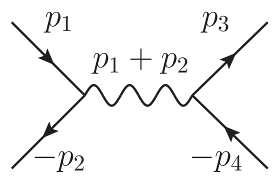
Calculating the $e^+e^- \to \mu^+\mu^-$ cross section using Schoonschip

In electron-positron colliders, one of the fundamental processes is $e^+e^- \to f^+f^-$, where f is a spin 1/2 fermion. The prototype Feynman diagram for these processes is $e^+e^- \to \mu^+\mu^-$, illustrated below



This diagram can be expressed as a complex amplitude of the form

$$\mathcal{M} = \frac{e^2\,\bar{u}(\vec{p}_3)\gamma_\mu v(\vec{p}_4)\bar{v}(\vec{p}_2)\gamma_\mu u(\vec{p}_1)}{(p_1+p_2)^2}\,, \tag{1}$$

where $(p_1+p_2)^2 = -s$ and the invariant s is the square of the center of mass energy. If we imagine and $e^+\,e^-$ collider with beam energies large compared with any mass, $\sqrt{s} = 2p$, where p is the beam momentum.

The Schoonschip code for the evaluation of $|\mathcal{M}|^2$ in the center of mass frame is

```
Date: Mon Aug  6 2001 17:59:53.  Memory: start 00020008, length 476860.

        A me,mf,cos,p,pp,Ee,Ef,s
        V p1,p2,p3,p4
        I mu,nu,i1,i2
        F M=u
        Z T=M(mu)*Conjg(M(nu))
L 2     Id,M(mu~)=Ubg(i1,mf,p3)*G(i1,mu)*Ug(i1,-mf,p4)*
           Ubg(i2,-me,p2)*G(i2,mu)*Ug(i2,me,p1)/s
L 3     Id,Spin,i1,i2
L 4     Id,Trick,Trace,i1,i2
> P out
        *yep

T =  + 64*me^2*mf^2*s^-2 - 32*me^2*s^-2*p3Dp4 - 32*mf^2*s^-2*p1Dp2
   + 32*s^-2*p1Dp3*p2Dp4 + 32*s^-2*p1Dp4*p2Dp3 + 0.

C p1(mu~)=(0,0,p,iEe)
C p2(mu~)=(0,0,-p,iEe)
C p3(mu~)=(pp*sin,0,pp*cos,iEf)
C p4(mu~)=(-pp*sin,0,-pp*cos,iEf)
L 1     Id,p1Dp=-p^2-Ee^2
L 2     Id,p3Dp4=-pp^2-Ef^2
L 3     Id,p1Dp3=p*pp*cos-Ee*Ef
L 4     Id,p2Dp4=p*pp*cos-Ee*Ef
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```
L 5     Id,p1Dp4=-p*pp*cos-Ee*Ef
L 6     Id,p2Dp3=-p*pp*cos-Ee*Ef
C Ignore masses
L 7     Id,me=0
L 7     Al,mf=0
L 8     Id,Ee=p
L 8     Al,Ef=p
L 9     Id,pp=p
        B s,p
> P out
        *end
```

and the output file is

```
Schoonschip, 68000 version of June 27, 1991. Public version.
Date: Mon Aug  6 2001 18:58:13.  Memory: start 00020008, length 476860.

Command line: e+e-2mu+mu-.e e+e-2mu+mu-.txt

A me,mf,cos,p,pp,Ee,Ef,s
V p1,p2,p3,p4
I mu,nu,i1,i2
F M=u
Z T=M(mu)*Conjg(M(nu))
L 2 Id,M(mu~)=Ubg(i1,mf,p3)*G(i1,mu)*Ug(i1,-mf,p4)*
   Ubg(i2,-me,p2)*G(i2,mu)*Ug(i2,me,p1)/s
L 3 Id,Spin,i1,i2
L 4 Id,Trick,Trace,i1,i2
> P out
*yep

T =  + 64*me^2*mf^2*s^-2 - 32*me^2*s^-2*p3Dp4 - 32*mf^2*s^-2*p1Dp2
   + 32*s^-2*p1Dp3*p2Dp4 + 32*s^-2*p1Dp4*p2Dp3 + 0.

C p1(mu~)=(0,0,p,iEe)
C p2(mu~)=(0,0,-p,iEe)
C p3(mu~)=(pp*sin,0,pp*cos,iEf)
C p4(mu~)=(-pp*sin,0,-pp*cos,iEf)
L 1 Id,p1Dp=-p^2-Ee^2
L 2 Id,p3Dp4=-pp^2-Ef^2
L 3 Id,p1Dp3=p*pp*cos-Ee*Ef
L 4 Id,p2Dp4=p*pp*cos-Ee*Ef
L 5 Id,p1Dp4=-p*pp*cos-Ee*Ef
L 6 Id,p2Dp3=-p*pp*cos-Ee*Ef
C Ignore masses
L 7 Id,me=0
L 7 Al,mf=0
```

```
L 8 Id,Ee=p
L 8 Al,Ef=p
L 9 Id,pp=p
B s,p
> P out
*end

T = + s^-2*p^4
  * ( 64 + 64*cos^2 ) + 0.


End run. Time 0 sec.
```

