A Low-Noise Amplifier*

HENRY WALLMAN†, ASSOCIATE, I.R.E., ALAN B. MACNEE†, ASSOCIATE, I.R.E., AND C. P. GADSDEN‡, MEMBER, I.R.E.

Summary—**This paper describes an amplifier circuit which yields very low noise factor, consisting of a grounded-cathode triode followed by a grounded-grid triode. The combination is entirely noncritical and provides the low noise factor of a triode with the high amplification and stability of a pentode. Noise factors averaging 0.25 db at a carrier frequency of 6 Mc. and 1.35 db at 30 Mc. have been achieved. Typical circuit details are given.**

I. Introduction

In COMMUNICATIONS systems in which the minimum usable signal is determined by receiver noise, as distinct from noise arising from atmospherics,[1] jamming, etc., improving receiver noise factor[2,3] is as valuable as increasing transmitter power. The amplifying arrangement described in this paper affords a reduction of about 2 db in the minimum perceptible signal of certain microwave radar receivers; this is equivalent to increasing peak power in the associated transmitters from 1.0 to 1.6 megawatts, at vastly less cost.

It is well known that the random division of cathode current between plate and screen in a pentode makes the shot-noise current of a given pentode about three to five times that of the same tube connected as a triode with screen strapped to plate.[4] This effect is called "partition noise"; because of it, many suggested arrangements for obtaining good amplifier noise factor have revolved around the use of a triode as a first stage.[5-7]

A triode can be employed in three ways; namely, as a grounded-cathode stage, a grounded-grid stage, or a grounded-plate stage (cathode-follower). For a given tube type, these three configurations can be shown to yield, very closely, the same noise factor. If a single triode is used to precede a pentode amplifier chain, and if any triode load resistors necessary for stability are considered to be part of the triode stage, then all these arrangements have the disadvantage in wide-band amplifiers (1 Mc. or wider) that the available gain[8] is low. The noise factor of the complete amplifier is, consequently, materially affected by the noise factor of the remainder of the amplifier, in accordance with the relation[3]

$$F_{12} = F_1 + \frac{F_2 - 1}{G_1} \tag{1}$$

where

F_{12} = noise factor of entire amplifier
F_1 = noise factor of first stage
F_2 = noise factor of balance of amplifier with source resistance equal to the output resistance of the first stage
G_1 = available gain of first stage.

This circumstance suggests the desirability of using a triode as a second stage also, either to make F_2 small or, as is done in the circuit described below, to permit stability to be maintained with a large value of G_1.

Two triodes can be cascaded in nine possible ways. Theoretical and experimental investigation of these nine possibilities led us to the one described below as being the best combination, with regard to noise factor, stability, and gain.

The arrangement in question was devised by the authors in 1944 at the M.I.T. Radiation Laboratory. It consists of a grounded-cathode triode first stage, followed by a grounded-grid triode second stage. An a.c. diagram is shown in Fig. 1. The various coils are midband resonant with their associated capacitances.

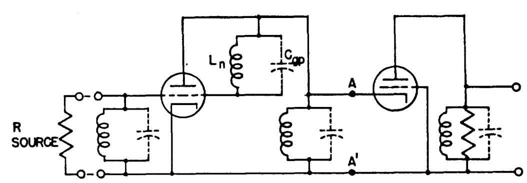

Fig. 1—A.c. diagram of cascode low-noise amplifier.

The coil L_n in parallel with the grid-plate capacitance C_{gp} is a neutralizing coil whose purpose is, however, not to obtain stability but to achieve low noise factor. Even in amplifiers operating at a midband frequency as high as 180 Mc., L_n can be omitted with complete preservation of stability, although the noise factor is increased from 5.5 to 8.0 db.

Search for a concise name for the grounded-cathode, grounded-grid combination led to the designation "cas-

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† Massachusetts Institute of Technology, Cambridge, Mass.
‡ Tulane University, New Orleans, La.

[1] K. G. Jansky, "Minimum noise levels obtained on short-wave radio receiving systems," Proc. I.R.E., vol. 25, pp. 1517-1530; December, 1937.

[2] D. O. North, "The absolute sensitivity of radio receivers," *RCA Rev.*, vol. 6, pp. 332–344; January, 1942.

[3] H. T. Friis, "Noise figures of radio receivers," Proc. I.R.E., vol. 32, pp. 419–422; July, 1944.

[4] B. J. Thompson, D. O. North, and W. A. Harris, "Fluctuations in space-charge-limited currents at moderately high frequencies," *RCA Rev.*, vols. 4 and 5; January, 1940, to July, 1941.

[5] M. C. Jones, "Grounded-grid radio-frequency voltage amplifiers," Proc. I.R.E., vol. 32, pp. 423–429; July, 1944.

[6] Milton Dishal, "Theoretical gain and signal-to-noise ratio of the grounded-grid amplifier at ultra-high frequencies," Proc. I.R.E., vol. 32, pp. 276–284; May, 1944.

[7] G. C. Sziklai and A. C. Schroeder, "Cathode-coupled wide-band amplifiers," Proc. I.R.E., vol. 33, pp. 701–708; October, 1945.

[8] Throughout this paper, gain refers to power ratios, and amplification to voltage ratios.

code," after a somewhat similar arrangement employed by Hunt and Hickman.[9]

The cascode amplifier shown in Fig. 1 provides (a) the stability and noncriticalness of a pentode, (b) the amplification and gain of a pentode, and (c) the low noise factor of the first triode.

It is the two triodes together that have the amplification of a single pentode; thus the cost of the improvement in noise factor is one additional tube. It is possible, however, that a high-quality double triode with separate cathodes, such as the recently announced type 2C51, may yield the advantages of this circuit with only one envelope.[10]

Conventional pentode amplifier stages follow the cascode to provide the bulk of the amplifier gain.

The cascode low-noise amplifier was used for wide-band band-pass amplifiers, and can be applied a fortiori to narrow-band amplifiers; with this technique, it should be possible to build 30-Mc. communications receivers with noise factors of 1.4 db.

It is believed that the low-noise cascode circuit can also be adapted to low-pass amplifiers.

