PHY481 - Lecture 30: Magnetic materials
Griffiths: Chapter 6

Magnetostatics of materials

No applied current or external field

The magnetostatics treatment of magnetic materials is based on the concept of the magnetic moment density or magnetization $\vec{M}$. The magnetic field produced by a given magnetization is treated by finding the current sources that produce the magnetization $\vec{j}_b = \vec{\nabla} \wedge \vec{M}$, $\vec{K}_b = \vec{M} \wedge \hat{n}$. Once we have these bound currents we can find the magnetic fields inside and outside magnetized domains using the standard methods of magnetostatics. Three examples are:

(i) Magnetized cylinder of radius R and length l, with $l >> R$ (needle), with $\vec{M} = M_0\hat{z}$ and $\hat{z}$ the cylinder axis.

In that case, the $\vec{j}_b = 0$ and $\vec{K}_b = M_0\hat{\phi}$. We can then use Ampere's law to find the field: $Bl = \mu_0 K_b l$, so that $\vec{B} = \mu_0 K_b \hat{z} = \mu_0 \vec{M}$. Outside the cylinder and for distances $r >> l$, the magnetic field looks like that of a magnetic dipole, with dipole moment $\vec{m} = \vec{M}\pi R^2 l$.

(ii) Magnetized disc of radius R and thickness $l << R$, with $\vec{M} = M_0\hat{z}$ and $\hat{z}$ the cylinder axis.

Again $\vec{j}_b = 0$ and $\vec{K}_b = M_0\hat{\phi}$. The magnetic field is not like that of a current ring, with the current in the ring is $I = K_b l$. At long distances the magnetic field is like that of a dipole, with dipole moment again $\vec{m} = \vec{M}\pi R^2 l$, while the magnetic field at the center of the disc is $\mu_0 K_b l/2R$. If the disc is really thin ($l \to 0$), the magnetic field at the center of the disc goes to zero. This means that $\vec{B} \neq \mu_0 \vec{M}$.

(iii) Magnetized sphere of radius R and magnetization $\vec{M} = M_0\hat{z}$.

In this case we need to carry out a more detailed calculation (see last lecture). The magnetic field inside the sphere turns out to be a constant $\vec{B}_{in} = \frac{2}{3}\mu_0\vec{M}$. The magnetic field outside the sphere is like that of a dipole, with dipole moment $\vec{m} = \frac{4\pi}{3}R^3\vec{M}$.

With applied field or applied current

In this case bound currents still occur, but we also have to take into account the external current or magnetic field. This is most easily carried out for the case of an external current i_f, that generates a field as described by Ampere's law. Actually the field generated by the external (or free) current is usually called the magnetic intensity $\vec{H}$ and is found using $\oint \vec{H} \cdot d\vec{l} = i_f$, or $\vec{\nabla} \wedge \vec{H} = \vec{j}_f$. All of the calculations we did before using Ampere's law can be carried out to calculate $\vec{H}$. We also need to find the magnetic field, which requires that we treat both the bound currents and the free currents, using;

$$\oint \vec{B} \cdot d\vec{l} = \mu_0(i_f + i_b); \quad \text{or} \quad \vec{\nabla} \wedge \vec{B} = \mu_0(\vec{\nabla} \wedge \vec{H} + \vec{\nabla} \wedge \vec{M}) \quad \text{so that} \quad \vec{B} = \mu_0(\vec{H} + \vec{M}) \tag{1}$$

Note that the equation $\vec{B} = \mu_0(\vec{H} + \vec{M})$ applies even in cases where there is magnetization, but no applied current ($i_f = 0$), however we cannot simply set $\vec{H} = 0$. Instead we only have $\oint \vec{H} \cdot d\vec{l} = 0$ and this integral can be zero even though $\vec{H}$ is not zero everywhere. This can be checked by writing $\vec{H} = \vec{B}/\mu_0 + \vec{M}$ and checking that the resulting expression for $\vec{H}$ is zero for all contours.

Given the external current we find the magnetic intensity, $\vec{H}$ it produces but we don't know either the bound currents or the magnetization so we can't go further without a relation between the applied field and either of these quantities. The simplest and most often discussed case is linear isotropic response - this is the only case we treat in this course.

