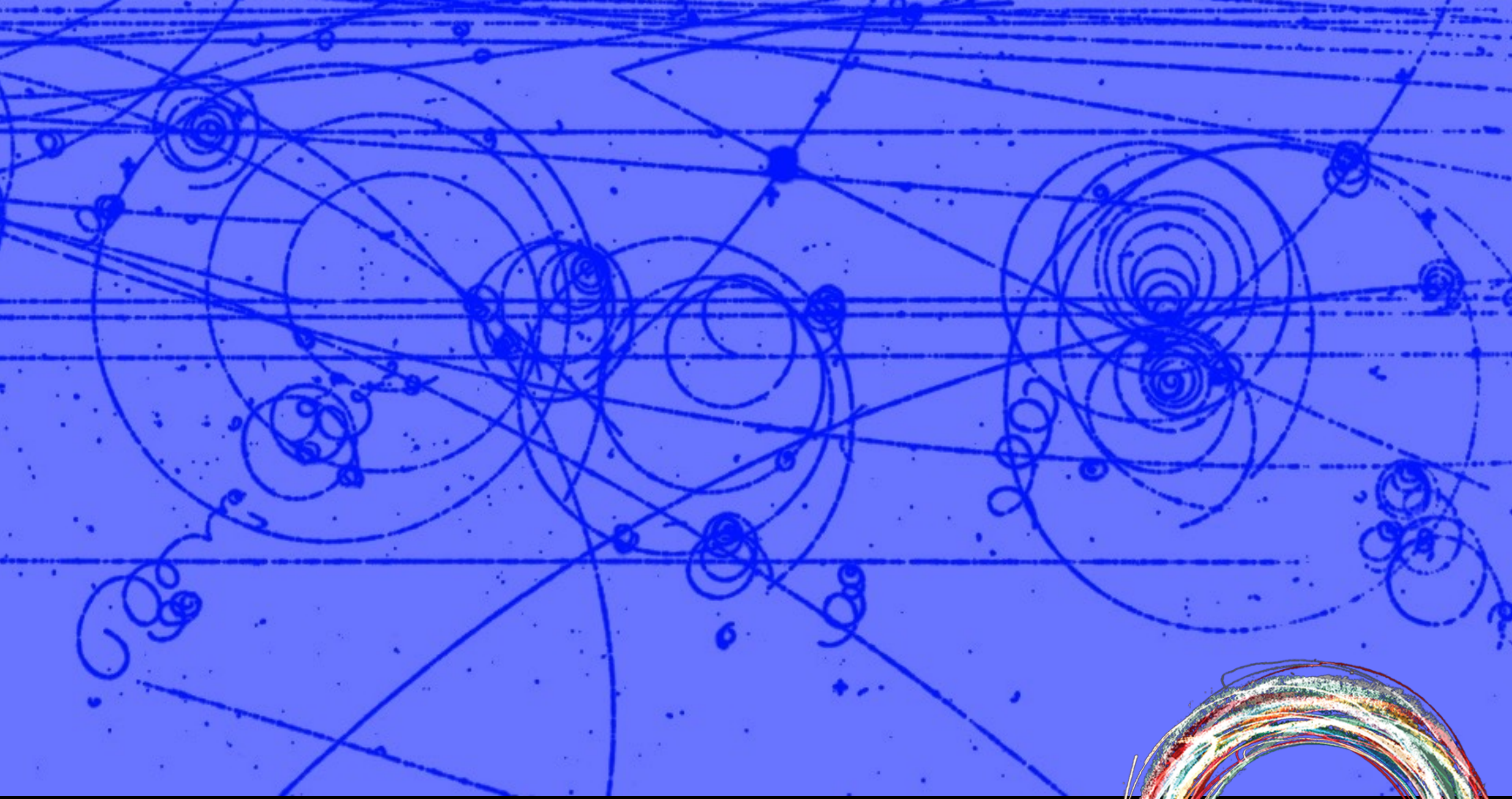
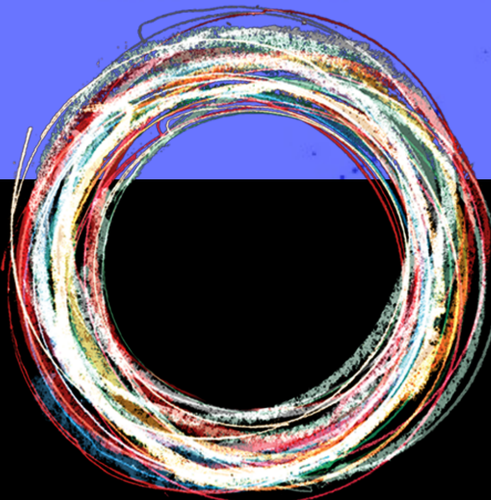
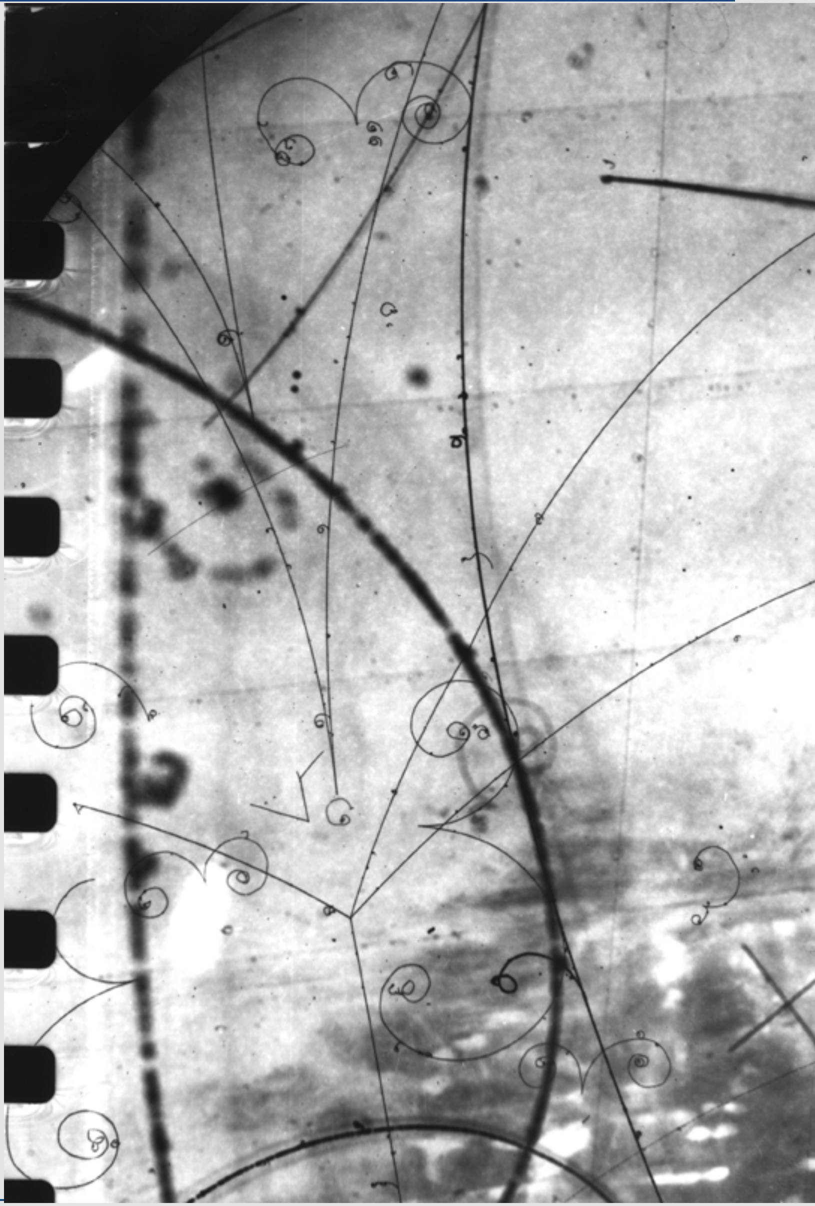




LECTURE 14
QUANTUM FLAVOR DYNAMICS

PHY 493/803, 2017

Recap / Up Next



Last time:

Quantum Chromodynamics

- Quarks in QED
- QCD Feynman Rules
- Asymptotic Freedom

This time:

Quantum Flavor Dynamics

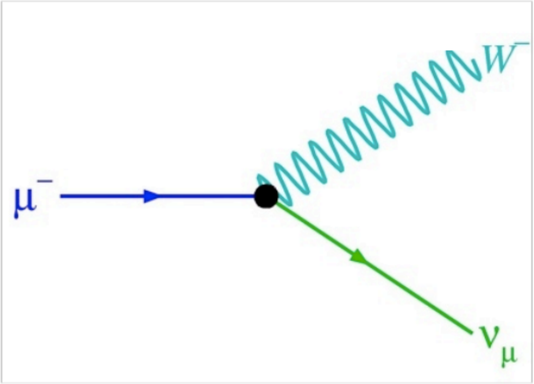
- Charged Currents
- QFD Feynman Rules
- Weak Decays
- Neutral Currents

Weak Interactions

Like QED and QCD, the weak interaction is mediated by spin-1 (vector) particle exchange.

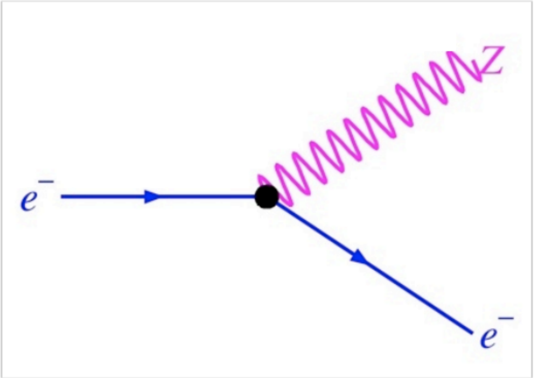
Any fermion (quark, lepton) may emit or absorb a W-boson.