II. Grounded-Cathode Amplifiers

For use in the subsequent discussion, we now summarize the noise-factor analysis of a grounded-cathode amplifier stage.[4,11] The analysis is made for midband frequency, at which the various tube and circuit capacitances are assumed to be resonated out.

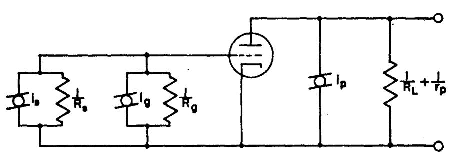


Fig. 2—Equivalent circuit for band-center noise-factor analysis of grounded-cathode stage.

The amplifier can then be analyzed in terms of the equivalent circuit of Fig. 2, where

k = Maxwell-Boltzmann constant (1.38×10^{-23} joule/°K.)
T = absolute temperature of source resistance (usually taken as 290°K., "room temperature")
B = noise bandwidth
R_s = transformed source resistance
R_g = input resistance due to tube and coupling circuits
g_m = tube mutual transconductance
r_p = tube plate resistance
R_L = load resistance
$\overline{i_s^2}$ = mean-squared thermal-agitation-noise current, $4kTB/R_s$, of R_s
$\overline{i_g^2}$ = mean-squared grid-noise current
T_g = effective absolute temperature of the input loading, defined as $\overline{i_g^2}R_g/4kB$
$\overline{i_{ns}^2}$ = mean-squared tube-shot-noise current
$\overline{i_{nr}^2}$ = mean-squared thermal-agitation-noise current, $4kTB/R_L$ of R_L
$\overline{i_p^2} = \overline{i_{ns}^2} + \overline{i_{nr}^2}$ = mean-squared plate-circuit noise current.

Although the tube in Fig. 2 is shown as a triode, it can also be a tetrode or pentode, provided all electrodes other than the control grid and plate are by-passed to ground over the frequency range to be amplified.

For convenience of notation, it is common in noise-factor analysis to replace an actual tube with plate-circuit noise current i_p by a fictitious noiseless tube, having in series with its signal grid lead a noise voltage

$$e_{eq} = i_p/g_m \tag{2}$$

which, when amplified by the tube, produces in its plate circuit the noise current i_p. It is then possible to define a purely fictitious resistance R_{eq} according to the relation

$$R_{eq} = \overline{e_{eq}^2}/4kTB, \tag{3}$$

called the equivalent noise resistance of the grounded-cathode stage.[12] As a measure of the noisiness of the tube and its load resistor, R_{eq} is a schematic substitute for the plate-noise current i_p.

If there is a coupling circuit between the signal source and the tube grid, R_s and i_s are regarded as the source resistance and the corresponding noise current referred to the output terminals of the coupling circuit.

The shunt input loading $1/R_g$ is made up of three components: tube loading due to cathode-lead-inductance feedback, transit-time loading, and loading due to losses in the input circuit. In noise analyses, the cathode-lead inductance can be considered to be zero or tuned out by a suitable series capacitor, for it has been shown[13] that if this were not the case the resulting input loading would have only second-order effect on amplifier noise factor, because it degenerates tube noise as well as input noise. Cathode-lead-inductance loading does, however, affect the input bandwidth.

Tube loading due to transit time is, on the other hand, of utmost importance in noise-factor considerations because it is found to have a large noise current associated with it. For transit angles less than one radian, corresponding to frequencies less than about 200 Mc. for a

[9] F. V. Hunt and R. W. Hickman, "On electronic voltage stabilizers," *Rev. Sci. Instr.*, vol. 10, pp. 6–21; January, 1941. The low-noise property that forms the feature of the present circuit is, however, entirely unconnected with the original use of the cascode as a d.c. amplifier in a voltage stabilizer.

[10] Results obtained in 1944 with the type 7F8 were variable and disappointing, but this may have been a vicissitude of early 7F8 production.

[11] E. W. Herold, "An analysis of the signal-to-noise ratio of ultra-high-frequency receivers," *RCA Rev.*, vol. 6, pp. 302–331; January, 1942.

[12] The equivalent resistance r_{eq} of the tube itself neglects i_{nr} and is defined by $r_{eq} = \overline{(i_{ns}/g_m)^2}/4kTB$ (cf. (3)); it is related to R_{eq} by $R_{eq} = r_{eq} + [1/(g_m^2 R_L)]$, and because the bracketed term is usually negligible, R_{eq} and r_{eq} are usually very closely equal.

[13] M. J. O. Strutt and A. Van der Ziel, "Methods for the compensation of the effects of shot noise in tubes and associated circuits," *Physica*, vol. 8, pp. 1–22; January, 1941.

tube such as the 6AK5, it has been observed[14,15] that the effects of electron-transit time can be represented by a shunt resistance R_t from grid to cathode in parallel with a noise current i_t such that

(a) the ratio T_t of $\overline{i_t^2}$ to $4kB/R_t$ is constant, that is, independent of frequency, and

(b) $$T_t \approx 5T. \tag{4}$$

This representation assumes that transit-time grid noise is statistically independent of plate noise, or, in any event, that their effects add in the mean square. For large transit angles, this assumption is incorrect.[16]

By careful design, the third source of input loading, circuit losses, can be kept small enough to have negligible effect on amplifier noise factor compared to transit-time loading, except possibly at very low frequencies where the noise factor is extremely good in any case. For example, the transit-time loading for a 6AK5 at 30 Mc. is about 15 μmho. Because of its high effective temperature, the effect on noise factor of the transit-time loading completely dominates that of circuit losses even if circuit losses introduce equal loading, corresponding to an input coil Q of about 120, which is very moderate. At higher frequencies, the coil Q required to make coil losses negligible is even smaller.

In the following discussion of noise performance, therefore, the grid loading can be assumed to consist of transit-time loading only, and i_g to be equal to i_t.

