per volt" γ are the physical parameters introduced and it is shown that these are related to coercivity E_c and mobility μ, respectively. The dynamics of polarization reversal can be derived from $f(E,t)$ regardless of the form of the driving field. The driving fields chosen as illustrations are of the pulse and sinusoidal form. The activation field and displacement velocity depend on temperature, crystal thickness, and vary with impurity content or crystal imperfection.

The mathematical formulation presented here has been successfully applied to the switching dynamics of polarization reversal in triglycine sulfate and triglycine fluoberyllate. It may also prove to be useful in the study of other materials presenting hysteresis properties.

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Gain, Band Width, and Noise Characteristics of the Variable-Parameter Amplifier*

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The variable parameter (or parametric) principle of amplification is characterized by a typical arrangement in which a variable energy storage element, such as an inductor or a capacitor, is suitably coupled to two resonant circuits. If the value of the energy storage element is made to vary in the proper way, energy is fed from the source which drives the element (that is, the pump) to the fields of both the resonant circuits. This paper describes the behavior relative to gain, band width, and noise of this type of amplifier.

Specifically, it is shown that to increase gain, the Q of one of the resonant circuits, the one commonly called the idling circuit, must be increased or the variation in the variable reactance must be increased. The band width is inversely proportional to this Q and to the voltage gain. Hence, for high gain, the amplifier is normally a narrow band device. One of the most important sources of noise is the thermal noise originating in the idling circuit. However, in principle this source can be reduced indefinitely by making the idling frequency approach the pumping frequency or by artificially cooling the idling circuit. In this fashion very low noise figures should be possible.

The parametric principle can also be applied to producing frequency conversion with large conversion gain. The appendix presents the expressions for gain, band width, and noise figure for this application. The behavior of the converter relative to gain, band width, and noise is quite similar to that of the amplifier.

INTRODUCTION

UNTIL recently microwave amplifiers have all been characterized by the use of electron beams in which dc kinetic or potential energy possessed by free electrons is converted into ac energy for amplifying signals. One limitation of these amplifiers lies in the amount of noise they add to the signals being amplified. Great improvement has been made in reducing this noise but further reduction appears to be increasingly difficult. The reason for this can be seen from the following considerations. Beams of free electrons must be emitted from hot cathodes, and the high temperature results in high initial beam noise. The operation of the amplifiers is such that noise reduction beyond a certain point by means of beam smoothing, noise cancellation, or other techniques becomes more and more difficult.

Recently several other principles have been proposed which are applicable to microwave amplification. One such principle rests on the fact that when a resonant circuit is suitably coupled to an energy storage element whose value is made to vary in the proper way, energy may be extracted from the source which drives the energy storage element and transferred to the fields of the resonant circuit. Such an energy transfer can be used for amplifying signals. Amplifiers based on this principle are called variable parameter or parametric amplifiers. Obviously they do not necessarily require the use of electron beams. In fact Landon[1] and Suhl[2] have independently suggested amplifiers of this type using ferrimagnetic material. Landon was thinking in terms of amplification at lower frequencies, but Suhl had in mind microwave frequencies. Both amplifiers employ the bound electrons which surround the atoms of material rather than the free electrons in a beam.

The principle of operation of these amplifiers is by no means new. Lord Rayleigh[3] in the last century showed that oscillations could be sustained in a single mechanical resonant system by the energy extracted from a source which suitably drives an energy storage element. Many of Rayleigh's concepts are pertinent to the particular variable parameter amplifier which will be treated in this paper, that is, one in which two resonant circuits are coupled together by a variable energy storage element. The fact that classical concepts

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[1] V. D. Landon, RCA Rev. **10**, 387 (1949).
[2] H. Suhl, J. Appl. Phys. **28**, 1225 (1957).
[3] Lord Rayleigh, Phil. Mag. S. **5** (1883).

are involved suggests the possibility of a number of different physical embodiments, and indeed this is so. In addition to the versions using ferrimagnetic materials, versions using semiconductor, ferroelectric materials, and electron beams have been suggested.[4–6]

Two facts regarding the variable parameter amplifier invite attention relative to the possibility of low-noise characteristics. First of all, the versions which employ ferrite or semiconductor material as the active element permit elimination of high temperature as the basic cause of the noise problem. The active element can be kept cool if need be. Second, even in the electron beam version where high temperature remains a factor, since the principle of operation is quite different from that of the conventional beam amplifier, noise cancellation or beam smoothing might well be more effective than in the conventional amplifier. Preliminary calculations have given some encouragement in this regard. Thus, the new amplifier opens up the important possibility of noise-free amplification to a degree previously unattainable.

Since the principle of operation of the parametric amplifier does not involve quantum mechanical phenomena in any essential way, it is not surprising that the amplification process can be explained in terms of classical circuit concepts. Although quantum effects may be involved in the details of how the classical circuit elements are related to the material properties in any particular physical embodiment, nevertheless the method of amplification is essentially classical.

This paper presents an illustration of the mechanism for energy transfer, a description of the basic circuit and how it operates, and a discussion, including expressions, of the gain, band width, and noise characteristics. Appendix I discusses gain, band width and noise for the system operating as a frequency converter.[7] Appendix II presents the details of the derivation of the noise figure for the amplifier, and Appendix III discusses the modifications necessary when the variable element has a nonlinear characteristic rather than a simple time variation.

MECHANISM OF ENERGY TRANSFER

To illustrate the mechanism by means of which energy can be transferred from a "pump," which drives an energy storage element, into the fields of a resonant tank, consider the simple resonant circuit of Fig. 1. Imagine that it is possible to pull the condenser plates apart and push them together again at will. Suppose that at the time the voltage across the condenser goes through a positive or a negative maximum the condenser plates are suddenly pulled apart. Work must be done in separating the charge on the two plates. The energy goes into the electric fields existing across the plates. The capacitance is reduced and, since $V=Q/C$, the voltage is amplified. Each time the voltage goes through zero the plates are suddenly pushed back together again. When the plates are pushed together there is no charge on the capacitor, so no work is done or required in this operation. The net result is amplification of the voltage across the capacitor, the flow of energy being from whatever pumps the plates into the fields of the resonant tank. This is illustrated in the voltage curve labelled (a). The crosses indicate a sudden pulling apart of the plates, the circles a sudden pushing together of the plates. Note that for this circuit the pumping is repeated periodically at twice the frequency of the signal. Note also that here a phase condition is necessary for amplifying the tank voltage. If the plates are pushed together when the voltage is high and pulled apart when it is zero the energy flow is in the opposite direction. The voltage is then attenuated as shown in curve (b).