- To conserve electric charge that fermion must change flavor!
- To conserve lepton number $e \leftrightarrow \nu_e$, $\mu \leftrightarrow \nu_\mu$, $\tau \leftrightarrow \nu_\tau$
- To conserve baryon number (d, s, b) $\leftrightarrow$ (u, c, t)
- W boson mass = 80.385 GeV



Any fermion (quark, lepton) may emit or absorb a Z-boson.

- That fermion will remain the same flavor.
- Very similar to QED, but neutrinos can interact with a Z boson too.
- Z boson mass = 91.1876 GeV



QFT Feynman Rules

<u>W/Z Propagators:</u>

The form of the propagator tells us a lot about the structure of the interaction

We need to generalize the photon propagator

$$-\frac{ig_{\mu\nu}}{q^2}$$

in order to account for the mass, M, of the intermediate vector bosons.

This problem is more subtle than it looks. We will use the so-called *unitary gauge* propagator:

$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)}{q^2 - M^2}$$

Weak Boson Propagators

<u>W/Z Bosons:</u>

$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)}{q^2 - M^2}$$

<u>Photons:</u>

$$-\frac{i g_{\mu\nu}}{q^2}$$

$$\mathcal{M} \propto g^2 \left(J_1^\dagger\right) \left[\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M^2}\right)}{q^2 - M^2}\right] \left(J_2\right)$$

<u>Differences:</u>

1) For q≪M, the weak boson propagator leads to a much smaller matrix element than the QED matrix element.
   - This is indistinguishable from a smaller coupling constant!

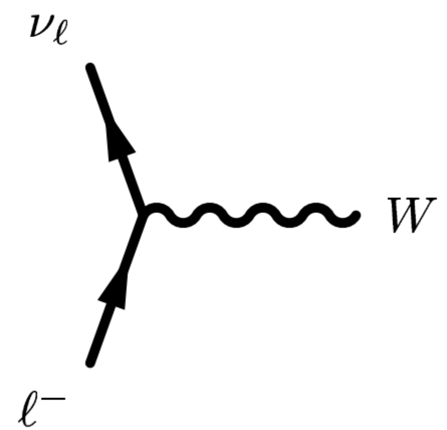
2) The weak boson propagator has a pole at q=M, causing the matrix element to blow up. We refer to this as "on resonance" particle production.

3) For q>>M, the QED and QFD propagators look very similar. A consequence is that Z bosons and photons have similar contributions to many processes.

Charged Current Vertex

The $W^{\pm}$ bosons mediate charged current (CC) weak interactions.
They couple to leptons via:

$$\frac{-ig_w}{2\sqrt{2}}\gamma^{\mu}(1-\gamma^5)$$



- The coupling constant is specific to the weak force and we'll calculate this later on.
- We'll also come back to Z bosons and quarks soon.

Notice that this interaction mixes vector and axial vector terms. We call this a (V-A) interaction and it leads to parity violation.

* (V-A) = "V minus A"

Back to Gamma Matrices

We had 3 conditions, including an anti-commutation relation.

Can't do this with numbers since they commute (AB=BA always) but we can do it with matrices (which do not, in general, commute).

$$(\gamma^0)^2 = 1 \quad (\gamma^{1,2,3})^2 = -1 \quad \{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 0 \quad \text{For } \mu \neq \nu$$

Dirac's clever idea was to let γ represent a set of 4x4 matrices

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix} \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

γ^5: The Black Sheep of the Family

Define an additional γ-matrix by

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$$

In the Bjorken and Drell representation:

$$\gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

<u>Note:</u>
$(\gamma^5)^2=1$ and anti-commutes with every other γ:

$$\{\gamma^\mu, \gamma^5\} = 0 \qquad \Rightarrow \qquad \gamma^\mu\gamma^5 = -\gamma^5\gamma^\mu$$

Bilinear Covariants

There are 16 possible products of the form $\psi^*_i\psi_j$. These 16 products can be grouped together into bilinear covariants:

$\bar{\psi}\psi$	Scalar	1 component
$\bar{\psi}\gamma^5\psi$	Pseudoscalar	1 component
$\bar{\psi}\gamma^\mu\psi$	Vector	4 components
$\bar{\psi}\gamma^\mu\gamma^5\psi$	Axial Vector	4 components
$\bar{\psi}\sigma^{\mu\nu}\psi$	Antisymmetric tensor	6 components

Note that:

$$\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

Bilinear Covariants: Why??

We now have a simple basis set { 1, γ^μ, γ^5, $\gamma^\mu\gamma^5$, $\sigma^{\mu\nu}$ } for any 4x4 matrix, therefore we can always simplify more complicated combinations of γ matrices.

The tensorial and parity character of each bilinear is evident. This makes it easy to see why the QED interaction Lagrangian

$$-eA_\mu\bar{\psi}\gamma^\mu\psi$$

leads to a parity-conserving electromagnetic force mediated by a vector (spin-1) boson.

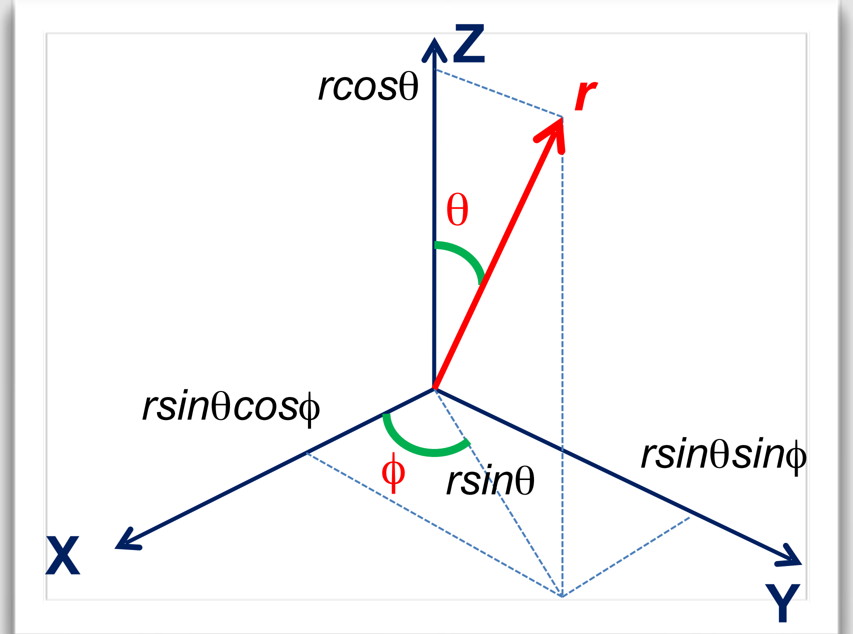
To describe the parity-violating weak interaction, we could (and will!) mix vector and axial interactions.

$$(\bar{\psi}\gamma^\mu\psi) \pm (\bar{\psi}\gamma^\mu\gamma^5\psi)$$

Spatial Reflection: Parity

Spherical polar co-ordinates:
$(x,y,z) \rightarrow (r,\theta,\phi)$

$$X = r \sin\theta \cos\phi$$
$$Y = r \sin\theta \sin\phi$$
$$Z = r \cos\theta$$



In these co-ordinates, ***r* → - *r*** implies:

$$\left\{ \begin{array}{lll} r & \rightarrow & r \\ \theta & \rightarrow & \pi - \theta \\ \phi & \rightarrow & \pi + \phi \end{array} \right.$$

It can be shown for the spherical harmonics function that
$\hat{P}\, Y^l m(\theta,\phi) = (-1)^l\, Y^l m(\theta,\phi)$

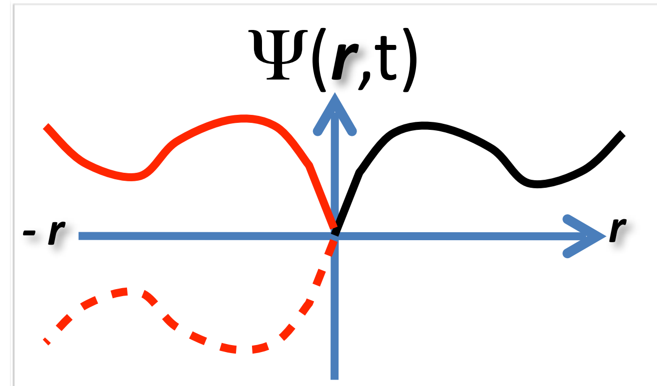
Intrinsic Parity (P)

The behavior of a state under a coordinate transformation $\boldsymbol{r} \rightarrow -\boldsymbol{r}$

Particles have intrinsic parity, & it's conserved!

Particles are in definite eigenstates of the parity operator:

$$\hat{P}\,\Psi(\boldsymbol{r},t) = P\,\Psi(-\boldsymbol{r},t)$$



$$\left\{ \begin{array}{l} \hat{P}^2\,\Psi(\boldsymbol{r},t) = \Psi(\boldsymbol{r},t) \\ \hat{P}^2\,\Psi(\boldsymbol{r},t) = P\hat{P}\,\Psi(-\boldsymbol{r},t) = P^2\,\Psi(\boldsymbol{r},t) \end{array} \right.$$

$$P^2 = 1, \qquad P = \pm 1$$

Parity: Putting it Together

Particles and bounds states can thus be in complicated parity eigenstates

- Eigenstates of intrinsic parity
- Eigenstates of angular momentum parity

$$\hat{P}\psi(a) = P_a(-1)^\ell \psi(a)$$

$$N(938): \quad I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$$

$$N(1520): \quad I(J^P) = \frac{1}{2}(\frac{3}{2}^-)$$

Parity Transformations

Scalar	***P(s) = s***
Pseudoscalar	***P(p) = -p***
Vector	***P(v) = -v***
Pseudovector (or axial vector)	***P(a) = a***

$$J = S + L$$

$$|L - S| \leq J \leq L + S$$

Parity Violation?