The noise factor of the circuit of Fig. 2 is

$$F = 1 + \frac{\overline{i_g^2}}{\overline{i_s^2}} + \frac{\overline{i_p^2}}{\overline{i_s^2}}\left(\frac{R_s + R_g}{R_sR_g}\right)^2 \frac{1}{g_m^2}$$

or

$$F = 1 + \frac{R_s}{R_g}\,\frac{T_g}{T} + \frac{R_{eq}}{R_s}\left(\frac{R_s + R_g}{R_g}\right)^2. \tag{5}$$

Because the source resistance R_s can usually be varied by suitable impedance-transforming schemes, it is desirable to determine the value of R_s that makes the noise factor of the amplifying stage a minimum. Differentiating (5) shows that this is[11]

$$R_{s,\mathrm{opt}} = \sqrt{\frac{\overline{i_p^2}R_g^2}{\overline{i_g^2}g_m^2R_g^2 + \overline{i_p^2}}} = \sqrt{\frac{R_g^2R_{eq}}{R_g\left(\dfrac{T_g}{T}\right) + R_{eq}}}\,. \tag{6}$$

For most cases of interest,

$$\overline{i_g^2}g_m^2R_g^2 \gg \overline{i_p^2}, \tag{7}$$

which is equivalent to the condition

$$(T_g/T)R_g \gg R_{eq}. \tag{8}$$

This permits simplifying (6) to the form

$$R_{s,\mathrm{opt}} \approx \frac{i_p}{i_g g_m} = \sqrt{R_{eq}\,\frac{R_g}{T_g/T}}\,. \tag{9}$$

Equation (9) shows that the optimum source resistance is considerably lower than the value matching the input resistance of the tube; the optimum source resistance is rather that value that makes *the plate current resulting from the grid-noise voltage equal to the plate-noise current.* For R_s approximately adjusted to its optimum value, Fig. 2 simplifies to Fig. 3.

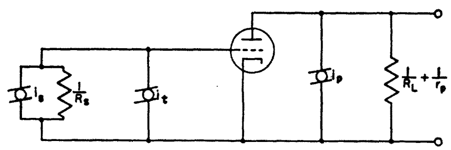


Fig. 3—Simplified equivalent circuit for band-center noise-factor analysis with R_s approximately adjusted to its optimum value, (9), so that one can neglect the transit-time loading $1/R_t$ but not the transit-time noise current i_t. Cathode-lead inductance loading and input-circuit loss are also neglected. The noise voltage at the grid, due to electron-transit time, is i_tR_s, and R_s is optimum when $g_m(i_tR_s) = i_p$.

Substituting (6) into (5) gives an expression for the optimum noise factor,

$$F_{\mathrm{opt}} = 1 + 2\left[\frac{R_{eq}}{R_g} + \sqrt{\left(\frac{R_{eq}}{R_g}\right)^2 + \frac{T_g}{T}\,\frac{R_{eq}}{R_g}}\right],$$

which can be simplified under the condition of (8) to

$$F_{\mathrm{opt}} \approx 1 + 2\sqrt{\frac{T_g}{T}\,\frac{R_{eq}}{R_g}}, \tag{10}$$

or

$$F_{\mathrm{opt}} = 1 + 2\,\frac{R_{eq}}{R_{s,\mathrm{opt}}}\,. \tag{11}$$

Because T_g/T is constant (see (4)) for the frequency range under consideration, (10) permits one to evaluate the potential noise performance of a tube without considering circuit details, but knowing only two tube properties, namely, equivalent noise resistance and input resistance due to transit time. It also indicates the manner in which optimum noise factor increases with increasing frequency. Because

$$R_g \propto 1/f^2, \tag{12}$$

it follows that

$$R_{s,\mathrm{opt}} \propto 1/f, \tag{13}$$

and hence

$$(F_{\mathrm{opt}} - 1) \propto f; \tag{14}$$

that is, *if as the center frequency is varied the source resistance of a grounded-cathode amplifier is continually ad-*

[14] C. J. Bakker, "Fluctuations and electron inertia," *Physica,* vol. 8, pp. 23–43; January, 1941.

[15] D. O. North and W. R. Ferris, "Fluctuations induced in vacuum-tube grids at high frequencies," Proc. I.R.E., vol. 32, pp. 419–423; July, 1941.

[16] There is some evidence, both theoretical and experimental, that the noise factor is slightly improved by tuning the input circuit somewhat below band center, about 2 Mc. at 30 Mc. and 15 Mc. at 180 Mc. The reason is the existence of a certain amount of coherence between grid and plate noise. This point is discussed in sec. 13.13 of vol. 18, "Vacuum-Tube Amplifiers," Radiation Laboratory Series, McGraw-Hill Book Co., New York, N. Y., 1948.

justed to yield the best noise factor, then the excess noise factor will be proportional to frequency.

Experimental corroboration of (14) is shown in Fig. 4, in which are displayed measured noise factors at 6, 30, and 180 Mc. (see Table 1).

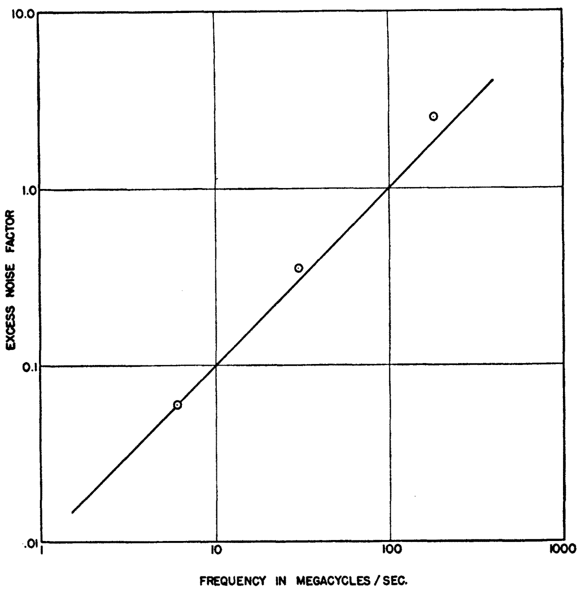


Fig. 4—Measured excess noise factors F_1 of low-noise cascode amplifiers at 6, 30, and 180 Mc. Source resistance adjusted at each frequency for optimum noise factor (see Table 1).