For the two-tank circuit of Fig. 2 there is no such phase restiriction. Here the variable capacitance serves to couple together two different tank circuits of resonant frequencies Ω_1 and Ω_2, respectively. The variable capacitor is driven sinusoidally at a rate $\omega_3=\Omega_1+\Omega_2$.[8] If a voltage exists across one of the tanks at its resonant frequency, a second voltage is developed

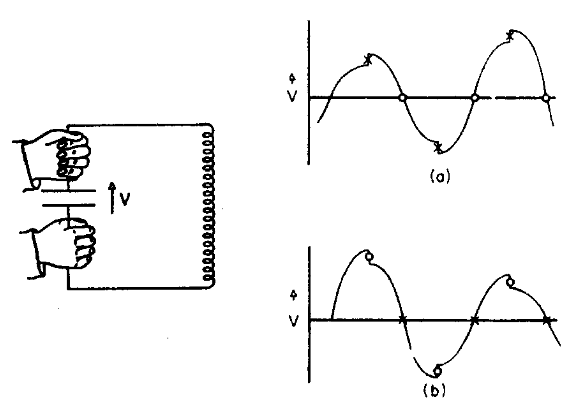


FIG. 1. Simple resonant circuit and pump to illustrate the mechanism of energy transfer for the variable parameter principle.

[4] M. E. Hines, "Amplification in non-linear reactance modulators," Fifteenth Annual Conference on Electron Tube Research, Berkeley, California (June, 1957).

[5] R. Adler, "A new principle of signal amplification," Fifteenth Annual Conference on Electron Tube Research, Berkeley, California (June, 1957).

[6] H. Heffner and G. Wade, "Noise, gain and bandwidth characteristics of the variable parameter amplifier," 1957 Electron Devices Meeting, Washington, D. C. (October, 1957).

[7] In connection with frequency conversion, reference should be made to the work of J. M. Manley and H. E. Rowe, Proc. Inst. Radio Engrs. 44, 904 (1956), which deals with the derivation of some general power relations which govern nonlinear reactor modulators.

[8] Such a capacitance variation with the amplitude going positive and negative is nonphysical but can be considered to be the variation about some average value $C_0>C_3$. If the Q's of the two resonant circuits are sufficiently high, the effect of the average coupling capacitance C_0 can be taken into account by lumping it in parallel with both of the resonant circuits leaving only the time varying portion as a coupling element. For further discussion of the nature of the variable capacitance see Appendix III.

across the second tank at its resonant frequency by the mixing action in the variable capacitance. The phase of the second voltage is automatically adjusted so that net energy flows into the tank circuits from the pumped capacitor.

Thus far, what has been described is obviously suitable for setting up and maintaining oscillations. The energy transfer mechanism is also suitable for amplification. For example, in Fig. 2 assume we couple a signal generator and an output load into one of the tanks, say tank 1. The signal generator, of course, should be tuned as closely as possible to Ω_1. The magnitude of the capacitor variation, that is C_3, should be reduced to a value just below the point where oscillations occur. An amplified version of the input signal will then appear at the output load.

EQUIVALENT CIRCUIT OF THE TWO-TANK AMPLIFIER

In the analysis which follows the two-tank circuit of Fig. 2 will serve as the model. The dual of this circuit, of course, is also a variable parameter circuit. However, only this circuit need be analyzed in detail, the behavior of the dual being obtained by interchanging symbols for capacitance and inductance, resistance and conductance, and voltage and current.

In Fig. 2, two parallel resonant circuits are coupled by a time varying capacitance C_c where

$$C_c=C_3\sin(\omega_3t+\phi_3) \tag{1}$$

and

$$\omega_3=\Omega_1+\Omega_2. \tag{2}$$

We shall assume that both resonant circuits have sufficiently high Q's that they act effectively as short circuits to signals at frequencies other than their respective resonant frequencies.

Let the voltage across tank 1 be

$$V(\omega_1)=V_1\sin(\omega_1t+\phi_1)=\mathrm{Re}\{-jV_1e^{j\phi_1}e^{j\omega_1t}\}, \tag{3}$$

where ω_1 is very nearly Ω_1, the resonant frequency of tank 1. Let the voltage across tank 2 be

$$V(\omega_2)=V_2\sin(\omega_2t+\phi_2)=\mathrm{Re}\{-jV_2e^{j\phi_2}e^{j\omega_2t}\}, \tag{4}$$

where

$$\omega_2=\omega_3-\omega_1. \tag{5}$$

Because of the high Q assumption the voltage appearing

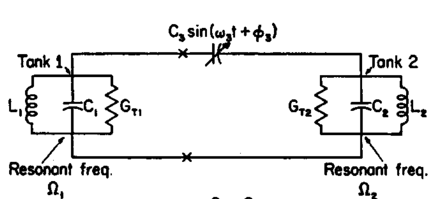
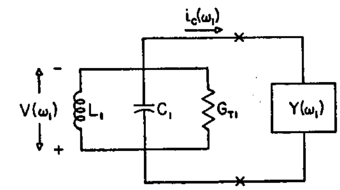


FIG. 2. An equivalent lumped circuit for the two-tank variable-parameter system.

FIG. 3. Equivalent circuit specifying tank 1 and $Y(\omega_1)$ and showing the voltage and current convention used in the analysis. The terminals marked with x's here correspond to those marked with x's in Fig. 2.



across the variable coupling capacitance is

$$V_c=V_1\sin(\omega_1+t\phi_1)+V_2\sin(\omega_2t+\phi_2). \tag{6}$$

The assumed voltage convention is that V_c is positive when in Fig. 2 the right-hand plate is at a higher potential than the left. The current flowing out of C_c to the right is then

$$i_c=-(d/dt)(C_cV_c). \tag{7}$$

This current contains components at $(\omega_3\pm\omega_1)$ and $(\omega_3\pm\omega_2)$. Because of the high Q assumption only the current component at $(\omega_3-\omega_1)=\omega_2$ produces a voltage across tank 2 and only the current component at $(\omega_3-\omega_2)=\omega_1$ produces a voltage across tank 1. These two components of current are

$$\begin{aligned}i_c(\omega_1)&=\frac{\omega_1C_3}{2}V_2\sin(\omega_1t+\phi_3-\phi_2)\\&=\mathrm{Re}\left\{-j\frac{\omega_1C_3}{2}V_2e^{j(\phi_3-\phi_2)}e^{j\omega_1t}\right\}\end{aligned} \tag{8}$$

and

$$\begin{aligned}i_c(\omega_2)&=\frac{\omega_2C_3}{2}V_1\sin(\omega_2t+\phi_3-\phi_1)\\&=\mathrm{Re}\left\{-j\frac{\omega_2C_3}{2}V_1e^{j(\phi_3-\phi_1)}e^{j\omega_2t}\right\}.\end{aligned} \tag{9}$$

Let us now ask what is the effective admittance, $Y(\omega_1)$, presented across tank 1 at ω_1 by the combination of the variable capacitance and tank 2. The equivalent circuit is shown in Fig. 3.

$$\begin{aligned}Y(\omega_1)&=\frac{\text{complex representation of } i_c(\omega_1)}{-\text{complex representation of } V(\omega_1)}\\&=\frac{-j(\omega_1C_3/2)V_2e^{j(\phi_3-\phi_2)}}{jV_1e^{j\phi_1}}\\&=-\frac{\omega_1C_3}{2}\frac{V_2}{V_1}e^{j(\phi_3-\phi_2-\phi_1)}.\end{aligned} \tag{10}$$

This result shows that the effective admittance presented to tank 1 depends on the relative amplitude of V_2, the voltage across tank 2 which Suhl has termed the idling circuit. Now let us examine V_2 and its relationship to V_1. The equivalent circuit is shown in

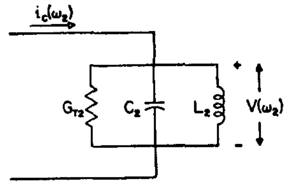


FIG. 4. Equivalent circuit specifying tank 2 and showing the voltage and current convention used in the analysis.