The first indication for parity violation

: K decays (final states have opposite parities) $K \to \pi\pi$, $K \to \pi\pi\pi$

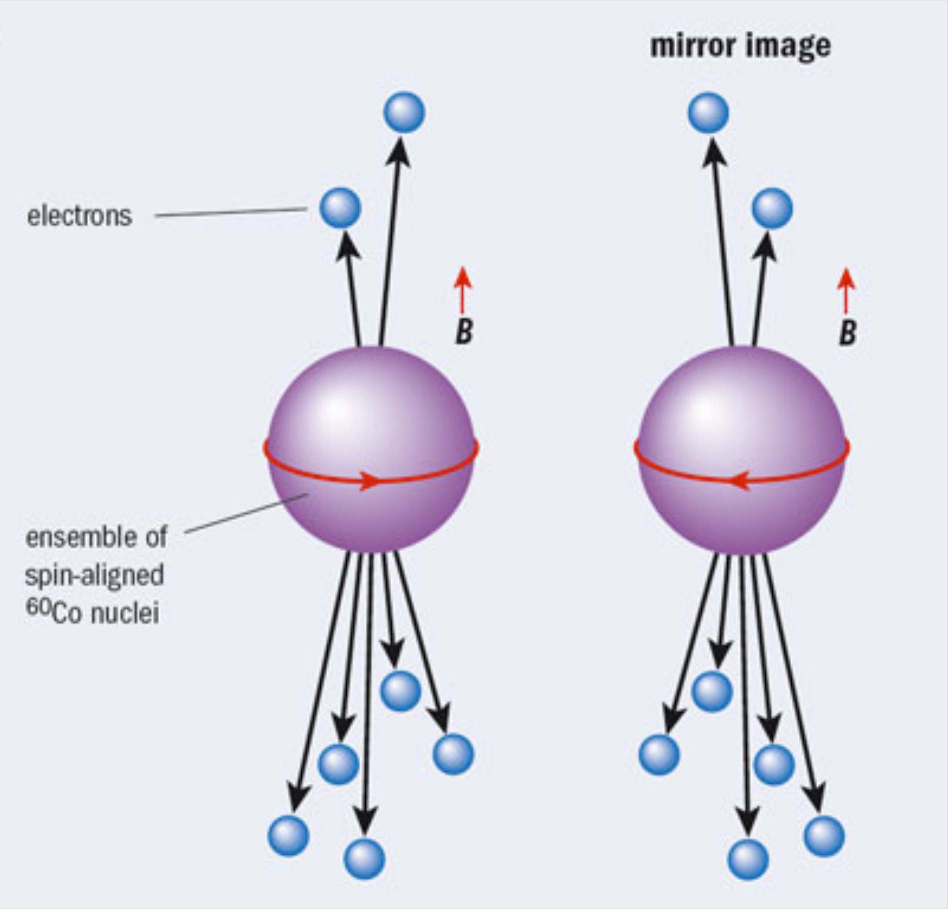
The experiment performed by Wu and co-workers (1957)

$^{60}Co \to {}^{60}Ni + e^{-} + \overline{\nu}_e$

1. **Place a sample of ^{60}Co inside a magnetic solenoid at a temperature of 0.01K (^{60}Co polarized along the magnetic field)**

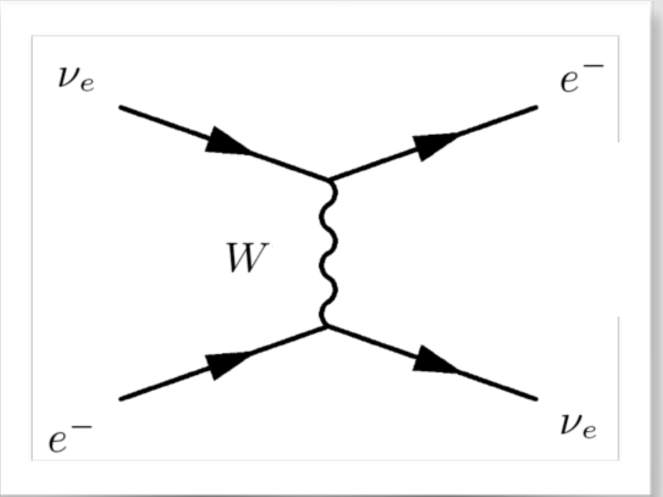
2. **Observe beta-rays by changing the direction of the magnetic field**

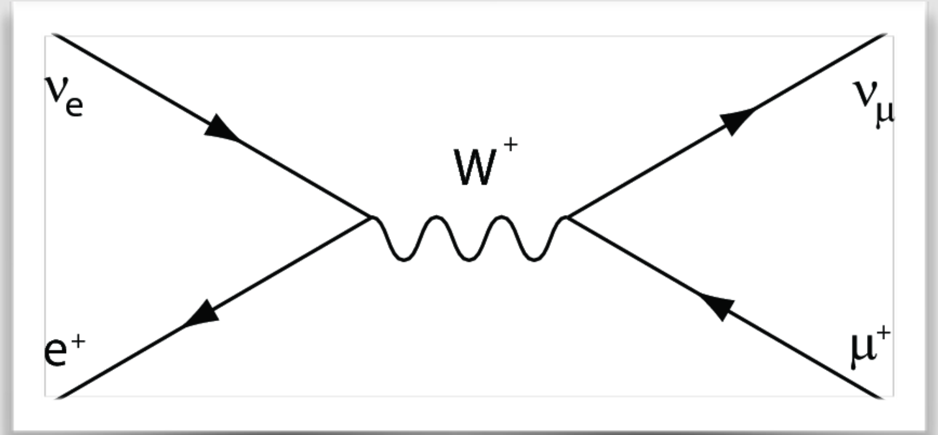
3. **beta-asymmetry observed indicates parity violation (if P conserved, no correlation between electron spin and momentum)**

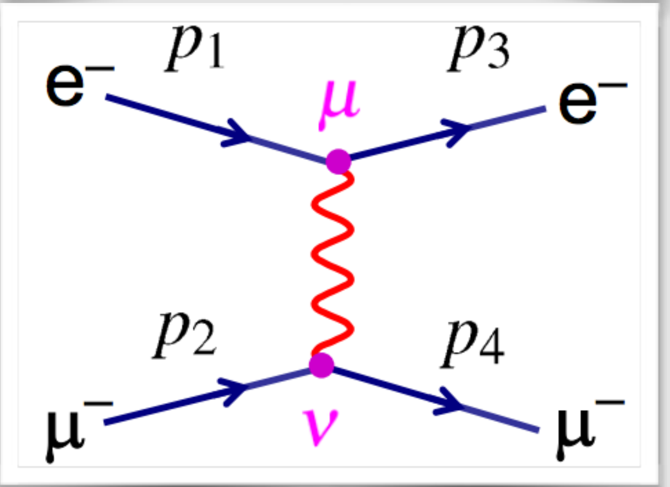


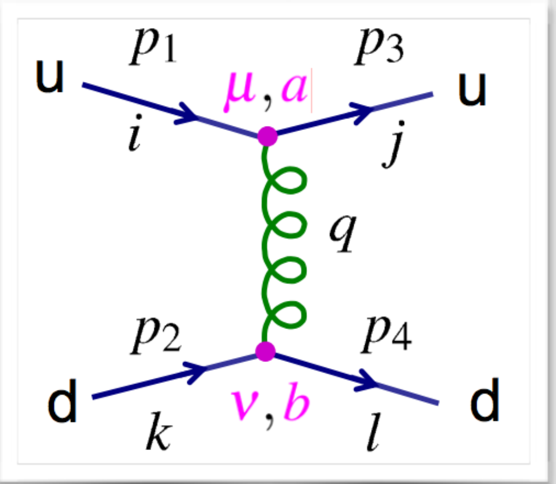
QFD Matrix Elements

Consider electron-neutrino scattering. For now we'll study the t-channel diagram.





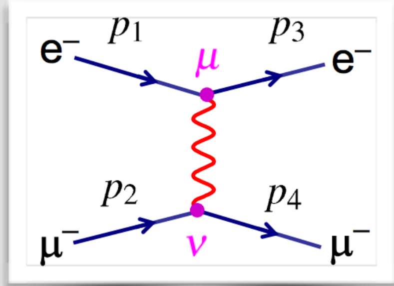




QFD Matrix Elements

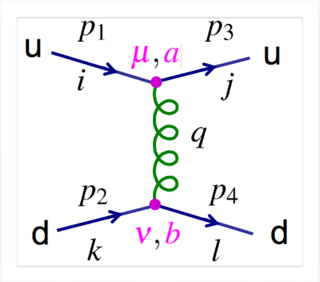
QED Lepton Scattering:

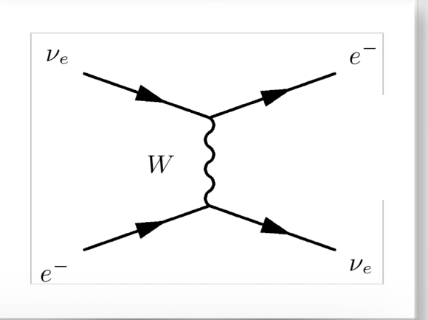
$$\mathcal{M} = -\frac{g_e^2}{q^2}[\bar{u}_e\gamma^\alpha u_e][\bar{u}_\mu\gamma_\beta u_\mu]$$



QCD Quark Scattering:

$$\mathcal{M} = -\sqrt{\frac{2}{9}}\frac{g_s^2}{q^2}[\bar{u}_u\gamma^\mu u_u][\bar{u}_d\gamma_\mu u_d]$$





QFD Lepton-Neutrino Scattering:

$$\mathcal{M} = -\frac{g_w^2}{8}\left(\frac{g_{\mu\nu} - q_\mu q_\nu/M_W^2}{q^2 - M_W^2}\right)[\bar{u}_\nu\gamma^\mu(1-\gamma^5)u_e][\bar{u}_e\gamma^\mu(1-\gamma^5)u_\nu]$$

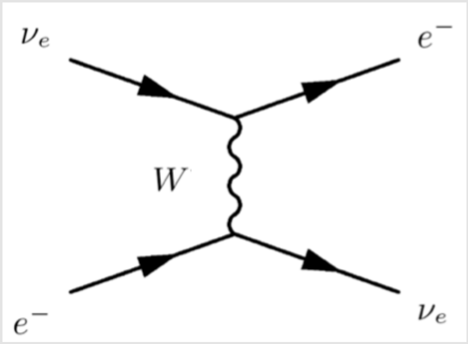
Electron-Neutrino Scattering

Example worked in class.