Although the bandwidth of the input circuit, when adjusted for optimum noise factor, does not explicitly appear in (6) and (9), it is, nevertheless, not arbitrary, but is determined for a given type of input circuit by the value of $R_{s,\,opt}$ together with the tube and circuit capacitances, and is proportional to frequency (13). For many tubes having good noise performance the input-circuit bandwidth obtained by adjusting the source resistance for optimum noise factor is moderately wide. An example is the 6AK5 pentode at 30 Mc., where $R_{s,\,opt} \approx 6000$ ohms and the bandwidth for a single-tuned input circuit with an input capacitance of 10 $\mu\mu$fd. is about 3 Mc., permitting an over-all bandwidth of about 1.5 Mc. Bandwidths about twice as wide can be achieved for the same transformed source resistance by using a double-tuned input circuit; if, in order to obtain even wider bandwidths, the value of R_s is reduced below $R_{s,\,opt}$, the noise factor will be degraded. If, for image rejection or other reasons, it is desirable to have a narrower input bandwidth, it should be obtained by increasing the input capacitance, leaving the source resistance at its optimum value.

For the grounded-cathode amplifier of Fig. 2, the available gain is

$$G = g_m^2\left(\frac{R_L r_p}{R_L + r_p}\right)\frac{R_s R_g^2}{(R_s + R_g)^2}\cdot \tag{15}$$

The various equations derived in this section are subject to the assumption that the tube and circuit capacitances are tuned out. Therefore, the noise-factor expression of (5) is accurate only for the single frequency at the center of the amplifier pass band. Some discussion of how the noise factor varies over the pass band is given in Part IV. One usually finds that variations in noise factor over the pass band are small enough to be neglected in rough design calculations. The notable exception to this statement occurs in the case of the grounded-grid amplifier.

Noise Factor of 6AK5 Pentode

One of the best pentodes now available as a first-stage amplifier is the type 6AK5. For this tube, $g_m = 5000$ μmho, $1/R_t = 16 \times 10^{-21} f^2$ μmho/(c.p.s.)^2, and $r_{eq} = 2500$ ohms. If the load resistor for the stage is chosen to be 2000 ohms, $R_{eq} = 2520$ ohms.[12] At 30 Mc. (9) shows, taking $R_g = R_t$, that

$$R_{s,opt} = 5900 \text{ ohms.} \tag{16}$$

Using the value of (16) in (11), the optimum noise factor is found to be

$$F_{opt} \approx 1 + 2\left(\frac{2520}{5900}\right) = 1.85, \quad \text{or} \quad 2.7 \text{ db.} \tag{17}$$

Although the noise factor of (17) was derived for band center, it is in fairly close agreement with measured integrated noise factors of 6-Mc.-wide amplifiers at 30 Mc. using 6AK5 pentodes; these averaged about 3.2 db.

For the 2000-ohm load resistor, the available gain (15) is

$$G \approx (5000)^2 \times 10^{-12}(2000)(5900) \approx 300. \tag{18}$$

The gain given by (18) is so high that even a second-stage noise factor of 10 times will not appreciably degrade the over-all noise factor of the amplifier.

Noise Factor of Hypothetical Triode

Let us now consider the performance that could be achieved if it were possible to operate the same 6AK5 tube as first stage of the 30-Mc. amplifier, but connected as a triode. For the triode connection, $R_{eq} = 400$ ohms and $g_m = 6700$ μmho, but the transit-time loading remains unchanged. Using these data, one finds that

$$R_{s,opt} = 2350 \text{ ohms,} \tag{19}$$

$$F_{opt} = 1.35, \quad \text{or} \quad 1.3 \text{ db,} \tag{20}$$

$$G = 137. \tag{21}$$

Comparing these values with those calculated for the pentode connection (16), (17), and (18), we see that the noise factor is substantially improved. Moreover, the optimum source resistance is appreciably lowered; this permits wider input bandwidths at the optimum noise factor. The available gain remains high enough to render negligible the noise contribution of all but the noisiest second stage.

The difficulty with this plan lies in the fact that a triode-connected 6AK5 operated at 30 Mc. with the indicated source and load resistances would oscillate, and thus would be valueless.

It will be shown, however, that, through suitable connection of two triode tubes, it is possible to achieve the stability of a single pentode stage. Furthermore, the noise factor, optimum source resistance, and available gain of this combination are determined almost entirely by the first triode alone. In particular, if a triode-connected 6AK5 is used for first tube, the values given in (19), (20), and (21) can be achieved.

III. The Cascode Low-Noise Circuit—Qualitative Explanation

In preparation for the precise discussion of the cascode low-noise circuit contained in Part IV, a qualitative exposition will now be given. This discussion is restricted to band center.

Referring to Fig. 1, let g_{m1}, r_{p1}, g_{m2}, r_{p2} be the transconductance and plate resistance of the first and second tubes, and let R_2 be the load resistance of the second tube. Assuming, for simplicity, that R_2 is considerably smaller than r_{p2}, as in wide-band amplifiers, then one knows that the input resistance of the grounded-grid stage is approximately $1/g_{m2}$; this is the resistance looking to the right at points AA' of Fig. 1.

The resistance looking to the left at AA' is r_{p1}. Typical values are about 200 ohms for $1/g_{m2}$ and 4500 ohms for r_{p1}. It is this combination of very low resistance to the right and very high resistance to the left that is the crucial characteristic of the grounded-cathode, grounded-grid combination, with regard to both stability and noise factor.

a. *Stability*

The amplification from the grid of tube 1 to its plate is g_{m1}/g_{m2}. If g_{m1} and g_{m2} are about equal, as is usually the case, the amplification is about unity. This low amplification makes the grounded-cathode stage stable. If g_{m2} is twice g_{m1}, say (for tube 1 a triode-connected 6AK5 and tube 2 a 6J4), the amplification of tube 1 is only one-half.

b. *Amplification*

For a 1-volt signal applied to the input grid, the plate current of tube 1 is g_{m1} ampere. Because this current flows through the plate circuit of tube 2, the voltage across R_2, and hence the amplification of the cascode, is approximately $g_{m1}R_2$. With regard to amplification, the cascode circuit is thus equivalent to a pentode of transconductance g_{m1}.

Observe that the amplification of the cascode does not depend on g_{m2}. The motivation for large g_{m2} is essentially only this: the larger g_{m2}, the smaller is the amplification of the grounded-cathode stage, and hence the greater the stability of the grounded-cathode stage.