Fig. 4. If we call the admittance of this circuit Y_2, then

$$Y_2=G_{T2}+j\left(\omega_2C_2-\frac{1}{\omega_2L_2}\right)$$

$$=\frac{\text{complex representation of } i_c(\omega_2)}{\text{complex representation of } V(\omega_2)}. \quad (11)$$

Using (4) and (9), we have

$$V_2=\frac{(\omega_2C_3/2)V_1e^{j(\phi_3-\phi_2-\phi_1)}}{Y_2}. \quad (12)$$

If we take the complex conjugate of this expression and insert it into Eq. (10) to give the effective admittance presented to the driving circuit, we obtain

$$Y(\omega_1)=-\omega_1\omega_2C_3^2/4Y_2^*. \quad (13)$$

Note that Eqs. (1) through (13) are applicable regardless of what produces the voltages $V(\omega_1)$ and $V(\omega_2)$ across the tanks. Thus these equations are general enough to apply whether the system is used for oscillations or amplification. Let us consider briefly what happens for oscillation. Fig. 2 then illustrates the complete system with G_{T1} and G_{T2} designating the internal shunt conductances of the tanks. The frequencies of the voltages set up across the tanks are precisely the resonant frequencies of the tanks, so that $\omega_1=\Omega_1$ and $\omega_2=\Omega_2$. Then $Y(\omega_1)$ is a negative real number representing a purely negative conductance. It must be equal in magnitude to G_{T1} to sustain oscillations. This specifies the amplitude of the capacitance variation, C_3. Tank 1 then appears to be facing a conductance equal in magnitude and opposite in sign to its own internal shunt conductance. It is easily shown that the same is true for tank 2. Consequently, the pumping of the variable capacitor results in sustaining the oscillations in both tanks, the oscillations in each being at the corresponding resonant frequency.

Consider now the conditions for amplification. Assume that we couple a signal generator, tuned to $\omega_1\cong\Omega_1$, and an output load into tank 1, and that C_3 is reduced to a value below the point where oscillations would occur. The signal generator is then responsible for the existence of $V(\omega_1)$ across tank 1. The mixing action in the variable capacitor will then result in producing $V(\omega_2)$ across tank 2. Equations (1) through (13) still hold with G_{T1} representing the total shunt conductance across tank 1, including the conductances introduced by the generator and the load. Define now

Q_1 = the quality factor (including the loading) of tank 1, the amplifying tank,

Q_2 = the quality factor of tank 2, the idling tank,

$$\omega_1=\Omega_1+\Delta\omega, \quad (14)$$

and

$$\delta=\Delta\omega/\Omega_1.$$

From (2), (5), and (14) we have

$$\omega_2=\Omega_2-\Delta\omega. \quad (15)$$

Thus the admittance of the idling circuit becomes

$$Y_2=G_{T2}[1-j2\delta(\Omega_1/\Omega_2)Q_2],$$

and the admittance presented to the amplifying circuit is

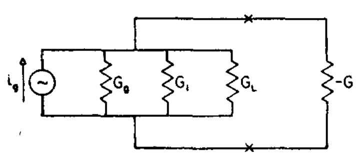


FIG. 6. Equivalent circuit for the variable parameter amplifier when the signal frequency is on resonance.

$$Y(\omega_1)=-\frac{\omega_1\omega_2C_3^2}{4G_{T2}[1+j2\delta(\Omega_1/\Omega_2)Q_2]}. \quad (16)$$

Let us now split up the total conductance of the amplifying circuit, G_{T1}, into the sum of a generator internal conductance, G_g, a load conductance, G_L, and a circuit loss conductance, G_1, so that

$$G_{T1}=G_g+G_L+G_1. \quad (17)$$

Then the equivalent circuit for the variable-parameter amplifier at the signal frequency ω_1 is shown in Fig. 5. The equivalent circuit is considerably simpler if the signal frequency ω_1 equals precisely Ω_1, the resonant frequency. As previously stated, the admittance then presented to the driving circuit is a pure negative conductance,

$$Y(\omega_1)=-G=-(\omega_1\omega_2C_3^2/4G_{T2}), \quad (18)$$

and the equivalent circuit is shown in Fig. 6.

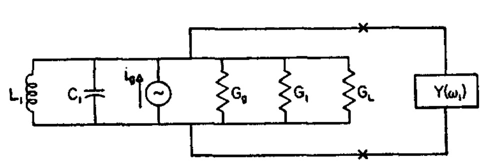


FIG. 5. Equivalent circuit for the variable-parameter amplifier specifying tank 1 and its loading.

GAIN OF THE TWO-TANK AMPLIFIER

With the aid of the equivalent circuit of Fig. 6, one can immediately write the gain of the two-tank amplifier circuit for the signal at resonance. We shall define the gain to be the ratio of the power dissipated in the load conductance to the available power from the generator. With this definition one finds

$$\text{power gain}=\frac{[i_g^2/(G_{T1}-G)^2]G_L}{[i_g^2/4G_g]}=\frac{4G_gG_L}{(G_{T1}-G)^2}. \quad (19)$$

The gain expression has the familiar form exhibited by all regenerative amplifiers and indicates that for large gains

$$G\cong G_{T1} \quad (20)$$

for which the capacitance variation requirement is

$$C_3\cong 2(G_{T1}G_{T2}/\omega_1\omega_2)^{\frac{1}{2}}. \quad (21)$$

The general gain expression for signals which may be off resonance is somewhat more complicated. Using the equivalent circuit of Fig. 5 one finds that the general gain expression is

$$\text{power gain}=\frac{4G_gG_L}{\left\{G_{T1}-\dfrac{G}{[1+[2\delta Q_2(\Omega_1/\Omega_2)]^2]}\right\}^2+4\delta^2\left\{G_{T1}Q_1+\dfrac{G(\Omega_1/\Omega_2)Q_2}{[1+[2\delta Q_2(\Omega_1/\Omega_2)]^2]}\right\}^2}. \quad (22)$$

