PHY481 - Lecture 31: Some final remarks
Griffiths: Chapters 4,6

Boundary value problems for magnetostatics with materials
The equations are,

$$\oint \vec{H}\cdot d\vec{l} = i_f; \quad \oint \vec{B}\cdot d\vec{l} = \mu_0(i_f + i_b); \quad \oint \vec{B}\cdot d\vec{a} = 0 \tag{1}$$

so that the boundary conditions across a surface that may carry both free and bound currents are;

$$H^{\|}_{above} - H^{\|}_{below} = K_f; \quad B^{\|}_{above} - B^{\|}_{below} = \mu_0(K_f + K_b); \quad B^{\perp}_{above} - B^{\perp}_{below} = 0 \tag{2}$$

In addition if there are no free currents, $\vec{\nabla}\wedge\vec{H} = 0$, so that $\vec{H} = -\vec{\nabla}\phi_m$, with ϕ_m the scalar magnetic potential and is continuous across the surface.

A uniform sphere of radius R and permeability μ in a constant, uniform magnetic field Take the external field to be $\vec{B}_0 = \mu_0\vec{H}_0 = \mu_0 H_0\hat{z}$. Since there are no free currents we have $\vec{\nabla}\wedge\vec{H} = 0$, so that $\vec{H} = -\vec{\nabla}\phi_m$, where ϕ_m is the magnetic potential. Using $\vec{\nabla}\cdot\vec{H} = 0$ (which applies for a uniform magnetic sphere), we find that $\nabla^2\phi_m = 0$. Since the external field is uniform, we can write a solution that is of the same form as for the analogous electrical case,

$$\phi_m^{in} = -C_0 r cos\theta; \quad \text{and} \quad \phi_m^{out} = -H_0 r cos\theta + C_1\frac{R^3 cos\theta}{r^2} \tag{3}$$

To proceed further we need to know the boundary conditions on $\vec{H}$, $\vec{B}$ and on ϕ_m. We need ϕ_m to be continuous at $r = R$, so that, $-C_0 = C_1 - H_0$. From Ampere's law for $\vec{H}$ we find that at the sphere surface, $H^{\|}_{out} - H^{\|}_{in} = K_f$, where K_f is the free current density at the surface which in this case is zero. Gauss's law for magnetism indicates that $B^{\perp}_{out} = B^{\perp}_{in}$ at the sphere surface. Using this equation we have,

$$-\frac{1}{\mu_0}\frac{\partial\phi_m^{out}}{\partial r}|_R = -\frac{1}{\mu}\frac{\partial\phi_m^{in}}{\partial r}|_R \quad \text{so that} \tag{4}$$

which gives $\mu C_0 = \mu_0(H_0 + 2C_1)$. We then have,

$$C_0 = H_0 - C_1, \quad \text{and} \quad \mu C_0 = \mu_0(H_0 + 2C_1) \quad \text{so that} \quad C_1 = \frac{(1-\mu_r)H_0}{2\mu_r + 1}; C_0 = \frac{3H_0}{\mu_r + 2} \tag{5}$$

The magnetic intensity inside the sphere is $C_0\hat{z}$, so the magnetic field inside is $\mu C_0\hat{z}$ which gives,

$$\vec{B}_in = \mu\frac{3H_0}{\mu_r + 2}\hat{z} = \frac{3\mu_r}{\mu_r + 2}\vec{B}_0 = \frac{1 + \chi_m}{1 + \chi_m/3}\vec{B}_0 \tag{6}$$

Ferromagnets, non-linear magnetic materials, hysteresis
In ferromagnetic materials, the magnetic moments of the atoms in the material seek to align in the same direction. Examples are Fe and Permalloy (55% Fe, 45% Ni). It is actually quite difficult to find good ferromagnetic materials. There is a continuing search for ferromagnetic materials which have large local magnetic moments. A group at GM research in Detroit made a major breakthrough in this area about a decade ago. They helped develop the Niodymium, Iron, Boron magnets. The production of these magnets is now a multibillion dollar industry. Calculation of the fields around magnetics is carried out in a similar manner to the fixed magnetization case discussed above, e.g. for a uniformly magnetized sphere. A more general calculation uses a non-linear consitutive law. Sometimes ferromagnets are treated as a linear dielectric with a large positive value of χ_m - this is not completely correct, but it gives an indication of the expected behavior.

Ferromagnetic materials are very important in technology. For example the hard drives in most computers are made using small domains on ferromagnetic materials. A small sensor (or read head) scans the surface of the hard drive. On the hard drive surface are small domains of ferromagnetic material. These domains are oriented in the plane of the surface and they have a prefered direction. The read head measures a resistivity which is sensitive to the local magnetic field. The technology of magnetic storage (e.g. hard drives) relies on a particular property of ferromagnetic materials. This property is called hysteresis. Hysteresis is a property which occurs when a magnetic field is applied to a ferromagnet which is below its Curie temperature.

In order to describe hysteresis we must describe the way in which we vary the temperature and the magnetic field. Let us start at high temperatures and quench to a temperature well below the Curie temperature. The magnetic material is frozen in a domain structure by this process. Now we apply a positive external field. The domains now begin to align with the magnetic field. At sufficiently high magnetic field the atomic magnetic moments are all aligned with the applied field. This is called the saturation magnetization.

Now consider reducing the applied field until it is oriented in the opposite direction to the direction of the magnetic moments. However, the magnetic moments in a ferromagnetic material prefer to have the same orientation so they do not want to follow the direction of the magnetic field at first. The magnetic moment then remains oriented opposite the applied field until a sufficiently large opposite magnetic field is applied. At this point a sudden switching of the orientation of the magnetic moment occurs. This is the switching field H_c. In magnetic storage, when we write information onto the hard drive, we are switching the orientation of the magnetic domains. The read operation does not do this, instead it just senses the direction of the local field. This magnetic memory is non-volatile as it is not necessary to have a power source continually applied to the material in order to maintain the orientation of the spins.

Magnetic materials with very large reversal fields (H_c)are called magnetically hard materials, while those with small hysteresis loops are called soft magnetic materials.