The transconductance of a pentode is increased when it is connected as a triode, approximately in the ratio of cathode to plate current. For this reason, the amplification of a cascode employing a triode-connected 6AK5 as first stage is actually about a third larger than that of a single pentode 6AK5 stage.

c. *Noise factor*

The available gain of the first stage is

$$G_1 = \frac{\mu_1^2}{4r_{p1}} \bigg/ \frac{1}{4R_s} = \frac{\mu_1^2 R_s}{r_{p1}} \tag{22}$$

where μ_1 is the amplification factor of tube 1 and R_s is the source resistance.

The noise factor F_2 of the grounded-grid stage, regarding R_2 as part of that stage, can be shown to be

$$F_2 - 1 = \frac{T_{g2}}{T}\frac{r_{p1}}{R_{g2}} + \frac{r_{eq2}}{r_{p1}} + \frac{r_{p1}}{R_2} \tag{23}$$

where T_{g2} is the effective temperature of the second-tube grid-noise resistor R_{g2}, and r_{eq2} is the second-tube equivalent plate-noise resistance.[12] In the right side of (23) the first term represents the contribution of grid noise; the second term, plate-shot noise; and the third term, thermal-agitation noise in R_2.

Typical values, pertaining to a 6AK5–half-6J6 cascode (Fig. 7) at 30 Mc. are $\mu_1 = 30$, $R_s = 2350$ ohms, $T_g/T = 5$, $r_{p1} = 4500$ ohms, $R_{g2} = 60{,}000$ ohms, $r_{eq2} = 500$ ohms, and $R_2 = 2000$ ohms; for that case

$$G_1 = 470, \tag{24}$$

which is extremely high, and

$$F_2 - 1 = 0.38 + 0.11 + 2.25 = 2.74. \tag{25}$$

Equations (1), (24), and (25) show that, in this typical case, the noise factor F_{12} of the cascode is

$$F_{12} = F_1 + 0.01. \tag{26}$$

In the same typical case, F_1 is about 1.35. Equation (26) thus validates the assertion that the noise factor of the cascode is extremely close to that of the grounded-cathode stage alone.

It follows that, although the grounded-cathode tube type should be chosen for low noise factor, noise performance is irrelevant in the choice of tube type for the grounded-grid stage.

d. *Available Gain*

The available gain G_{12} of the cascode, regarding R_2 as part of it, is that of a pentode of transconductance g_{m1}, source resistance R_s, and load resistance R_2; i.e.,

$$G_{12} = \frac{(g_{m1}R_2)^2}{4R_2} \bigg/ \frac{1}{4R_s} = g_{m1}^2 R_s R_2. \tag{27}$$

In the typical case above, $G_{12} \approx 200$; this is enough to make any usual third-stage noise negligible.

IV. Single-Frequency Noise Factor of the Cascode

For small-signal analysis, the low-noise cascode can be replaced by the equivalent circuit of Fig. 5, where

Y_{gp1} = grid-to-plate admittance of tube 1

$$= j\omega_0 C_{gp1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

C_{gp1} = grid-to-plate capacitance of tube 1
f_0 = band-center frequency $= \omega_0/2\pi$
Y_{gc1} = grid-to-cathode admittance of tube 1

$$= g_s + j\omega_0 C_{gc1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

C_{gc1} = grid-to-cathode capacitance of tube 1
g_s = transformed signal conductance at grid of tube 1
Y_{inter} = interstage admittance

$$= \frac{1}{r_{p1}} + j\omega_0(C_{pc1} + C_{gc2})\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

C_{pc1} = plate-to-cathode capacitance of tube 1
C_{gc2} = grid-to-cathode capacitance of tube 2
r_{p1} = plate resistance of tube 1
Y_{pc2} = plate-to-cathode admittance of tube 2

$$= \frac{1}{r_{p2}} + j\omega_0 C_{pc2}$$

C_{pc2} = plate-to-cathode capacitance of tube 2
r_{p2} = plate resistance of tube 2
Y_{gp2} = grid-to-plate admittance of tube 2

$$= \frac{1}{R_L} + j\omega_0 C_{gp2}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

C_{gp2} = grid-to-plate capacitance of tube 2
R_L = load resistor of second stage of cascode
$\overline{i_s^2}$ = mean-squared thermal-noise current $4kTBg_s$ of g_s
$\overline{i_{n1}^2}$ = grid-noise current of tube 1
$\overline{i_{n2}^2}$ = shot-noise current of tube $1 = 4kTBr_{eq1}g_{m1}^2$
$\overline{i_{n3}^2}$ = shot-noise current of tube $2 = 4kTBr_{eq2}g_{m2}^2$

In analyzing the cascode by the equivalent circuit of Fig. 5, a number of approximations are made:

(a) The grid-noise current of tube 2 is neglected compared with the plate-noise current of tube 1. These two currents flow between the same two terminals and can thus be compared directly.

(b) The admittance of the grid loading of tube 1 is neglected, but the noise current associated with this loading is not.

(c) All tube and circuit capacitances with the exception of C_{pc2} are assumed to be tuned to parallel resonance by suitable parallel inductances.

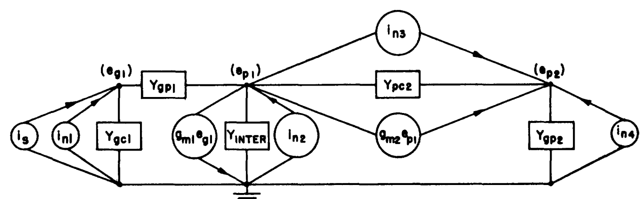


Fig. 5—Equivalent circuit for cascode noise-factor analysis.

(d) The losses associated with these inductances are assumed to be negligible, with regard to both loading and thermal noise.

(e) The grid-noise current of each tube is statistically independent of its plate-noise current.

Assumptions (a) and (e) are observed to hold for most receiving tubes at frequencies below 200 Mc. Assumption (b) is justified for most tubes by (7). Assumption (c) means that the input, interstage, and output circuits are single-tuned, and the grid-plate capacitance of tube 1 is neutralized at band center. The results obtained for this case are indicative of the behavior to be expected in general and are yet simple enough to be easily manipulated. Assumption (d) is met in practice provided care is taken to employ reasonably high-Q coils.