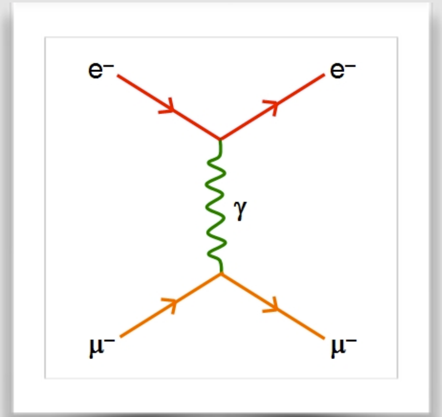
Electron-Neutrino Scattering

$$\sigma(e^-\nu_e \to e^-\nu_e) = \frac{1}{8\pi}\left(\frac{g_w^2 E}{M_W^2}\right)^2$$

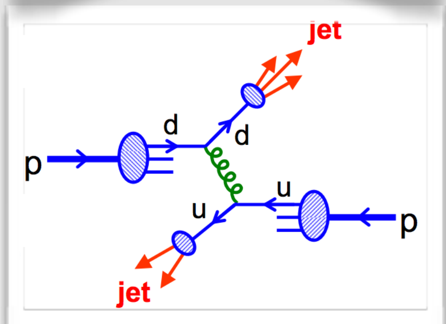
$$\sigma(e^-\nu_e \to e^-\nu_e) = \frac{2\pi\alpha_w^2 E^2}{M_W^4}$$



$$\sigma(e^-\mu^- \to e^-\mu^-) = \frac{4\pi\alpha_{em}^2}{E^4}$$



$$\sigma(ud \to ud) = \frac{2}{9}\frac{4\pi\alpha_s^2}{E^4}$$



Muon Decay

Example worked in class.

Differential Muon Decay Rate

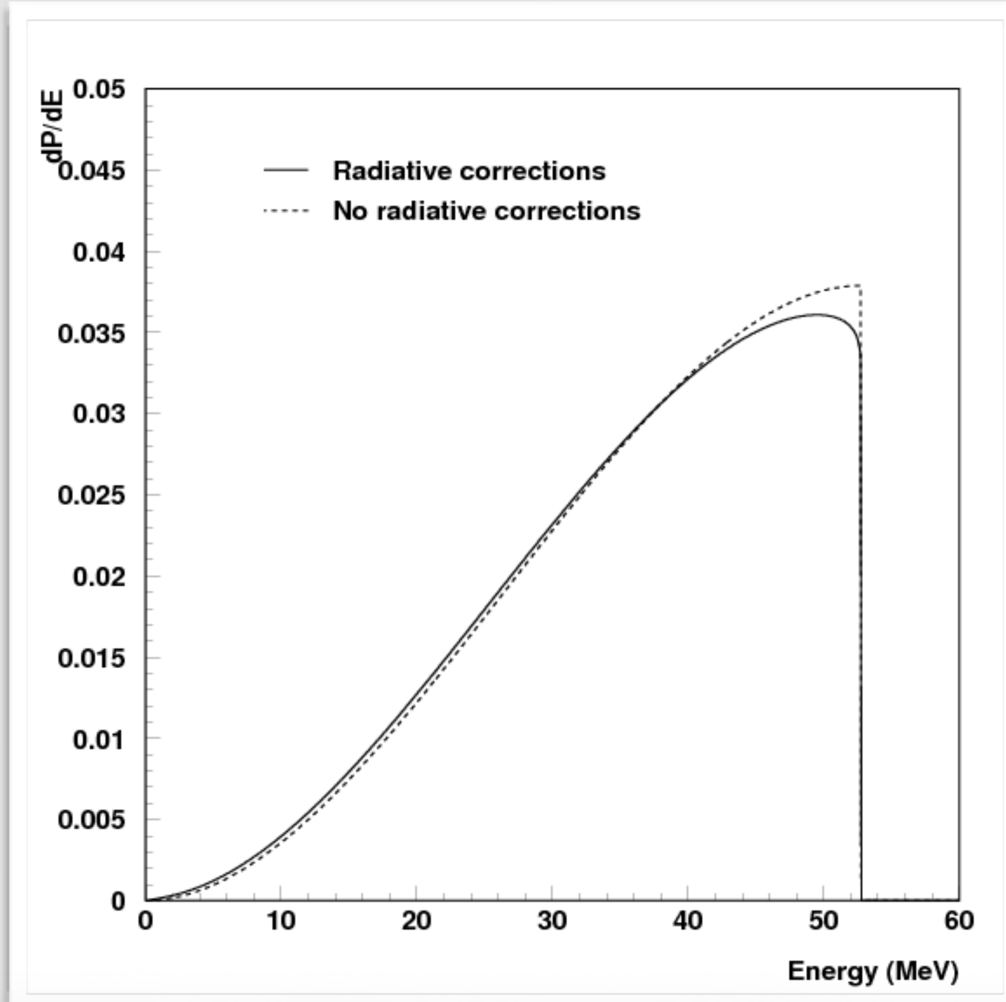
Starting from the in-class example

$$\begin{aligned} d\Gamma_\mu &= \frac{\left\langle |\mathcal{M}|^2 \right\rangle}{(4\pi)^4 m_\mu} \frac{d^3\mathbf{p}_4}{E_4^2}\, dE_2 \\ &= \left(\frac{g_w}{4\pi M_W}\right)^4 \frac{m_\mu\, d^3\mathbf{p}_4}{E_4^2} \int_{m_\mu/2-E_4}^{m_\mu/2} E_2(m_\mu - 2E_2)\, dE_2 \\ &= \left(\frac{g_w}{4\pi M_W}\right)^4 \frac{m_\mu\, d^3\mathbf{p}_4}{E_4^2} \left(\frac{m_\mu E_4^2}{2} - \frac{2E_4^3}{3}\right) \end{aligned}$$

Writing the volume integral in spherical coordinates and integrating over angles, we get:

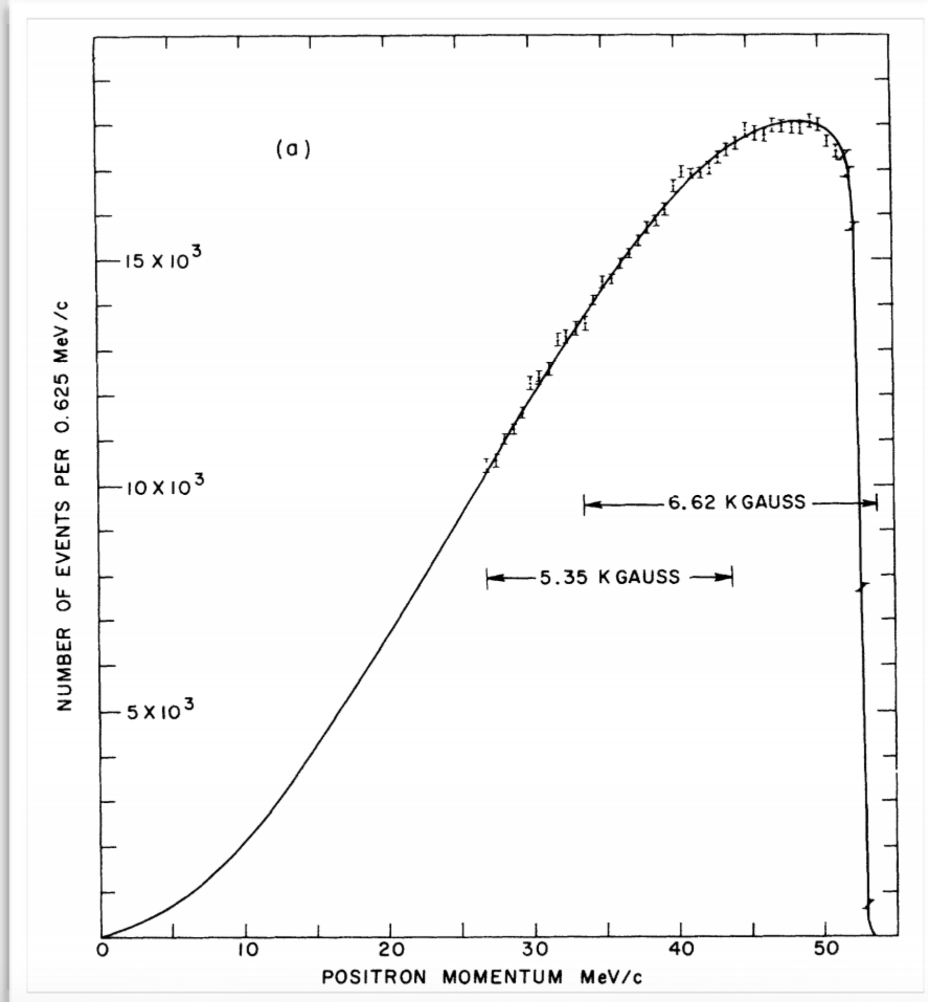
$$\begin{aligned} d\Gamma_\mu &= 4\pi \left(\frac{g_w}{4\pi M_W}\right)^4 m_\mu\, dE_4 \left(\frac{m_\mu E_4^2}{2} - \frac{2E_4^3}{3}\right) \\ \frac{d\Gamma_\mu}{dE} &= \left(\frac{g_w}{M_W}\right)^4 \frac{m_\mu^2 E^2}{2(4\pi)^3} \left(1 - \frac{4E}{3m_\mu}\right) \end{aligned}$$