BAND WIDTH OF THE TWO-TANK AMPLIFIER

As the signal frequency deviates from resonance, three effects tend to reduce the gain. First, as in any resonant circuit, the voltage across the circuit for a given excitation is reduced. Second, the effective negative conductance coupled into the amplifying tank is reduced. Third, and what turns out to be most important, an effective susceptance is coupled into the amplifying tank. To determine the band width we must investigate the general gain expression of Eq. (22) and find the values of δ, the fractional frequency deviation from resonance, which reduce the power gain to some fraction of its resonance value, say to one-half its value. Equating Eq. (22) to one-half the resonance gain of Eq. (19) gives an equation for the three db fractional band width of 2δ.

$$\left\{G_{T1}-\frac{G}{[1+[2\delta Q_2(\Omega_1/\Omega_2)]^2]}\right\}^2 +4\delta^2Q_1^2\left\{G_{T1}+\frac{G(\Omega_1Q_2/\Omega_2Q_1)}{[1+[2\delta Q_2(\Omega_1/\Omega_2)]^2]}\right\}^2 =2(G_{T1}-G)^2. \quad (23)$$

If the gain is large the second term representing the off-resonant susceptance increases more rapidly than the first so that the half-power fractional band width is

$$2\delta=\frac{G_{T1}-G}{Q_1[G_{T1}+G(\Omega_1Q_2/\Omega_2Q_1)]}. \quad (24)$$

Since $(G_{T1}-G)$ can be related to the square root of the power gain, we can write a gain band width product as

$$(\text{power gain})^{\frac{1}{2}}\times(\text{fractional band width}) =\frac{2\Omega_2(G_gG_L)^{\frac{1}{2}}}{\Omega_2Q_1G_{T1}+\Omega_1Q_2G}. \quad (25)$$

Usually the Q of the idling tank must be made to be considerably higher than that of the loaded amplifying tank and

$$\Omega_1Q_2G\gg\Omega_2Q_1G_{T1}. \quad (26)$$

Under these conditions,

$$(\text{power gain})^{\frac{1}{2}}\times(\text{fractional band width}) =\frac{1}{Q_2}\frac{\Omega_2}{\Omega_1}2\left(\frac{G_g}{G}\frac{G_L}{G}\right)^{\frac{1}{2}}. \quad (27)$$

For high gains where $G\cong G_{T1}$, the quantity $2(G_gG_L/GG)^{\frac{1}{2}}$ is at most unity so that this gain band width product is less than $\Omega_2/Q_2\Omega_1$. This implies that a two-tank variable-parameter amplifier with a power gain of 20 db and having a Q_2, the idling tank Q, of 1000 will have a band width of the order of 0.01%. Note that the gain band width product can be increased by increasing the ratio of idling frequency, ω_2, to amplifying frequency, ω_1.

NOISE FIGURE OF THE TWO-TANK AMPLIFIER

Consider now the noise figure under the assumption that the signal being amplified is precisely at the resonant frequency of tank 1. Figure 6 shows the equivalent circuit under this assumption. The noise figure can be written as follows:

$$F=\left(\frac{S_i}{N_i}\right)\left(\frac{N_0}{S_0}\right)=\frac{1}{\text{power gain}}\frac{1}{KTB}N_0 =\frac{1}{4KTB}\frac{(G_{T1}-G)^2}{G_gG_L}N_0, \quad (28)$$

where

S_i/N_i = the available signal-to-noise ratio at the input,
N_0/S_0 = the noise-to-signal ratio at the output,
K = Boltzmann's constant,
T = standard noise temperature (290°K),
B = the noise band width of the amplifier.

The significant noise sources which could possibly contribute to N_0 are[9]

1. Thermal noise at ω_1 in tank 1.
2. Thermal noise at ω_2 in tank 2.
3. Noise current at ω_1 emanating from C_3.
4. Noise current at ω_2 emanating from C_3.
5. Noise fluctuations at ω_3 in the value of the variable capacitor C_3.
6. Noise fluctuations at $2\omega_1$ in the value of the variable capacitor C_3.
7. Noise fluctuations at $2\omega_2$ in the value of the variable capacitor C_3.
8. Noise fluctuations at $(\omega_1-\omega_2)$ in the value of the variable capacitor C_3.

Since the total noise at the output, N_0, will be the summation of the noise due to each of the sources, we may write

$$F=\frac{1}{4KTB}\frac{(G_{T1}-G)^2}{G_gG_L}\sum_{n=1}^{8}N_{0n}=\sum_{n=1}^{8}F_n. \tag{29}$$

The determination of each N_{0n} will complete the derivation of the noise figure expression. The details of the steps involved in determining the N_{0n}'s are presented in Appendix II.

Using the results from Appendix II we have for the noise figure equation

$$F=1+\frac{G_1}{G_g}+\frac{G}{G_g}\frac{\omega_1}{\omega_2}+\frac{1}{4KTBG_g}\left[\langle i_3^2\rangle+\langle i_4^2\rangle\frac{G}{G_{T2}}\frac{\omega_1}{\omega_2}\right]+\frac{S_i}{N_i}\frac{\text{gain}}{4}\frac{G}{G_g}\left[\frac{G}{G_L}\langle\rho^2\rangle+\frac{G_{T2}}{G_L}\frac{\omega_1}{\omega_2}\langle\sigma^2\rangle+\frac{G}{G_{T2}}\frac{G}{G_L}\frac{\omega_1}{\omega_2}\langle\alpha^2\rangle\right], \tag{30}$$

where

$\langle i_3^2\rangle$ = the mean-squared value of the noise current at ω_1 emanating from the variable capacitance,
$\langle i_4^2\rangle$ = the mean-squared value of the noise current at ω_2 enamating from the variable capacitance,
$\langle\rho^2\rangle$ = the mean-squared value of the ratio of the noise variation at ω_3 to the coherent variation at ω_3 in the variable capacitance,
$\langle\sigma^2\rangle$ = the mean-squared value of the ratio of the noise variation at $2\omega_1$ to the coherent variation at ω_3 in the variable capacitance, and
$\langle\alpha^2\rangle$ = the mean-squared value of the ratio of the noise variation at $2\omega_2$ to the coherent variation at ω_3 in the variable capacitance.

[9] The particular physical embodiment used in applying this principle of amplification will affect the precise value of the contribution of a number of these sources. However, as the analysis shows, an expression for the noise figure can be derived including general terms representing these sources. Note that in keeping with the definition of noise figure the list of sources does not include thermal noise originating in the output load, G_L, although this source may contribute significiently to the noise appearing across G_L.