The node equations for the cascode equivalent circuit, written in matrix form, are

$$\begin{bmatrix} i_s + i_{n1} \\ i_{n2} - i_{n3} \\ i_{n3} + i_{n4} \end{bmatrix} = \begin{bmatrix} (Y_{gc1} + Y_{gp1}) & -Y_{gp1} & 0 \\ (g_{m1} - Y_{gp1}) & (g_{m2} + Y_{gp1} + Y_{\text{inter}} + Y_{pc2}) & -Y_{pc2} \\ 0 & -(g_{m2} + Y_{pc2}) & (Y_{pc2} + Y_{gp2}) \end{bmatrix} \times \begin{bmatrix} e_{g1} \\ e_{p1} \\ e_{p2} \end{bmatrix}. \tag{28}$$

$\overline{i_{n4}^2}$ = thermal-noise current of load resistor $= 4kTB/R_L$
T = absolute room temperature
T_g = effective absolute temperature of grid loading $= \overline{i_g^2}R_g/4kB$
B = noise bandwidth
k = Maxwell-Boltzmann constant
r_{eq1} = equivalent noise resistance of tube 1
r_{eq2} = equivalent noise resistance of tube 2
g_{m1} = transconductance of tube 1
g_{m2} = transconductance of tube 2.

We define
e_{p2n} = output voltage with all currents applied
e_{p2o} = output voltage with the noise currents i_{n1}, i_{n2}, i_{n3}, and i_{n4} set equal to zero.

Then the noise factor of the cascode circuit is

$$F = \frac{\overline{e_{p2n}^2}}{\overline{e_{p2o}^2}}. \tag{29}$$

The voltage e_{p2n} is

$$
e_{p2n} = \frac{\begin{vmatrix} (Y_{gc1} + Y_{gp1}) & -Y_{gp1} & (i_s + i_{n1}) \\ (g_{m1} - Y_{gp1}) & (g_{m2} + Y_{gp1} + Y_{\text{inter}} + Y_{pc2}) & (i_{n2} - i_{n3}) \\ 0 & -(g_{m2} + Y_{pc2}) & (i_{n3} + i_{n4}) \end{vmatrix}}{|Y|} \tag{30}
$$

where

$$
|Y| = \begin{vmatrix} (Y_{gc1} + Y_{gp1}) & -Y_{gp1} & 0 \\ (g_{m1} - Y_{gp1}) & (g_{m2} + Y_{gp1} + Y_{\text{inter}} + Y_{pc2}) & -Y_{pc2} \\ 0 & -(g_{m2} + Y_{pc2}) & (Y_{pc2} + Y_{gp2}) \end{vmatrix} \tag{31}
$$

and

$$
e_{p2o} = -\frac{i_s(g_{m1} - Y_{gp1})(g_{m2} + Y_{pc2})}{|Y|}. \tag{32}
$$

Usually,

$$
g_{m1} \gg Y_{gp1}, \tag{33}
$$

$$
g_{m2} \gg Y_{pc2} + Y_{\text{inter}} + Y_{gp1}. \tag{34}
$$

Then, from (30) and (32),

$$
\frac{e_{p2n}}{e_{p2o}} = \frac{\begin{vmatrix} (Y_{gc1}+Y_{gp1}) & -Y_{gp1} & (i_s+i_{n1}) \\ g_{m1} & (Y_{gp1}+Y_{\text{inter}}) & (i_{n2}+i_{n4}) \\ 0 & -g_{m2} & (i_{n3}+i_{n4}) \end{vmatrix}}{-i_s g_{m1} g_{m2}}, \tag{35}
$$

which, when expanded, becomes

$$
\begin{aligned}
\frac{e_{p2n}}{e_{p2o}} = 1 + \frac{i_{n1}}{i_s} - \frac{i_{n2}}{i_s}\left[\frac{Y_{gc1}+Y_{gp1}}{g_{m1}}\right] \\
- \frac{i_{n3}}{i_s}\left[\frac{(Y_{gc1}+Y_{gp1})(Y_{gp1}+Y_{\text{inter}})+g_{m1}Y_{gp1}}{g_{m1}g_{m2}}\right] \\
- \frac{i_{n4}}{i_s}\left[\frac{g_{m1}Y_{gp1}+g_{m2}(Y_{gc1}+Y_{gp1})}{g_{m1}g_{m2}}\right].
\end{aligned} \tag{36}
$$

Equation (36) can be written

$$
\begin{aligned}
\frac{e_{p2n}}{e_{p2o}} = 1 + \frac{i_{n1}}{i_s} - \frac{i_{n2}}{i_s}\frac{g_s}{g_{m1}}[1+jQ_1\alpha] \\
- \frac{i_{n3}}{i_s}\frac{g_s}{\mu_1 g_{m2}}[(1+jQ_1\alpha)(1+jQ_2\alpha)+jQ_3\alpha] \\
- \frac{i_{n4}}{i_s}\frac{g_s}{g_{m1}}[1+jQ_4\alpha]
\end{aligned} \tag{37}
$$

where

$$
Q_1 \doteq \frac{\omega_0(C_{gp1} + C_{gc1})}{g_s}, \tag{38}
$$

$$
Q_2 = \omega_0 r_{p1}(C_{gp1} + C_{gc2} + C_{pc1}), \tag{39}
$$

$$
Q_3 = \mu_1\frac{\omega_0 C_{gp1}}{g_s}, \tag{40}
$$

$$
Q_4 = Q_1 + Q_3/(g_{m2}r_{p1}), \tag{41}
$$

$$
\alpha = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}. \tag{42}
$$

In the mean square of e_{p2n}/e_{p2o}, which is the desired noise factor, all the cross products of noise currents are zero because of the incoherence[17] of the respective noise currents. After substitution of the appropriate values for the mean-squared noise currents, the single-frequency noise factor is found to be

$$
\begin{aligned}
F(f) = 1 + \left(\frac{T_g}{T}\right)\frac{1}{g_s R_g} + g_s R_{eq1}(1 + Q_1^2\alpha^2) \\
+ \frac{g_s R_{eq2}}{\mu_1^2}[(1 - Q_1Q_2\alpha^2)^2 + Q_5^2\alpha^2] \\
+ \frac{g_s}{g_{m1}^2 R_L}(1 + Q_4^2\alpha^2)
\end{aligned} \tag{43}
$$

where

$$
Q_5 = Q_1 + Q_2 + Q_3.
$$

A plot of $[F(f)-1]$ for a typical 30-Mc. band-pass cascode amplifier using a 6AK5 first stage and a 6J4 second stage is given in Fig. 6. The various excess noise-factor contributions due to the two tubes and the output load resistance are also plotted. One notes that, over an 11-Mc. band, the noise contribution of the grounded-grid tube is less than the noise contribution of the output load resistor. Inspection of curves such as those of Fig. 6 shows how the integrated noise factor can be expected to deviate from the band-center value.