Electron spectrum from muon decay



$$\frac{d\Gamma_\mu}{dE} \sim E^2\left(1 - \frac{4E}{3m_\mu}\right) \qquad (E \leq m_\mu/2)$$

Electron spectrum from muon decay



$$\frac{d\Gamma_\mu}{dE} \sim E^2 \left(1 - \frac{4E}{3m_\mu}\right) \qquad (E \leq m_\mu/2)$$

Muon Decay Rate

Integrating over the electron energy, we (finally) obtain the muon decay rate:

$$
\begin{aligned}
\Gamma_\mu &= \left(\frac{g_w}{M_W}\right)^4 \frac{m_\mu^2}{2(4\pi)^3} \int_0^{m_\mu/2} E^2 \left(1 - \frac{4E}{3m_\mu}\right) dE \\
&= \left(\frac{g_w}{M_W}\right)^4 \frac{m_\mu^2}{2(4\pi)^3} \left(\frac{m_\mu^3}{48}\right) \\
&= \frac{1}{6144\pi^3} \left(\frac{g_w}{M_W}\right)^4 m_\mu^5
\end{aligned}
$$

$$
\tau_\mu = \frac{1}{\Gamma} = \frac{6144\pi^3}{m_\mu^5} \left(\frac{M_W}{g_w}\right)^4
$$

This calculation yields 2.15 μsec, very close to reality!
τ_{exp} = 2.197 μsec

The Fermi Coupling Constant

In the limit of $q^2 \ll M_W^2$, our results always depend on the *ratio* of g_w and M_W, and not the two constants separately.

$$\tau_\mu = \frac{1}{\Gamma} = \frac{6144\pi^3}{m_\mu^5}\left(\frac{M_W}{g_w}\right)^4$$

We define the Fermi coupling constant, G_F, by:

$$G_F = \frac{\sqrt{2}g_w^2}{8M_W^2}$$

This allows us to write the muon lifetime as:

$$\tau_\mu = \frac{192\pi^3}{G_F^2 m_\mu^5}$$

Using τ_μ and m_μ, we actually determine G_F from this equation:

$$G_F = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

The "Weak" Interaction??

How "weak" is the Weak interaction?

With the muon lifetime measurement giving us

$$G_F = \frac{\sqrt{2} g_w^2}{8 M_W^2} = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

we can use the W mass measurement and $M_W = 80.4$ **GeV** to determine g_w.

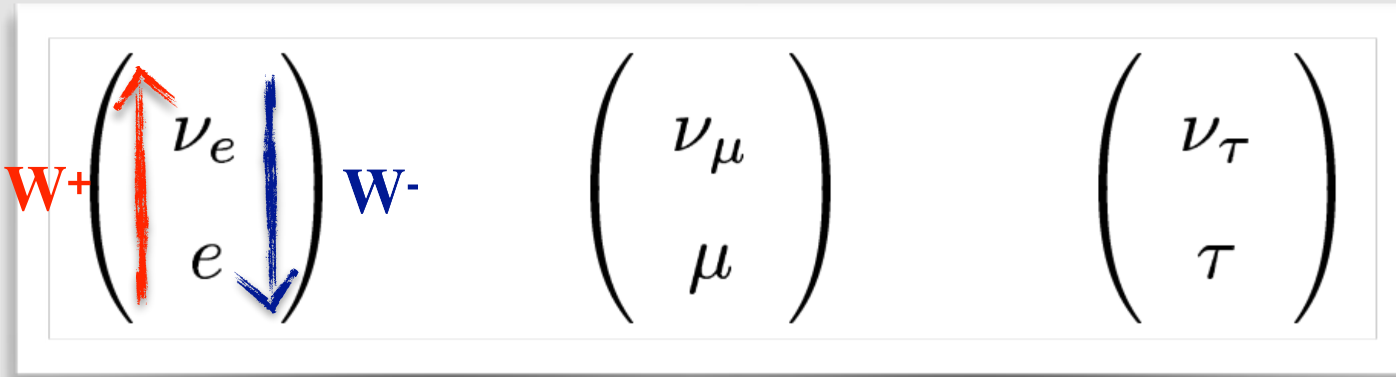
$$g_w = 0.65 \qquad \Rightarrow \alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29}$$

$$\alpha_{em} = \frac{1}{137}$$

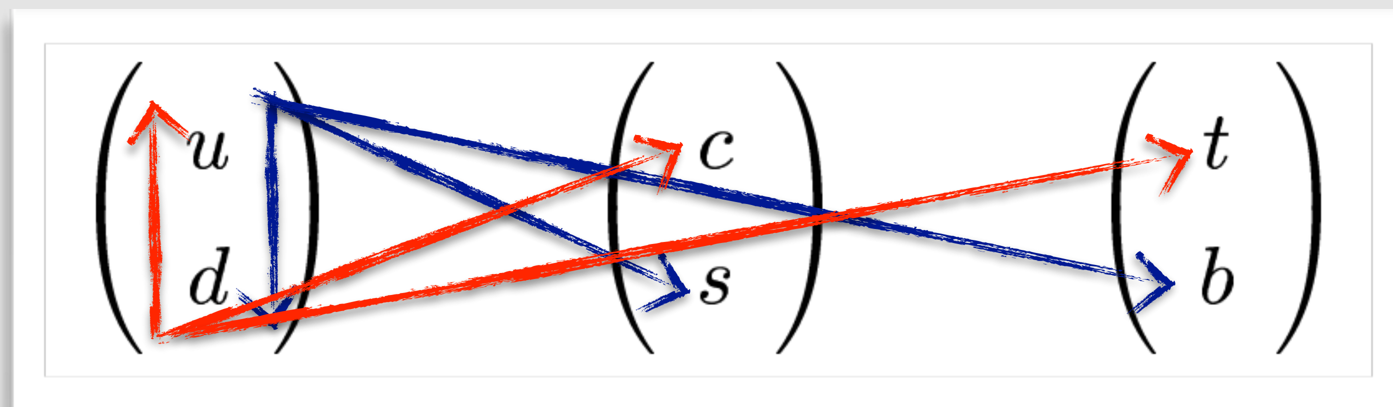
This indicates that the weak interaction is inherently ***stronger*** than the electromagnetic interaction! It is only the **suppression factor** E^2/M_W^2 that makes the weak force seem so feeble.

Quarks in Weak Interactions

For leptons, the W boson couples within a particular generation:

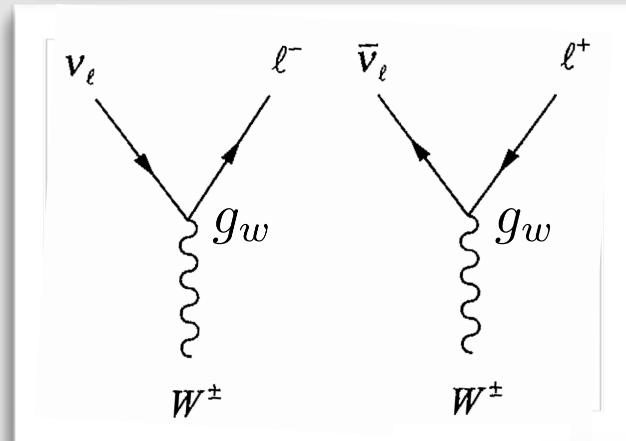


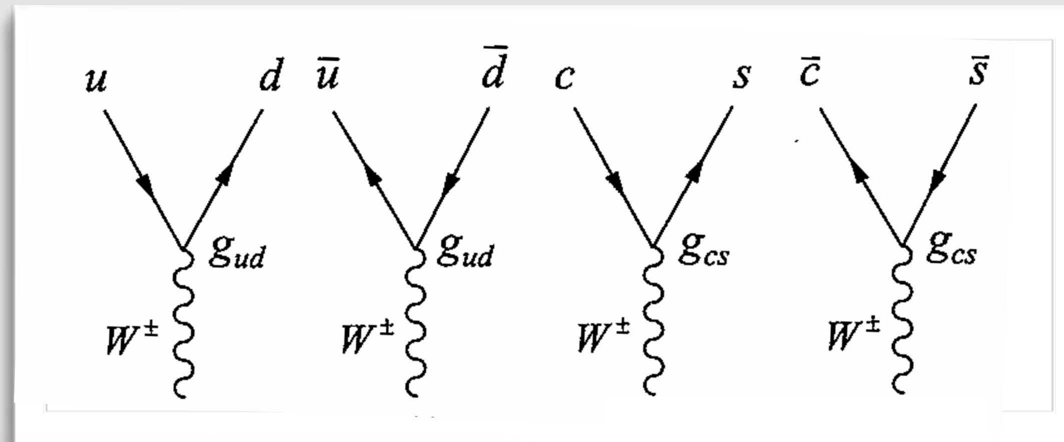
Things are more complicated for quarks, as the W boson couplings can mix generations:



Quark Interactions

A lepton-quark symmetry, if valid, would assume each quark and lepton generation should have identical weak coupling constants.





