Resonance terms in $R(\sqrt{s})$

In examining the graph of the R value for $e^+e^- \rightarrow hadrons$, which shows a region from about 1 GeV to 3 Gev that is consistent with light quark production, there is region at lower energy with several peaks. These peaks occur because of vector boson production. The ρ, ω and ϕ vector bosons

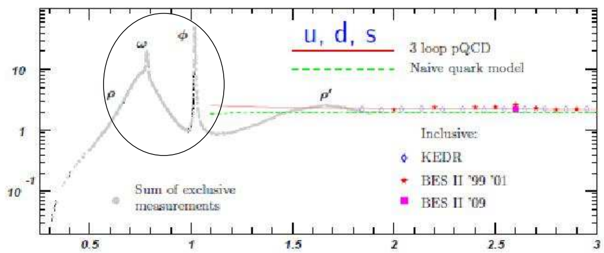

Figure 1: The R value for $\sqrt{s} \leq 3.0\,\mathrm{GeV}$ is shown.

have the same parity and charge conjugation quantum numbers as the (virtual) photon produced in e^+e^- annihilation and therefore couple to it. For arbitrary center of mass energy $\sqrt{s}$, this coupling can produce a virtual vector boson with very small probability. However, when $\sqrt{s}$ is near the vector boson's physical mass there is a high probability and a huge spike in the cross section. All these vector mesons decay by virtue of the strong interaction. What is seen in the detector at these values of $\sqrt{s}$ is the decay products of the vector meson that have the invariant mass of the vector meson. These decay products predominantly consist of two or three pions or a pair of kaons.

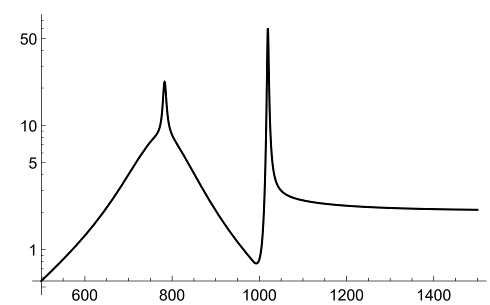

Figure 2: The effect of the ρ, ω and ϕ resonances on the $R(\sqrt{s})$ ratio is shown.

In addition to their dominant hadronic decay modes, all three vector bosons have a small decay width Γ_{e+e-} into e^+e^-. The strength of the photon-vector boson coupling, f_V, is determined from measuring Γ_{e+e-} and using the relation

$$\Gamma_{e+e-} = \frac{4\pi\alpha^2 m_V}{3f_v^2}\,. \tag{1}$$

In a similar way, the hadronic decay strength g_V can be obtained from its total width Γ_V. For the ρ meson, which decays almost exclusively into $\pi^+\pi^-$, Γ_ρ is

$$\Gamma_\rho = \frac{g_\rho^2 (m_\rho^2 - 4m_\pi^2)^{3/2}}{48\pi\, m_\rho^2} = 149.1\,\mathrm{MeV}\,. \tag{2}$$

Figure [2] shows the theoretical prediction for this resonance region. The primary decay modes are $\rho \to \pi^+\pi^-$, $\omega \to \pi^+\pi^-\pi^0$ and $\phi \to K\bar{K}$. In addition, both the ω and the ϕ have measurable $\pi^+\pi^-$ decay modes. This means that the ρ, ω and ϕ have $\pi^+\pi^-$ decay amplitudes that interfere with one another. The square of the scattering amplitude has the form

$$|\mathcal{M}|^2 = |M_\rho(\pi^+\pi^-) + M_\omega(\pi^+\pi^-) + M_\phi(\pi^+\pi^-)|^2 + |M_\omega(\pi^+\pi^-\pi^0)|^2 + |M_\phi(K\bar{K})|^2\,, \tag{3}$$

and each contribution is multiplied by the appropriate phase space factor. A typical vector boson amplitude looks like

$$M_\rho(\pi^+\pi^-) = \frac{e^2\, m_\rho^2}{f_\rho\, s} \frac{\bar{v}(p_2)\gamma_\mu\, u(p_1) H_\mu(k_1, k_2, \cdots)}{s - m_\rho^2 + i m_\rho \Gamma_\rho}\,, \tag{4}$$

where $H_\mu(k_1, k_2, \cdots)$ contains the kinematics of final state. The appearance of the factors like

$$\frac{1}{(s - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho}\,, \tag{5}$$

in the squared amplitude produces the peak when s is in the vicinity of m_ρ^2.