For the case plotted in Fig. 6, the 3-db bandwidth of the cascode circuit is about 6.5 Mc., so that at the 3-db frequencies the noise factor has increased about 0.1; if the balance of the amplifier has a bandwidth of about 7 Mc., one would expect the integrated noise factor to lie between 1.5 and 1.6 db.

If desired, the integrated noise factor can be exactly calculated, following Schremp[18]:

(a) Calculate the single-frequency noise factor $F(f)$.

(b) Calculate the transfer impedance $Z_{12}(f)$ of the cascode plus the balance of the amplifier. This is the voltage appearing at the output terminals of the amplifier when i_s is one ampere.

[17] See footnote 16.

[18] E. J. Schremp, "Vacuum-Tube Amplifiers," Radiation Laboratory Series, vol. 18, McGraw-Hill Book Co., New York, N. Y., 1948; sec. 12.7.

(c) Calculate a weighting factor $W_1^2(f)$ according to the relation

$$W_1^2(f) = \frac{Z_{12}^2(f)}{\int_0^\infty Z_{12}^2(f)df}. \tag{44}$$

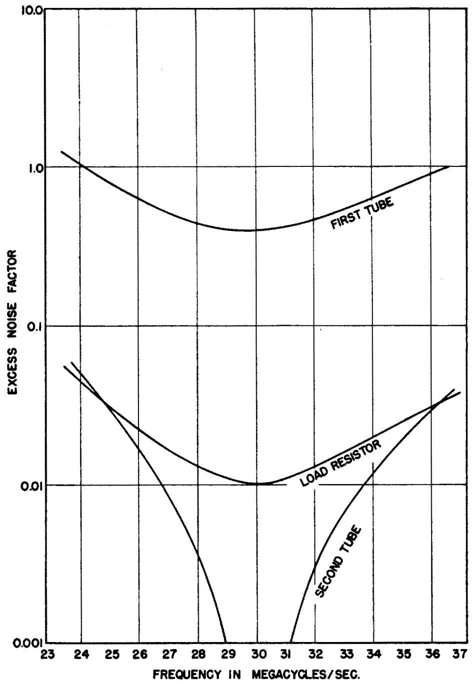


Fig. 6—Calculated single-frequency excess noise-factor components for a typical low-noise cascode. Tube data approximately those of a 6AK5-6J4 combination; source and load resistances equal to 2000 ohms.

(d) Determine the integrated noise factor from the relation

$$F = \int_0^\infty F(f)W_1^2(f)df. \tag{45}$$

V. Practical Circuits and Experimental Results

A practical embodiment of the low-noise cascode which gives excellent results is shown in Fig. 7.

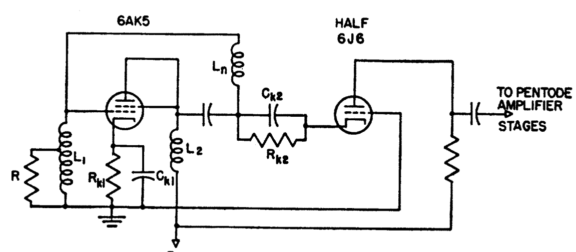


Fig. 7—Practical low-noise cascode circuit.

The grounded-cathode stage is a triode-connected[19] 6AK5. This type was found superior to the 6AG5 and 6J4 as first stage (for reasons that are not clear; there may be some connection with the fact that the 6AK5 has a gold-plated control grid, and hence small grid emission).

As the grounded-grid stage, half of a 6J6 is used, with socket pins connected as shown in Fig. 8. Strapping pins 1, 3, 5, and 6 to the center post of the socket in star fashion and grounding the combination furnishes a grounded-grid stage with a g_m of 5000 μmho and a cathode-to-plate capacitance of 0.25 $\mu\mu$fd.

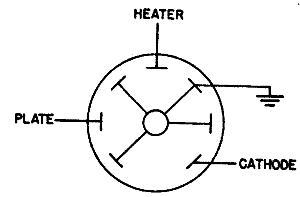


Fig. 8—Grounded-grid 6J6 socket connections.

A 6AK5 makes an unsatisfactory grounded-grid stage because the internal connection of suppressor grid-to-cathode leads to a very high cathode-to-plate capacitance, about 3.1 $\mu\mu$fd. A 6J4 is a very good grounded-grid stage but its expense is probably squandered in the cascode.

Resistors R_{k1} and R_{k2} are cathode-bias resistors of conventional magnitude, and C_{k1} and C_{k2} are their by-pass capacitors.

Coil L_1 resonates with the input-circuit capacitance at the desired band center.[16] It should be kept in mind that the input impedance of the first tube includes, by Miller effect, a capacitance $(1+A)$ C_{gp} in parallel with an inductance $L_n/(1+A)$, where A is the amplification of the first stage. In Fig. 7 the value of A is about unity.

The resistance R is stepped up to the value $R_{s,\,\mathrm{opt}}$ (see Table 1) by locating the tap on L_1. If the input-circuit bandwidth so obtained is inadequate, the single-tuned input circuit can be replaced by a double-tuned circuit; this is more complicated and critical and has no advantage in noise factor, but allows about twice the bandwidth for the same impedance step-up.