$$g_w = g_{e\nu} = g_{ud} = g_{cs}$$

But in a simple scheme like this, certain decays would be forbidden. Consider K and pion decays to muons. We observe both (!!), though the K^- decays are suppressed.

$$K^-(\bar{u}s) \rightarrow \mu + \bar{\nu}_\mu$$

$$\pi^-(\bar{u}d) \rightarrow \mu + \bar{\nu}_\mu$$

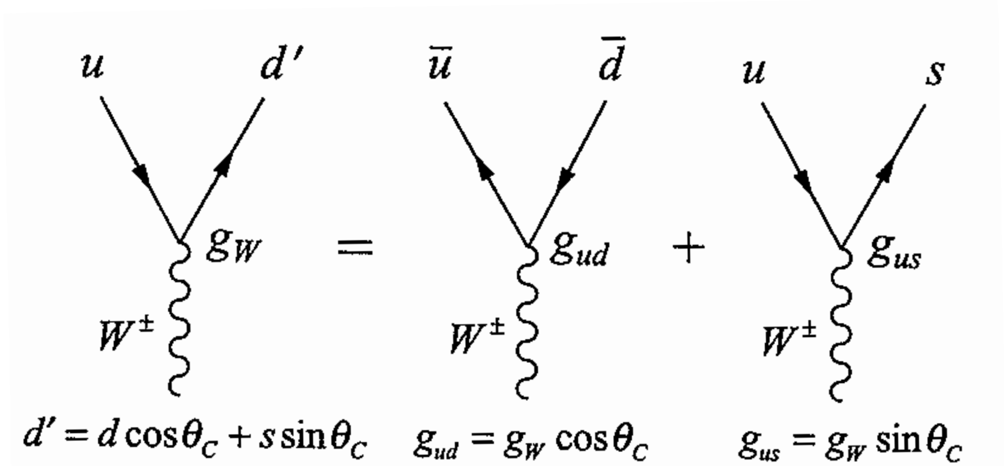
Quark Mixing

This can be explained by "Quark Flavor Mixing".

Basic idea: quark flavor/mass eigenstates are not weak eigenstates. The weak eigenstates are linear combinations of flavor eigenstates.

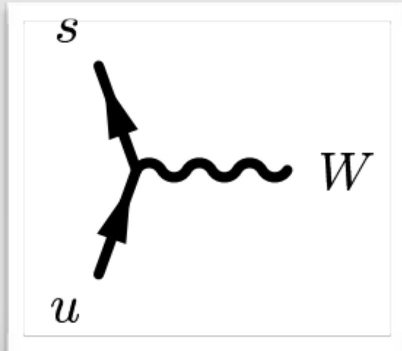
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

θ_c : referred to as the Cabibbo angle, after it's proposer.

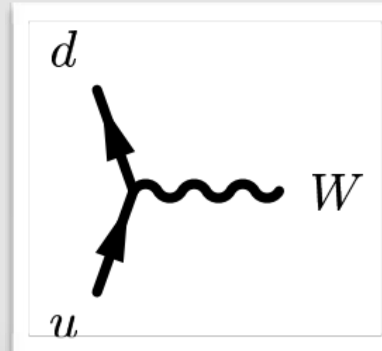


The Cabibbo Angle

View this as a basic modification to the vertex factor, not the coupling strength.



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)sin\theta_C$$



$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)cos\theta_C$$

$$\frac{\Gamma(K^- \to \mu^- + \bar{\nu}_\mu)}{\Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu)} \propto \frac{\sin^2\theta_c}{\cos^2\theta_c} \sim \frac{0.05}{0.95}$$

<u>Note:</u> The choice of down-type quarks for the mixing is arbitrary. We could just as easily have chosen the up-type quarks with the same results.

The CKM Matrix

This idea, as applied to the K/pi mystery, predicted the existence of a fourth quark: the charm quark.

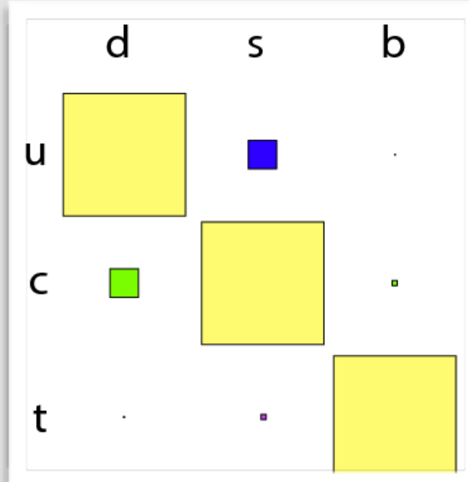
The charm quark was duly discovered shortly thereafter.

We now understand that all three quark generations mix with one another.

Described by the Cabibbo, Kobayashi, Masukawa (CKM) matrix.

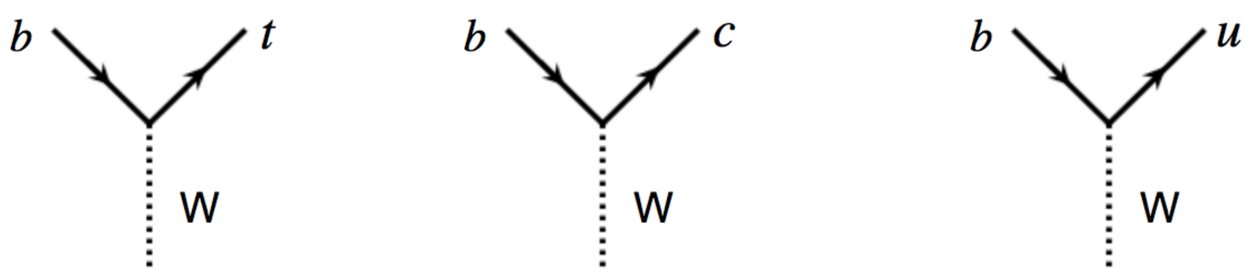
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$



The CKM Matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

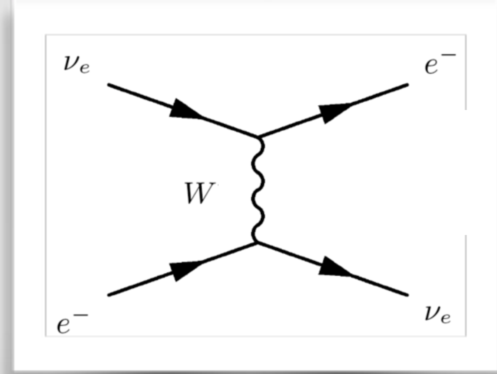


$V_{qq'} = V_{tb}$ $\quad$ $V_{qq'} = V_{cb}$ $\quad$ $V_{qq'} = V_{ub}$

Vertex Factor: $-\frac{g_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)V_{qq'}$

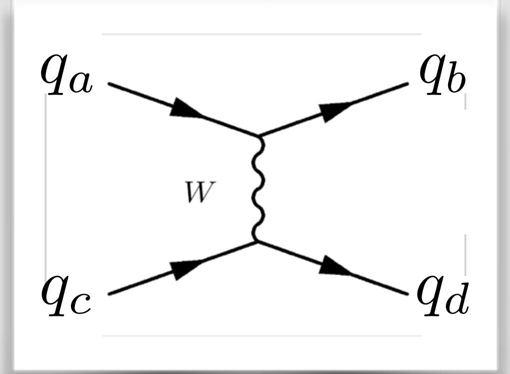
$$\langle|\mathcal{M}|^2\rangle \propto |V_{q_a q_a'}|^2\,|V_{q_b q_b'}|^2$$

W Boson Matrix Elements



QFD Lepton-Neutrino Scattering:

$$\mathcal{M} = -\frac{g_w^2}{8}\left(\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}\right)[\bar{u}_\nu \gamma^\mu (1-\gamma^5) u_e][\bar{u}_e \gamma^\mu (1-\gamma^5) u_\nu]$$

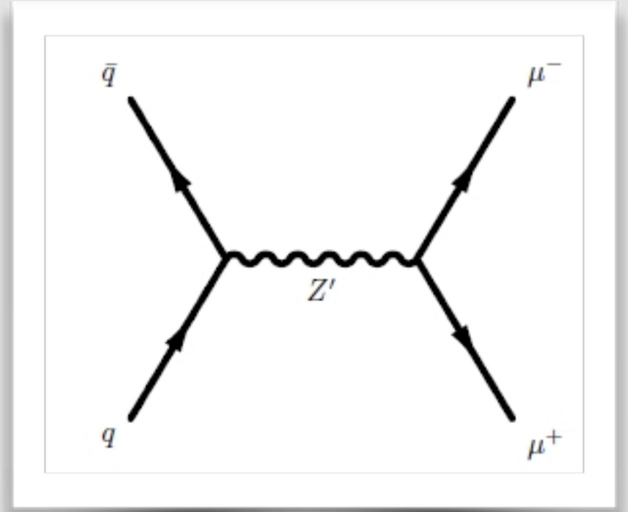


QFD Quark-Quark Scattering:

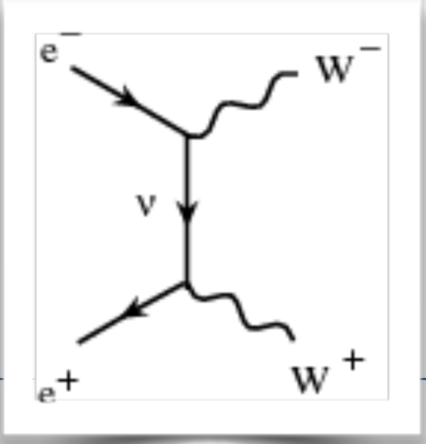
$$\mathcal{M} = -\frac{g_w^2 V_{ab} V_{cd}}{8}\left(\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}\right)[\bar{u}_b \gamma^\mu (1-\gamma^5) u_a][\bar{u}_d \gamma^\mu (1-\gamma^5) u_c]$$