Calculations show that the second term in the noise figure expression, i.e., the term due to thermal noise from G_1, is normally very small. However, the third term, that due to thermal noise from the idling tank can be significant. For high gain, it is nearly equal to the ratio of ω_1 to ω_2. Hence by choosing an amplifying frequency much less than the idling frequency this term can be made to be small. The noise figure was derived assuming the two tanks to be at the standard noise temperature. If the tanks were to be artifically cooled, both of these terms would decrease by a factor equal to the ratio of tank temperature to standard noise temperature. The value of the remaining terms depends upon the particular physical embodiment used. In the case cf electron-beam versions and semiconductor-diode versions, the noise currents i_3 and i_4 represent shot noise introduced at the amplifying and the idling frequencies, respectively. In electron-beam versions these terms can be significant. For back-biased semiconductor diodes these terms are negligible in comparison with the contributions due to thermal noise in the spreading resistance. If this resistance is properly taken into account in determining the tank loading conductances G_1 and G_{T2}, then the first three terms of Eq. (30) adequately represent the noise figure of the diode amplifier. In the case of the ferrite version there appears to be no noise contribution due to the equivalent cf i_3 and i_4. The remaining three terms are probably unimportant in any physical embodiment.

It is interesting to compare the noise figure expression for the parametric amplifier given here with that for the three-level maser.[10] The usual expression for the maser noise figure[11] includes only the first, second, and fourth terms of Eq. (30) representing amplified thermal noise originating at the signal frequency ω_1 (the first two terms) and the shot noise contributions (the fourth term) which in the maser come from spontaneous emission processes. In the maser this later contribution is usually expressed in terms of an effective temperature, typically a few degrees Kelvin.

Terms three and five in Eq. (30) representing respectively thermal noise at the idling frequency ω_2 and noise fluctuations in the value of the variable element have no counterparts in the maser noise figure. Upon reflection the reasons for their absence become clear. Thermal noise at the idling frequency does not appear because in the maser no circuit resonant to this frequency surrounds the active material. Thus noise fields which might induce transitions in tha maser at the frequency equivalent to the idling frequency are negligibly small. The last term of Eq. (30) representing noise fluctuations in the value of the variable element is analogous to fluctuations in the population difference of the three-level maser. This term is negligibly small in maser operation since the crystal is saturated. That is, above a certain amplitude, fluctuations of the

[10] N. Bloembergen, Phys. Rev. **104**, 324 (1956).
[11] R. V. Pound, Ann. Phys. **1**, 24 (1957).

pumping field have negligible effect in altering the virtual equality of populations in uppermost and lowermost energy levels.

Since the parametric amplifier shows noise contributions which do not appear in the three-level maser one sees that for the same effective temperature, i.e., the same contribution from term 3, the parametric amplifier will have a higher noise figure.

CONCLUSION

In conclusion, let us review briefly some of the inherent characteristics of the variable parameter amplifier as determined from the analysis. To increase the gain, the Q of the idling tank must be increased or the variation in the variable reactance must be increased. The band width is inversely proportional to this Q and to the voltage gain, and is directly proportional to the ratio of idling frequency to amplifying frequency. The device is essentially a narrow-band device. One of the most important sources of noise is the thermal noise from the idling tank. However, very low noise figures should be possible by choosing a large ratio of idling frequency to amplifying frequency or by artifically cooling the tanks. Thus a large ratio of idling to amplifying frequency will improve both the noise figure and gain band width.

APPENDIX I

Two-Tank Circuit as a Frequency Converter

The two tank parametric system can be used as a frequency converter with large conversion gain. This can be accomplished by a simple modification of

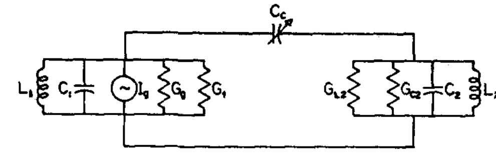


FIG. 7. Equivalent circuit illustrating the use of the parametric system as a frequency converter.

the system as an amplifier, that is, the output is taken from tank 2 rather than tank 1. Thus one can consider that the load conductance is removed from tank 1 and placed across tank 2. The converted signal, as is usual in mixer circuits, exhibits a modulation inversion since a signal $\omega_1+\Delta\omega$ applied to the first tank gives rise to a signal $\omega_2-\Delta\omega$ in the second tank. We can determine the conversion gain and band width rather simply from the results already developed for the amplifier. Let us split the conductance in the second tank, G_{T2}, into the two following portions: G_{C2} due to the circuit losses and G_{L2} due to the load (see Fig. 7).

Defining conversion gain in terms of the ratio of output power near ω_2 in G_{L2} to the available power from the generator at ω_1, we have

$$\text{conversion gain}=\frac{V_2V_2{}^*G_{L2}}{(I_gI_g{}^*/4G_g)}$$

$$=\frac{V_2V_2{}^*}{V_1V_1{}^*}\frac{4G_gG_{L2}}{[G_{T1}+j\omega C+(1/j\omega L)+Y(\omega_1)]^2}, \tag{31}$$

where the relationship between I_g and V_1 is apparent from Fig. 5. By using Eqs. (12), (14), (15), (16), and (18) we obtain

$$\text{conversion gain}=\frac{\omega_2G/\omega_1G_{T2}}{[1+[2\delta(\Omega_1/\Omega_2)Q_2]^2]}\times\frac{4G_gG_{L2}}{\left[G_{T1}-\dfrac{G}{1+[2\delta(\Omega_1/\Omega_2)Q_2]^2}\right]^2+4\delta^2\left[G_{T1}Q_1+\dfrac{G(\Omega_1/\Omega_2)Q_2}{1+[2\delta(\Omega_1/\Omega_2)Q_2]^2}\right]^2}. \tag{32}$$

At resonance this reduces to

$$\text{resonance conversion gain}=\frac{4(\omega_2G_{L2}/\omega_1G_{T2})GG_g}{(G_{T1}-G)^2}. \tag{33}$$

The factor to the right of the multiplication symbol in Eq. (32) is reminiscent of the gain expression of the variable parameter system as an amplifier [see Eq. (22)]. There is one difference, however; in (32) G_{T1} does not include the load conductance while in (22) it does. In spite of this difference, we can draw on the amplifier analysis to arrive at proper conclusions regarding band width. The band width representing the factor to the left of the multiplication symbol is just that of the second resonant tank and, under the conditions for which $G\rightarrow G_{T1}$, is considerably larger than that for the system as a whole. This is apparent from an examination of Eq. (24). Equation (24) gives the band width for the factor to the right in (32) taken alone, and becomes arbitrarily small as G approaches arbitrarily close to G_{T1}. For reasonable gains G must approach close to G_{T1} and the over-all band width of the converter may be regarded as that of the factor to the right taken alone. Thus Eq. (24) correctly gives the converter band width for high gain,

$$2\delta=\frac{G_{T1}-G}{Q_1[G_{T1}+G(\Omega_1Q_2/\Omega_2Q_1)]}, \tag{24}$$

where $G_{T1}=G_g+G_1$, $Q_1=$the quality factor of tank 1 (loaded only by the generator) and, $Q_2=$the quality factor (including the output loading) of tank 2. Note that because of the difference in the loading, the symbols have a slightly different meaning in the case of the converter than in the case of the amplifier.