The coil L_2 tunes the interstage capacitance of about 10 $\mu\mu$fd. The interstage circuit is extremely wide, about 80 Mc. in Fig. 7, because of the heavy input loading of the grounded-grid stage. For this reason, L_2 is extremely noncritical.[20]

The standing current of the grounded-grid stage flows to ground through R_{k2}, L_n, and L_1. With this arrangement, use of L_n requires no additional parts. The coil L_n tunes the grid-plate capacitance of the grounded-cathode state (1.2 $\mu\mu$fd. for the triode-connected 6AK5 of Fig. 7). It is not critical, as shown by the fact that stability is preserved if it is left out entirely, and noise factor is degraded only 0.2 db at 30 Mc. and 2.5 db at 180 Mc. However, if it is desired to adjust L_n accurately in a production prototype for resonance at band center, a signal generator can be applied to the grid terminal of

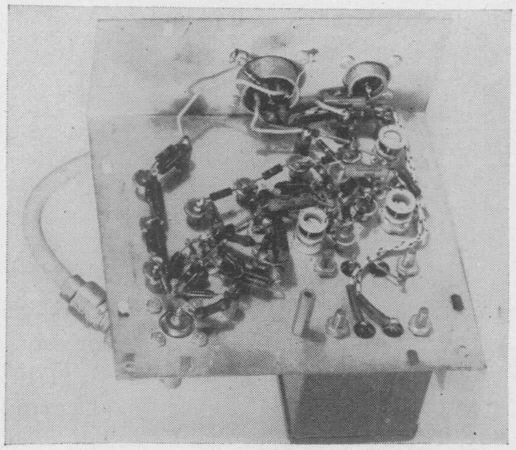
[19] Equal performance was obtained with a type 6AS6 when triode-connected by strapping grids 2 and 3 to the plate; this showed that no harm results from the grounded suppressor interposed between grid and plate of a triode-connected 6AK5.

[20] This is illustrated by the fact that, in an amplifier at 30 Mc., there was no noticeable change in noise factor, amplification, or bandwidth when the value of L_2 was inadvertently tripled.

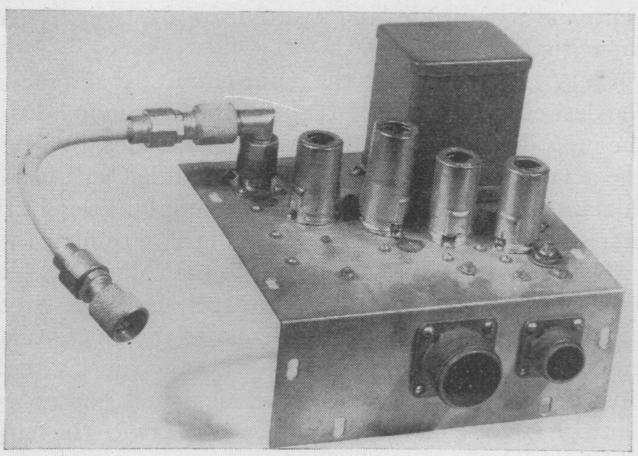
the cold[21] grounded-cathode stage and L_n proportioned for minimum transmission.

For best noise factor, L_1 and L_n should have fairly high Q's, about 200.

The photographs of Fig. 9 show a 30-Mc. amplifier consisting of a 6AK5–half-6J6 low-noise cascode followed by two 6AK5 pentode stages.



(a)



(b)

Fig. 9—Photographs of a 30-Mc. amplifier consisting of a 6AK5–half-6J6 low-noise cascode followed by two 6AK5 pentode stages. The 6AK5 of the cascode is at the left in (b).

having 100-db gains employing the cascode low-noise input circuit, at several frequencies. The noise factors were measured with noise diodes[22] and represent the integrated noise factor over the whole amplifier pass band.

The 6- and 30-Mc. amplifiers were constructed by Lawson and Nelson with every precaution to obtain least noise factor. The Q's of the input and neutralizing coils were over 200, and the 6AK5 bias was adjusted for best average noise factor ($R_{k1}=70$ ohms for 105 plate volts). Lawson and Nelson measured noise factors for 100 different 6AK5 first tubes in the 30-Mc. amplifier; the quoted 1.35-db noise factor was the median of these measurements, the range having extended from about 1.1 to 1.9 db. Changing tubes in the second stage did not affect noise factor, nor was stability affected by changes in either stage. In the 6-Mc. amplifier, there was very little variation of noise factor even with first-tube replacement.

Radar Receiver Noise Factor

By using a 30-Mc. intermediate-frequency amplifier with cascode low-noise input stage, the authors obtained, in 1945, a 3000-Mc. radar receiver with a radio-frequency noise factor F_{rf} of 7.4 ($=8.7$ db), as measured with a 3000-Mc. klystron noise source.[23] F_{rf} has the following form:

$$F_{rf} = L_{\text{crystal}} \times L_{tr} \times (F_{if} + T_{\text{crystal}} - 1) \qquad (46)$$

where L_{crystal} is the conversion loss and L_{tr} is the loss in the transmit-receive switch, given as ratios, and T_{crystal} is the crystal temperature index; i.e., the ratio of crystal intermediate-frequency noisiness to that of a resistor of equal intermediate-frequency resistance. In the case under discussion, L_{crystal} and T_{crystal} were about 3.6 and 1.1 (good values, but not the best ever obtained) and L_{tr} was about 1.4.

Newer transmit-receive switch designs permit reduction of L_{tr} to about 1.2. If the intermediate frequency were lowered to 6 Mc., as would be practical with a balanced mixer, 3000-Mc. radar receivers with noise factors of 5 ($=7$ db) could become common, even without improvement in crystal converters.

TABLE 1

NOISE FACTORS OF AMPLIFIERS HAVING LOW-NOISE CASCODE FIRST STAGE

Band center, Mc.	Over-all noise factor		$R_{s,\text{ opt}}$, ohms	Bandwidth, Mc.		Tubes used in cascode	Degradation of noise factor when L_n is omitted, db
	Ratio	db		Input circuit	Over-all		
6	1.06	0.25	15,000	2*	1	6AK5-6J4	not measured
30	1.35	1.35	2,500	12*	6	6AK5-6J4	0.2
180	3.5	5.5	400	30†	2.5	6J4-6J4	2.5

* Double-tuned.
† Single-tuned.

Results

In Table 1 are listed experimentally obtained noise factors and optimum source resistances of amplifiers

[21] Heaters disconnected.

[22] Radiation Laboratory Series, "Vacuum-Tube Amplifiers," "Measurement of Noise Figures," vol. 18, McGraw-Hill Book Co., New York, N. Y., 1948; chap. 14.

[23] M. C. Waltz and J. B. H. Kuper, M.I.T. Radiation Laboratory Report 443, September 17, 1943.