Linear magnetic materials

Linear magnetic materials are characterized by a linear relation between the magnetization and the magnetic field intensity, $\vec{M} = \chi_m\vec{H}$, which is similar to the definition of linear dielectrics, $\vec{P} = \epsilon_0\chi_e\vec{E}$ but not completely analogous. We then have,

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H} \tag{2}$$

where χ_m is the magnetic susceptibility and $\mu = \mu_0(1 + \chi_m)$ is the permeability. Sometimes the relative permeability $\mu_r = 1 + \chi_m$ is also used.

Paramagnets are linear materials with $\chi_m > 0$

Paramagnets do not exhibit spontaneous magnetic order, nevertheless they can have large magnetic susceptibilities. The magnetic moment of paramagnetic materials tries to align in the direction of the applied magnetic field. Actually many materials magnetically order at sufficiently low temperatures, but when the ordering temperature is very low, materials are called paramagnetic. The susceptibility of paramagnetic materials obeys the Curie Law,

$$\chi_m = \frac{\mu_0 C}{T} \tag{3}$$

Paramagnetic materials are attracted to magnets.

Diamagnets are linear materials with $-1 < \chi_m < 0$

If elementary particles did not have an intrinsic magnetic moment, then all materials would be diamagnetic. That is, the magnetic moment of materials would be opposite the direction of the applied field. Superconductors are the best diamagnets, and magnetic fields can be completely excluded from the interior of a superconductor. The phase is called the Meissner phase of a superconductor. From the expression,

$$\vec{B} = \mu_0(1 + \chi_m)\vec{H} \tag{4}$$

it is evident that in order for flux to be completely expelled so that $\vec{B} = 0$ inside the superconductor, we must have, $\chi_m = -1$. A measurement of χ_m is one of the first measurements that people do to determine if a material is in the superconducting state. Diamagnetic materials are repelled from magnets. This enables the possibility of magnetic levitation. Since superconductors are the best diamagnets, they are good candidates for possible magnetic levitation applications, such as maglev trains.

Magnetic field enhancement in a solenoid containing iron

For a solenoid with n turns per unit length and carrying current I, we found,

$$B_0 = \mu_0 n I \tag{5}$$

This is the result for a solenoid in air. Now if we place a material inside the solenoid, we use Ampere's law for the field intensity, and $\vec{B} = \mu\vec{H}$, to find,

$$H = ni; \quad \text{and} \quad B = \mu H = \mu_0(1 + \chi_m)ni \tag{6}$$

From this expression it is evident that the magnetic field inside the solenoid is greatly enhanced if the center (the core) of the solenoid is composed of a magnetic material which has large magnetic susceptibility χ_m, for example permalloy. Note that this seems different to dielectrics where the electric field is reduced when a dielectric is placed between the plates of an isolated capacitor. However, if a capacitor is connected to a battery, the electric field is unaltered by the addition of the dielectric, however the charge stored increases by a large amount. If we associate the charge stored on the capacitor with the flux in the solenoid, then the two devices appear more similar. This is the analogy that is often used.

The energy stored in the solenoid is still $Li^2/2$, but we need to calculated L again. L is defined through

$$Li = N\phi \quad \text{so that} \quad L = N^2\mu A/l \tag{7}$$

This is just the formula that we have for vacuum, but with $\mu_0 \rightarrow \mu$. The energy stored in the inductor thus increases dramatically when a large permeability material is used for the core of the inductor. As an example, consider an inductor containing permalloy with $\mu = 10000$ and with $N = 10,000$, $A = 0.1m^2$, $l = 1m$, carrying a current of $20A$, then $U = N^2\mu A i^2/l = 4 \times 10^{13} J$ (a gallon contains about $10^8 J$). The magnetic energy is a very large number and looks attractive for energy storage applications. However there are a number of critical limitations, ranging from hysteresis to resistive losses and the effects of large magnetic fields on materials and people.

The stored energy is $Li^2/2 = volume B^2/2\mu$, from which we deduce that the energy density is $\frac{1}{2}\vec{B} \cdot \vec{H}$.