Neutral Weak Interactions

- In the 1960s, there was no compelling experimental evidence for neutral weak currents.
- Theoretically, Fermi's four-fermion theory of the weak interaction suggested charged weak currents, but there was no neutral current analogue.
- Why, then, would we want to invent a particle without any experimental or theoretical justification?
- It turns out there was a subtle theoretical justification based on considering what happens at very high energies

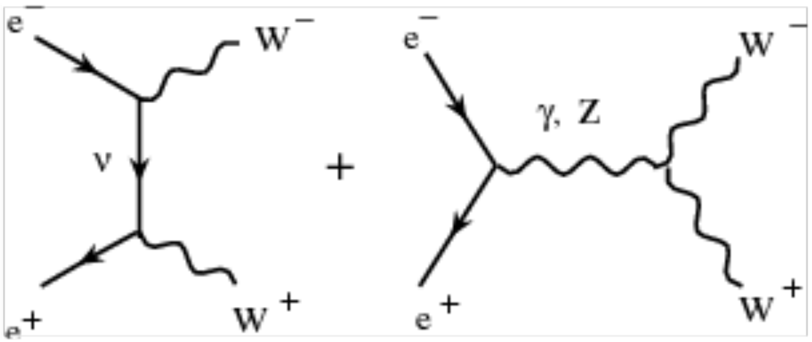


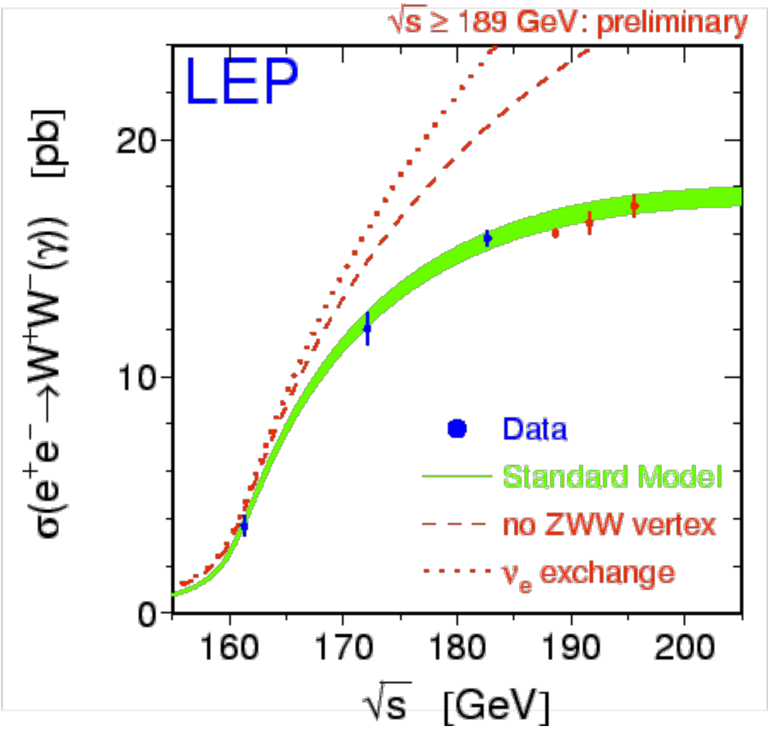
The problem arises when you consider the following diagram. The probability for the ee→WW process to occur surpasses unity if we assume that the it proceeds in this way.



Neutral Weak Interactions

In order to make the weak interaction self consistent, we require two additional contributions to the ee→WW scattering process:





Eventually, experimental evidence showed that the violation of unitarity didn't occur. The day was saved!

But we require a Z boson to allow this cancellation to happen.

The Weak Mixing Angle

The W and Z bosons are closely related, but not identical.

They are both massive, spin-1 bosons.

They both couple to weak hypercharge.

And their properties are linked via the Weak Mixing Angle.

We will explore the origin of the weak mixing angle later, but for now we can introduce the relationships. For starters,

$$M_W = M_Z \cos\theta_w$$

where θ_w is the weak mixing angle, also known as the Weinberg angle. This "mixing" arises from the *Glashow-Weinberg-Salam* (GWS) theory.

Experimentally, we've found:

$$\sin^2\theta_w(M_Z) = 0.23120(15)$$

The Weak Mixing Angle

The W and Z bosons are closely related, but not identical.

They are both massive, spin-1 bosons.
They both couple to weak hypercharge.
And their properties are linked via the Weak Mixing Angle.

The vertex factor for interactions with the Z boson will involve a coupling constant g_W. Just as the W and Z masses are related by the Weinberg angle, so are the coupling constants:

$$g_z = g_w / \cos\theta_w$$

It gets better. Both g_W and g_Z are related to the QED coupling constant g_e:

$$g_w = \frac{g_e}{\sin\theta_w} \qquad g_z = \frac{g_e}{\sin\theta_w \cos\theta_w}$$

This is why the weak force is inherently stronger than the electromagnetic force.

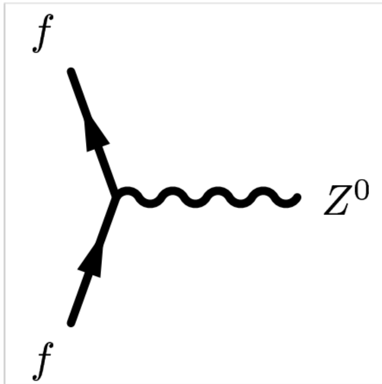
Feynman Rules for the Z

The Z boson propagator looks just like that of the W boson:

$$\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{q^2 - M_Z^2}$$

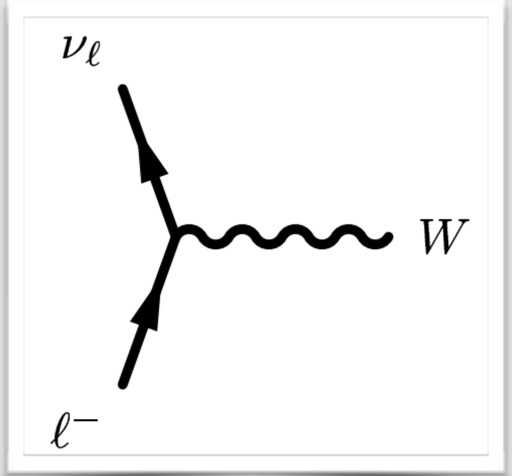
The Z bosons mediate neutral current (NC) weak interactions. They couple to fermions via the following vertex factor:

$$\frac{-ig_z}{2}\gamma^\mu(c_V - c_A\gamma^5)$$

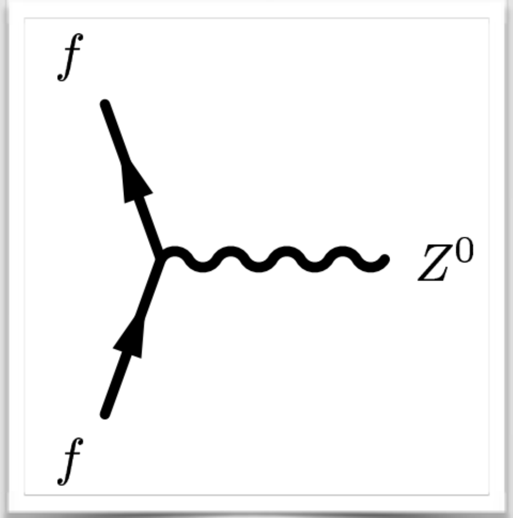


W vs Z: Vertex Factors

$$\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$



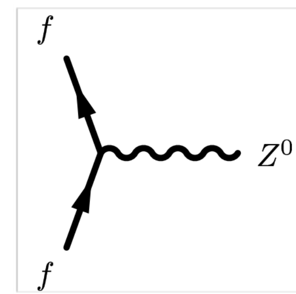
$$\frac{-ig_z}{2}\gamma^\mu(c_V - c_A\gamma^5)$$



Fermion Couplings to the Z

The vector and axial couplings c_V and c_A are specified by the GWS model:

f	c_V	c_A
ν_ℓ	$+\frac{1}{2}$	$+\frac{1}{2}$
ℓ^-	$-\frac{1}{2}+2\sin^2\theta_w$	$-\frac{1}{2}$
q_u	$+\frac{1}{2}-\frac{4}{3}\sin^2\theta_w$	$+\frac{1}{2}$
q_d	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

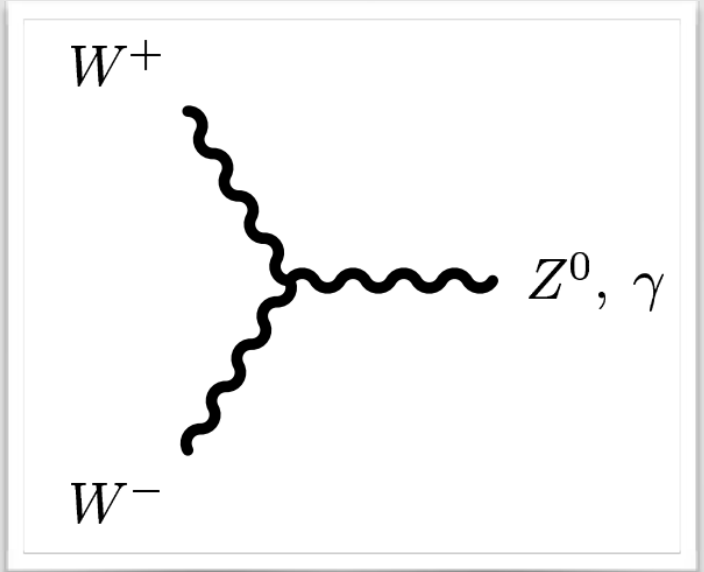


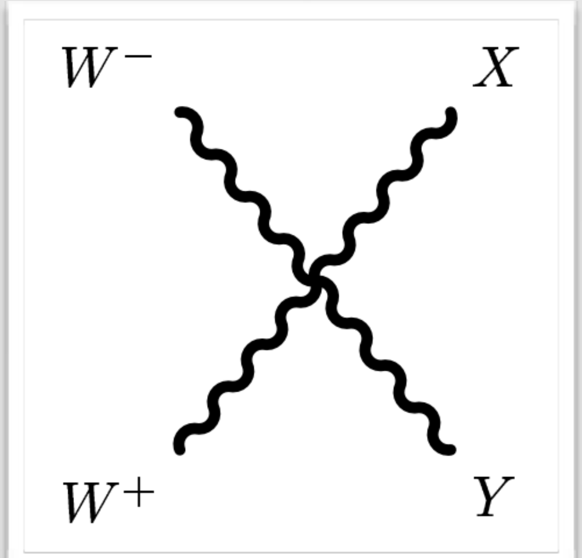
$$\frac{-ig_z}{2}\gamma^\mu(c_V - c_A\gamma^5)$$

The Z boson does not change the lepton or quark flavor. The SM has no flavor-changing neutral currents (FCNC) at tree level.