The noise figure of the converter is the ratio of the available signal-to-noise power ratio at the input at frequency ω_1, (S_i/N_i), to the actual signal-to-noise power ratio at the converter output at frequency ω_2, (S_{0c}/N_{0c});

$$F_{\text{conv}}=(S_i/N_i)/(S_{0c}/N_{0c}).$$

The amplifier noise figure already calculated involves the output signal-to-noise power ratio, S_0/N_0, at ω_1 rather than at ω_2;

$$F_{\text{amp}}=(S_i/N_i)/(S_0/N_0).$$

Thus the converter noise figure can be expressed as

$$F_{\text{conv}}=F_{\text{amp}}(S_0/S_{0c})(N_{0c}/N_0).$$

Since both noise and signal powers in the two circuits transform in the same way, we see that as far as any particular noise source is concerned the corresponding noise figure term is the same in the converter as in the amplifier.

Care must be taken in writing the converter noise figure expression directly from the amplifier expression since the definition of noise figure does not include noise generated in the load. Thus the third term of Eq. (30) must be modified slightly to exclude that portion of the total conductance of tank 2 which represents the load conductance. Under these conditions the noise figure of the converter shown in Fig. 7 is

$$F_{\text{conv}}=1+\frac{G_1}{G_g}+\left(\frac{G_{C2}}{G_{T2}}\right)\frac{G}{G_g}\frac{\omega_1}{\omega_2}+\frac{1}{4KTBG_g}\times\left[\langle i_3^2\rangle+\langle i_4^2\rangle\frac{G}{G_{T2}}\frac{w_1}{w_2}\right]$$
$$+\frac{S_i}{N_i}\frac{\text{conv gain}}{4}\frac{\omega_1}{\omega_2}\frac{G_{T2}}{G_g}\times\left[\frac{G}{G_{L2}}\langle\rho^2\rangle+\frac{G_{T2}}{G_{L2}}\frac{\omega_1}{\omega_2}\langle\sigma^2\rangle+\frac{G}{G_{T2}}\frac{G}{G_{L2}}\frac{\omega_2}{\omega_1}\langle\alpha^2\rangle\right].$$

Here the last term of the equation has been modified to express the result in terms of conversion gain rather than the amplifier gain which appears in Eq. (30).

APPENDIX II

Derivation of the Terms in the Noise Figure Expression for the Amplifier

This appendix presents the details in deriving the effect of the eight noise sources listed in the noise figure section. Consider noise source 1. Figure 8 shows

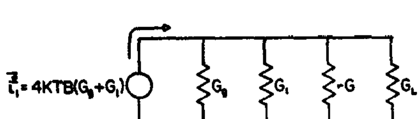


FIG. 8. Circuit diagram showing system for noise source 1.

schematically the equivalent circuit. Thermal noise from the generator and tank conductances both assumed to be at a temperature T are included here. (The noise emanating from the load conductance is not held against the amplifier.)

The thermal noise power absorbed by the load is

$$N_{01}=\frac{\langle i_1^2\rangle}{(G_{T1}-G)^2}G_L=\frac{4KTB(G_g+G_1)}{(G_{T1}-G)^2}G_L. \quad (34)$$

Therefore, from (29) we have

$$F_1=1+(G_1/G_g). \quad (35)$$

Consider noise source 2, that is noise due to thermal noise at ω_2 arising from G_{T2}. This current produces a voltage at ω_2 across tank 2 which in turn results in a noise voltage at ω_1 across tank 1. Figure 9 illustrates

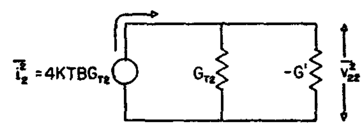


FIG. 9. The circuit to illustrate how thermal noise from G_{T2} produces a noise voltage across tank 2.

the equivalent circuit as far as tank 2 is concerned. The conductance $-G'$ represents the effect of the remainder of the circuit with respect to tank 2 and is analogous to the effective conductance $-G$ which pertains to tank 1. By analogy with Eq. (18) we may write

$$G'=\omega_1\omega_2C_3^2/4G_{T1}=GG_{T2}/G_{T1}. \quad (36)$$

Thus the noise voltage across tank 2 due to noise source 2 is

$$v_{22}=\frac{i_2}{(G_{T2}-G')}\frac{G_{T1}}{G_{T2}}. \quad (37)$$

By analogy with Eq. (12) we can solve for the corresponding voltage at ω_1 across tank 1 due to this noise source 2

$$v_{12}=\frac{\omega_1C_3}{2G_{T1}}v_{22}=\frac{\omega_1C_3i_2}{2G_{T2}(G_{T1}-G)}. \quad (38)$$

Thus we have

$$N_{02}=\langle v_{12}^2G_L\rangle=\frac{\omega_1^2C_3^2\langle i_2^2\rangle}{4G_{T2}^2(G_{T1}-G)^2}G_L \quad (39)$$

and

$$F_2=\frac{\langle i_2^2\rangle}{4KTBG_g}\frac{G}{G_{T2}}\frac{\omega_1}{\omega_2}. \quad (40)$$

Substituting the expression for $\langle i_2^2\rangle$ into (40) gives

$$F_2=\frac{G}{G_g}\frac{\omega_1}{\omega_2}. \quad (41)$$

For noise source 3, that is noise currents at ω_1 emanating from the variable capacitance C_c, we may refer to the circuit diagram of Fig. 8. The value of the noise current, $\langle i_3^2\rangle$, cannot be stated until the physical embodiment is designated. By inspection of (34) we may write

$$N_{03}=\frac{\langle i_3^2\rangle}{(G_{T1}-G)^2}G_L \tag{42}$$

and

$$F_3=\langle i_3^2\rangle/4KTBGg. \tag{43}$$

Figure 9 can be used to illustrate the equivalent circuit for noise source 4, that is noise currents at ω_2 emanating from the variable capacitance C_c. The derivation is similar to that for noise source 2, and we can write from inspection of (40) that

$$F_4=\frac{\langle i_4^2\rangle}{4KTBG_g}\frac{G}{G_{T2}}\frac{\omega_1}{\omega_2}. \tag{44}$$

Consider noise source 5; in addition to the desirable coherent pumping of C_c at ω_3 there are undesirable noise fluctuations in the value of C_c at ω_3. To include this in an equation for the total variable capacitance we might write

$$C_c+\rho C_c=C_3\sin(\omega_3t+\phi_3)+\rho C_3\sin(\omega_3t+\phi_3'), \tag{45}$$

where ρ and ϕ_3' are random functions of time. Equation (45) indicates a noise-pumped capacitance in parallel with the coherently pumped capacitance of Fig. 2. This situation can be illustrated as in Fig. 10.