Including the coupling $e^4 = (4\pi\alpha)^2$, the average over initial electron spins 1/4 and the fact that $s = 4\,p^2$, $|\mathcal{M}|^2$ is

$$|\mathcal{M}|^2 = 16\pi^2\alpha^2(1+\cos^2(\theta))\,. \tag{2}$$

In this case, the differential phase space is

$$dPS = \frac{d\Omega}{2(4\,\pi)^2}\,, \tag{3}$$

and the flux factor is

$$4E_e^2|\vec{v}_1 - \vec{v}_2| = 4E_e^2\left(\frac{p}{E_e} + \frac{p}{E_e}\right) = 8\,p\,E_e \to 8\,p^2 = 2\,s\,. \tag{4}$$

Thus,$d\sigma_{\mu\bar{\mu}}/d\Omega$ is

$$\frac{d\sigma_{\mu\bar{\mu}}}{d\Omega} = \frac{\alpha^2\,(1+\cos^2(\theta))}{4\,s}\,. \tag{5}$$

The total cross section is

$$\sigma_{\mu\bar{\mu}} = \frac{4\,\pi\,\alpha^2}{3\,s}\,. \tag{6}$$

In the Spear collider, the center of mass energy $\sqrt{s}$ was 3.1 GeV, which is much larger than $m_\mu = 105.66$ MeV. This energy is also much larger than the light quarks u, d and s. The u, d and s electric charges are $2/3, -1/3$ and $-1/3$ and each quark comes in three colors. Hence, the effective coupling for light quark production is

$$3\left(\frac{2^2}{3^2} + \frac{(-1)^2}{3^2} + \frac{(-1)^2}{3^2}\right)\alpha^2 = 2\,\alpha^2\,, \tag{7}$$

Thus, we would expect to see a region where $q\,\bar{q}$ production was twice as large as $\mu\,\bar{\mu}$, or

$$R = \frac{\sigma_{q\,\bar{q}}}{\sigma_{\mu\,\bar{\mu}}} = 2\,. \tag{8}$$

The experimental data for R in the light quark region is shown below. At lower energies, the data shows the ρ, ω and ϕ vector mesons. Above about 1.1 GeV, there is a flat R value at $R = 2.0$. This was the first strong evidence for the quark hypothesis.

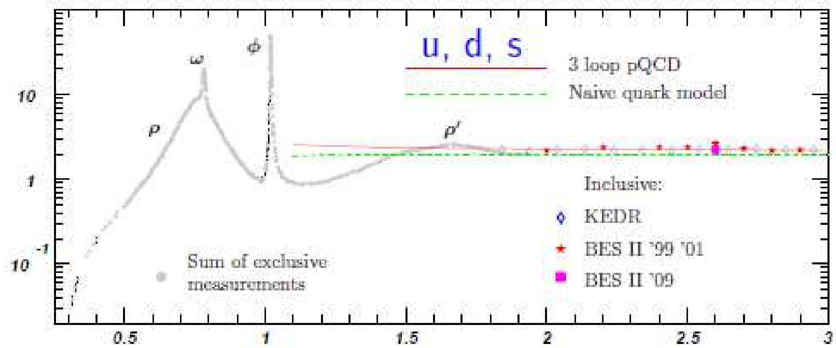

Figure 1: The R value for $\sqrt{s} \leq 3.0$ GeV is shown.