves perpendicular to the magnetic field it experiences the maximum force.
(iv) Since the force is perpendicular to the magnetic field lines and to the velocity vector, the particle "spirals" around the magnetic field lines. The larger the magnetic field the larger the magnetic force and the tighter the spiral.
(v) No work is done by the magnetic field on the charged particle. The kinetic energy of the particle is therefore a constant, if no other forces act on the charged particle.

An important special case motion in a constant $\vec{B}$ field
First consider a particle of charge q with velocity $\vec{v}$ that is perpendicular to the direction of the magnetic field, then, $F_B = qvB$. The direction of this force is perpendicular to the velocity and it has constant magnitude, the conditions for circular motion. We therefore have,

$$m\frac{v^2}{R} = qvB \tag{4}$$

This is a very important type of motion and underlies a great deal of science and technology, ranging from accelerators to the quantum hall effect, to mass spectrometers etc. Note however that a more precise theory finds that power is dissipated by an accelerating charge, including charges in uniform circular motion. This is why high energy particle accelerators have very large circumference. The radiation from an accelerated charged particle is proportional to the acceleration squared. An important use of this effect is for producing high intensity x-rays for a variety of purposes. Now look at various quantities that are important in the circular motion of a charge in a magnetic field:
(i) Radius of the orbit - $R = mv/qB$.
(ii) Period of the orbit - $\tau = 2\pi R/v = 2\pi m/qB$.
(iii) Frequency (Cylotron frequency) of the orbit $f = qB/2\pi m$. The angular frequency is $\omega = qB/m$.
Notice that the period and frequency of the orbit do not depend on the radius. They only depend on the charge to mass ratio q/m and the applied magnetic field B. In addition, if we know the speed of a charged particle and the magnetic field, we can find the charge to mass ratio. This is used in mass spectrometers. Often the frequency of this orbit is called the cyclotron frequency.
The circular motion described above is the basis of understanding more complex motion. For example
(i) If a charged particle has a component of its velocity parallel to the magnetic field lines, then it spirals around the magnetic field lines in a helical motion.
(ii) If there is a non-uniform magnetic field, then the radius of the spiral is larger in regions of weak field and smaller in regions of high field.

The Lorentz force
If both electric and magnetic fields are present, the total force on a charged particle is given by summing the electrical and magnetic forces (as vectors) to find the famous Lorentz force,

$$\vec{F} = q(\vec{E} + \vec{v} \wedge \vec{B}). \tag{5}$$

There are a number of standard Lorentz force problems. We shall go through a couple of them.

A velocity selector
A charged particle in crossed electric and magnetic fields can still have constant velocity motion. This occurs if the electric and magnetic forces balance perfectly. This special case can be used as a velocity selector. That is, if we want to select particles with speed v from a set of particles with different speeds, we can do so by arranging crossed electric and magnetic fields in the correct manner. Here is how to do it: Choose $\vec{v}$, $\vec{B}$ and $\vec{E}$ all perpendicular to each other. The direction of the magnetic force is then either parallel or antiparallel with the electric force. If we choose the electric force to be antiparallel to the magnetic force, we can arrange for them to cancel each other. This is achieved if,

$$qE = qvB \tag{6}$$

Therefore particles with speed v will travel without deflection, provided,

$$v = E/B \tag{7}$$

Notice that this does not depend on the charge or the mass of the particles. However the selectivity does depend on the charge and the mass.

Cyclotrons

One of the most notable features of motion in a constant magnetic field is that the classical angular frequency $\omega_c = qB/m$ does not depend on radius of the orbit. Cyclotron's take advantage of this fact to accelerate charged particles using a constant frequency electric field. The radius of the orbit is given by, $R = v/\omega$, so as the particle's velocity is increased, the radius increases, however the frequency does not change. The cyclotron works by having an electric field across a gap to accelerate charged particles. The electric field has opposite signs on the two sides of the cyclotron "D"'s. Note that once the particles becomes relativistic the frequency is no longer constant as there is a relativistic correction, $\omega = qB/(\gamma m) = \omega_c(1-(v/c)^2)^{1/2}$.