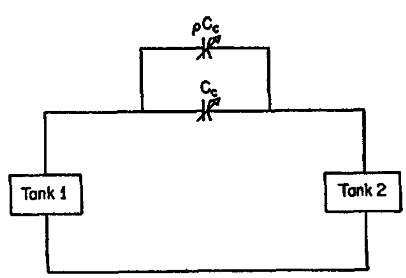


FIG. 10. Diagram to illustrate the effect of noise fluctuations in C_c at an ω_3 rate.

There are two ways by which the incoherent capacitance variation at ω_3 can give rise to noise voltages at the amplifying frequency ω_1. One of these involves a direct mixing action between the coherent voltage V_2 in tank 2 to give a noise voltage at $\omega_3-\omega_2=\omega_1$. The other is a more roundabout process whereby a coherent voltage V_1 at ω_1 mixes with the incoherent capacitance variation at ω_3 to give rise to a noise voltage at frequency $\omega_3-\omega_1=\omega_2$ in tank 2. This noise voltage in turn mixes with the coherent capacitance variation at ω_3 to give a second noise voltage at $\omega_3-\omega_2=\omega_1$ in tank 1. Since both noise voltages in tank 1 arise from the same incoherent source, they are correlated and must be added together directly to give the total noise voltage at ω_1. By inspection of Eq. (8) we may write an expression for the noise current emanating from the capacitor ρC_c at ω_1 as the result of interaction with the coherent voltage V_2 across tank 2;

$$i_{15}=\frac{\omega_1\rho C_3}{2}V_2\sin(\omega_1t+\phi_3'-\phi_2). \tag{46}$$

This current will produce a voltage across tank 1 at ω_1 which can be computed from inspection of Fig. 8. Thus one of the components of the noise voltage across tank 1 is

$$v_{15}'=\frac{i_{15}}{(G_{T1}-G)}=\frac{\omega_1\rho C_3V_2}{2(G_{T1}-G)}\sin(\omega_1t+\phi_3'-\phi_2). \tag{47}$$

Now consider the noise current out of ρC_c at ω_2 due to the coherent voltage V_1 across tank 1

$$i_{25}=\frac{\omega_2\rho C_3}{2}V_1\sin(\omega_2t+\phi_3'-\phi_1). \tag{48}$$

This current will produce a voltage across tank 1 at ω_1 which can be computed much as the v_{12} of Eq. (38) was computed;

$$v_{15}''=\frac{\omega_1C_3\omega_2\rho C_3}{4G_{T2}(G_{T1}-G)}V_1\sin(\omega_1t+\phi_3-\phi_3'+\phi_1). \tag{49}$$

Noting that at resonance the phase angle of the exponential in Eq. (12) is zero, we may write

$$V_2=\omega_2C_3V_1/2G_{T2}. \tag{50}$$

The total voltage across tank 1 due to this source of noise is

$$v_{15}=v_{15}'=v_{15}''=\frac{\omega_1\omega_2C_3^2}{4G_{T2}}\frac{\rho}{(G_{T1}-G)}\times V_1[\sin(\omega_1t+\phi_3'-\phi_2)+\sin(\omega_1t+\phi_3-\phi_3'+\phi_1)]. \tag{51}$$

Taking the mean square of (51) we obtain

$$\langle v_{15}^2\rangle=\frac{G^2}{(G_{T1}-G)^2}\langle\rho^2\rangle V_1^2=\frac{G^2}{(G_{T1}-G)^4}\langle\rho^2\rangle i_g^2. \tag{52}$$

This gives

$$N_{05}=\langle v_{15}^2\rangle G_L=\frac{G^2}{(G_{T1}-G)^4}\langle\rho^2\rangle i_g^2G_L \tag{53}$$

and

$$F_5=\frac{i_g^2}{4G_g}\frac{1}{KTB}\frac{4G_gG_L}{(G_{T1}-G)^2}\frac{G^2}{4G_gG_L}\langle\rho^2\rangle$$

$$=\frac{S_i}{N_i}\frac{\text{gain}}{4}\frac{G}{G_g}\frac{G}{G_L}\langle\rho^2\rangle. \tag{54}$$

Now consider noise source 6 arising from fluctuations of the variable capacitance at twice the signal frequency, $2\omega_1$. This involves an additional noise-pumped variable

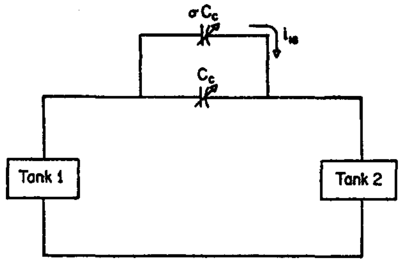


FIG. 11. Diagram to illustrate the effect of noise fluctuations in C_c at a $2\omega_1$ rate.

capacitance. An important current component will result from interaction between this variable capacitance and the coherent signal voltage, V_1, to give noise at $2\omega_1-\omega_1=\omega_1$. Let us call this incoherent capacitance σC_c. Its variation is expressed by

$$\sigma C_c=\sigma C_3 \sin(2\omega_1 t+\phi_1'). \tag{55}$$

By following the procedure used in deriving (8) and (9) we find that the above-mentioned noise current component at ω_1 flowing out of σC_c is given by

$$i_{16}=\frac{\omega_1\sigma C_3}{2}V_1 \sin(\omega_1 t+\phi_1'-\phi_1). \tag{56}$$

This situation is illustrated in Fig. 11. The output noise is given by

$$N_{06}=\langle v_{16}^2\rangle G_L=\frac{\langle i_{16}^2\rangle}{(G_{T1}-G)^2}G_L$$
$$=\frac{\omega_1^2 C_3^2}{4}\langle\sigma^2\rangle\frac{i_g^2}{(G_{T1}-G)^4}G_L. \tag{57}$$

From (28) the corresponding noise figure term is

$$F_6=\frac{S_i}{N_i}\frac{\text{gain}}{4}\frac{G}{G_g}\frac{G_2}{G_L}\frac{\omega_1}{\omega_2}\langle\sigma^2\rangle. \tag{58}$$

Noise source 7 involves fluctuations in the variable capacitance at a frequency of twice that of the resonant frequency of tank 2. We may take this source into account as before by adding a third noise-pumped variable capacitance,

$$\alpha C_c=\alpha C_3 \sin(2\omega_2 t+\phi_2'). \tag{59}$$

Interaction between this capacitance varying at $2\omega_2$ and a coherent voltage V_2 at ω_2 will produce a noise current at $2\omega_2-\omega_2=\omega_2$, which we can write by following the procedure for deriving (8) and (9),

$$i_{27}=\frac{\omega_2\alpha C_3}{2}V_2 \sin(\omega_2 t+\phi_2'-\phi_2). \tag{60}$$

This current may now be treated precisely as was treated the current i_2 of noise source 2. From inspection of (40) we may write

$$F_7=\frac{S_i}{N_i}\frac{\text{gain}}{4}\frac{G}{G_g}\frac{G}{G_2}\frac{G}{G_L}\frac{\omega_2}{\omega_1}\langle\alpha^2\rangle. \tag{61}$$

Noise source 8, arising from fluctuations in the variable capacitance at a frequency $\omega_1-\omega_2$, involves still another noise-pumped variable capacitance given as follows:

$$\delta C_c=\delta C_3 \sin[(\omega_1-\omega_2)t+\phi]. \tag{62}$$

Interaction with a coherent voltage V_2 in tank 2 will produce a current at $(\omega_1-\omega_2)+\omega_2=\omega_1$ flowing out of the capacitor. The procedure used in treating noise source 5 gives for the value of that current

$$i_{18}=-\frac{\omega_1\delta C_3}{2}V_2 \sin(\omega_1 t+\phi_2+\phi)$$
$$=-G\delta V_1 \sin(\omega_1 t+\phi_2+\phi). \tag{63}$$

This current will produce the following voltage across tank 1

$$v_{18}'=\frac{i_{18}}{(G_{T1}-G)}=-\frac{G\delta V_1}{(G_{T1}-G)}\sin(\omega_1 t+\phi_2+\phi). \tag{64}$$

Interaction between δC_c varying at $(\omega_1-\omega_2)$ and a coherent voltage V_1 at ω_1 will produce a current i_{28} at $\omega_1-(\omega_1-\omega_2)=\omega_2$ flowing into tank 2. This current may be treated in precisely the same way as i_{25} of noise source 5 or i_2 of noise source 2. From inspection of (38) and (12) we may write for the voltage v_{18}'' (including phase) which is produced across tank 1,

$$v_{18}''=\frac{\omega_1 C_3 i_{28}}{2G_2(G_{T1}-G)}\sin(\omega_1 t+\phi_3-\phi_1+\phi)$$
$$=\frac{G\delta V_1}{(G_{T1}-G)}\sin(\omega_1 t+\phi_2+\phi). \tag{65}$$

The total voltage across tank 1 resulting from this source is

$$v_{1b}=v_{18}'+v_{18}''=0. \tag{66}$$

Thus no noise appears at the output due to this source!

We now may write the complete noise figure expression from (29), (35) (41), (43), (44), (54), (58), (61), and (66),

$$F=1+\frac{G_1}{G_g}+\frac{G}{G_g}\frac{\omega_1}{\omega_2}+\frac{1}{4KTBG_g}\left[\langle i_3^2\rangle+\langle i_4^2\rangle\frac{G}{G_{T2}}\frac{\omega_1}{\omega_2}\right]$$
$$+\frac{S_i}{N_i}\frac{\text{gain}}{4}\frac{G}{G_g}\left[\frac{G}{G_L}\langle\rho^2\rangle+\frac{G_{T2}}{G_L}\frac{\omega_1}{\omega_2}\langle\sigma^2\rangle+\frac{G}{G_{T2}}\frac{G}{G_L}\frac{\omega_1}{\omega_2}\langle\alpha^2\rangle\right].$$

APPENDIX III

Modification of the Analysis for a General Nonlinear Variable Element

For electron-beam parametric amplifiers and for certain forms of bridged-diode amplifiers the treatment of the coupling capacitance as a simple time-varying element is proper. For most other forms, however, the coupling capacitance should be treated as a nonlinear element whose capacitance is a function of the voltage applied. Thus there will appear capacitance changes not only at the frequency of the applied pump voltage, but also at the amplifying and idling frequencies when these voltages are present. If the nonlinear capacitance varies with voltage as

$$C=Q/V=kV^n, \tag{67}$$

and if rf voltages $V_1\sin(\omega_1t+\phi_1)$, $V_2\sin(\omega_2t+\phi_2)$, and $V_3\sin(\omega_3t+\phi_3)$ are applied, there will be corresponding changes in capacitance $C'\sin(\omega_1t+\phi_1)$, $C''\times\sin(\omega_2t+\phi_2)$, and $C'''\sin(\omega_3t+\phi_3)$ such that

$$\frac{C'}{C'''}=\frac{V_1}{V_3} \quad \text{and} \quad \frac{C''}{C'''}=\frac{V_2}{V_3}. \tag{68}$$

The current flowing into the coupling capacitor is the time rate of change of the product of total voltage across the capacitor and the total capacitance. The component of current at ω_1 is made up of two parts, one due to the capacitance change at ω_3 and the voltage at ω_2, the other due to the capacitance change at ω_2 and the voltage at ω_3;

$$i(\omega_1)=\frac{\omega_1C'''}{2}V_2\sin(\omega_1t+\phi_3-\phi_2) + \frac{\omega_1C''}{2}V_3\sin(\omega_1t+\phi_3-\phi_2). \tag{69}$$

From Eq. (68) we see that these terms are identical, so

$$i(\omega_1)=\omega_1C'''V_2\sin(\omega_1t+\phi_3-\phi_2). \tag{70}$$

This is just the current computed from Eq. (8) for an effective value of capacitance C_3 equal to twice C''', that produced by the pump alone. Carrying through the derivation of the negative conductance presented to the amplifying tank, one finds

$$-G=-\frac{\omega_1\omega_2(C''')^2}{G_2}. \tag{71}$$

This again reduces to Eq. (18) if an effective time varying capacitance C_3 is used which is twice the C''' produced by the pump voltage.

With a coherent pump voltage V_3 across the nonlinear capacitor there are now three additional noise sources to be considered.[12]

(a) An incoherent capacitance change at ω_2 produced by thermal noise voltage in tank 2 which interacts with the coherent pump voltage across the capacitance to produce noise current at ω_1.

(b) An incoherent capacitance change at ω_1 produced by thermal noise voltage in tank 1 which interacts with the coherent pump voltage across the capacitance to produce noise current at ω_2.

(c) An incoherent noise voltage at ω_3 across the capacitance which interacts with coherent capacitance changes at ω_1 and ω_2 to produce noise currents at ω_2 and ω_1.

The incoherent capacitance changes at ω_1 and ω_2 are in phase with the noise voltages producing them so that it is not surprising that a detailed analysis shows that these effects can be taken into account, just as in the case of signals, by considering an amplifier with a time-varying linear capacitance which is twice the value of that resulting from the pump voltage alone. Thus noise sources 1 and 2 from Appendix II together with sources a and b above give a noise figure

$$F_{1,2,a,b}=1+\frac{G_1}{G_g}+\frac{\omega_1}{\omega_2}\frac{G}{G_g}, \tag{72}$$

which is exactly the form previously obtained with $C_3=2C'''$.

Moreover, the incoherent pump voltage and capacitance change at ω_3 are in phase so that noise source c and noise source 5 of Appendix II give the same contributions. This can be taken into account again by defining the effective capacitance change to be twice that produced by the pump voltage.

The result then is that the previously derived expression for gain, band width, and noise figure for the time-varying capacitance are correct for the case of a general nonlinear capacitance if the effective capacitance change is taken to be twice that produced by the pump alone.

[12] The authors are indebted to K. Kotzebue for discussions of these noise contributions.