Gauge Boson Self Couplings

Just as we saw with gluons in QCD, the electroweak bosons carry (weak interaction) charge and can interact with each other:





where (X,Y) can be (γ,γ), (γ,Z), (W,W) or (Z,Z).
Consult Appendix D of Griffiths for the appropriate vertex factors.

Photon vs Z boson

The Z boson couples to every charged fermion, just like the photon does.

$$Z\,/\,\gamma \to f\,\overline{f}$$

This made it difficult to detect the Z boson because at low energies, the QED effects dominate. Nevertheless, there are always small weak effects in otherwise electromagnetic systems (e.g. atomic parity violation).

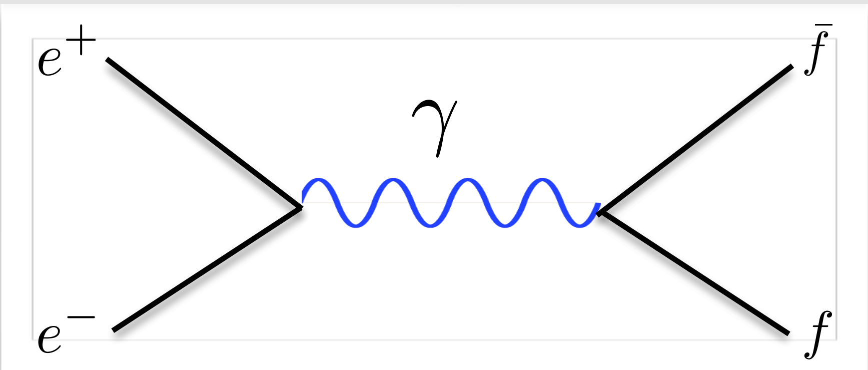
Unlike the photon, the Z boson also couples to neutrinos.

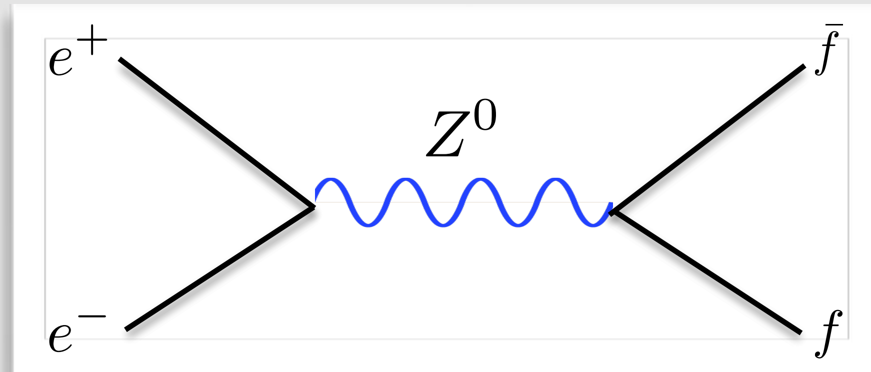
$$Z \to \nu\,\overline{\nu}$$

Neutrino experiments are never easy, but at least they allow us to isolate the weak interaction.

Photon vs Z boson: Example

We first considered this interaction in the context of extending QED in order to predict hadron production rates. Now we would like to see how the Z-mediated s-channel diagram compares to the corresponding γ-mediated diagram:





$$\sigma(e^+e^- \to \gamma \to \mu^+\mu^-) = \frac{\alpha_{em}^2 \pi}{3E^2}$$

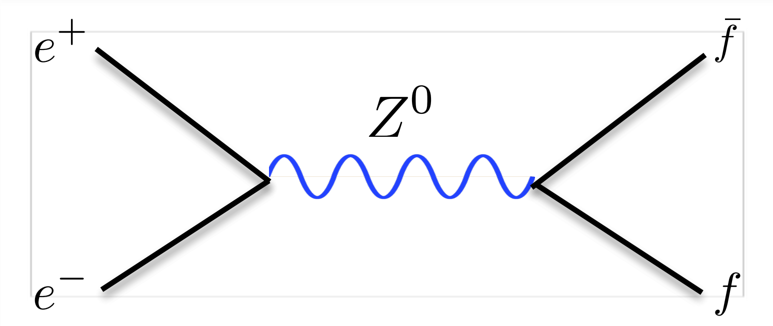
$$\sigma(e^+e^- \to Z^0 \to \mu^+\mu^-) = ??$$

The Scattering Amplitude

The amplitude is:

$$\mathcal{M} = i\left[\bar{u}_4\left(\frac{-ig_z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)\right)v_3\right]\left[\frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right)}{q^2 - M_Z^2}\right]$$

$$\times\left[\bar{v}_2\left(\frac{-ig_z}{2}\gamma^\nu(c_V^e - c_A^e\gamma^5)\right)u_1\right]$$



At low energies, $q^2 \ll M_Z^2$, and we would eventually find that, up to some factors of c_V, c_A, and $\sin^2\theta_w$, the Z-mediated diagram would be like the QED diagram only with α replaced by $G_F E^2$.

The Scattering Amplitude

If q^2 is not small, we can no longer simplify the Z-propagator term. Keeping the full propagator we have:

$$\mathcal{M} = -\frac{g_z^2}{4(q^2 - M_Z^2)} \left[\bar{u}_4 \gamma^\mu (c_V^f - c_A^f \gamma^5) v_3\right] \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right) \times \left[\bar{v}_2 \gamma^\nu (c_V^e - c_A^e \gamma^5) u_1\right]$$

Assuming that we can neglect all fermion masses:

$$q_\mu q_\nu = (p_1 + p_2)_\mu (p_1 + p_2)_\nu$$

$$\gamma^\mu (q_\mu q_\nu) \gamma^\nu = (\not{p}_1 + \not{p}_2)(\not{p}_1 + \not{p}_2)$$

$$\bar{u}_4 (\not{p}_1 + \not{p}_2)(\not{p}_1 + \not{p}_2) v_3 \Rightarrow (\bar{u}_4 \not{p}_1 X + \cdots + X \not{p}_2 v_3)$$

Recall the Dirac Eqn:

$$\bar{u}(\not{p} - m) = 0 \quad (\not{p} + m) v = 0$$

Thus, for massless fermions:

$$q_\mu q_\nu = 0$$

The Scattering Amplitude

$$\mathcal{M} = -\frac{g_z^2}{4(q^2 - M_Z^2)} \left[\bar{u}_4 \gamma^\mu (c_V^f - c_A^f \gamma^5) v_3\right] \times \left[\bar{v}_2 \gamma_\mu (c_V^e - c_A^e \gamma^5) u_1\right]$$

$$\left\langle |\mathcal{M}|^2 \right\rangle = \left[\frac{g_z^2}{8(q^2 - M_Z^2)}\right]^2 \mathrm{Tr}\left[\gamma^\mu (c_V^f - c_A^f \gamma^5) \not{p}_3 \gamma^\nu (c_V^f - c_A^f \gamma^5) \not{p}_4\right] \times \mathrm{Tr}\left[\gamma_\mu (c_V^e - c_A^e \gamma^5) \not{p}_1 \gamma_\nu (c_V^e - c_A^e \gamma^5) \not{p}_2\right]$$

The traces are best evaluated by first bringing the c_V and c_A terms together:

$$\begin{aligned}(c_V - c_A\gamma^5)\not{p}_3\gamma^\nu(c_V - c_A\gamma^5) &= (c_V - c_A\gamma^5)^2 \not{p}_3\gamma^\nu \\ &= (c_V^2 + c_A^2)\not{p}_3\gamma^\nu - 2c_V c_A \gamma^5 \not{p}_3 \gamma^\nu\end{aligned}$$

The Scattering Amplitude

A bit more algebra and one can show that after taking the traces, writing the momenta in terms of E and $\sin\theta$, and then using Fermi's Golden Rule, that the cross section for Z-mediated $e^+e^- \to f\bar{f}$ is

$$\sigma = \frac{1}{3\pi}\left(\frac{g_z^2 E}{4[(2E)^2 - M_Z^2]}\right)^2 [(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2]$$

As it stands, it looks like this cross section blows up when $E = M_Z/2$. This is much more serious than the infinite cross section for Rutherford scattering because this divergence can be traced all the way back to the amplitude.

$$\mathcal{M} = -\frac{g_z^2}{4(q^2 - M_Z^2)}\left[\bar{u}_4\gamma^\mu(c_V^f - c_A^f\gamma^5)v_3\right]\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_Z^2}\right) \times \left[\bar{v}_2\gamma^\nu(c_V^e - c_A^e\gamma^5)u_1\right]$$

An Aside….

Let's consider branching fractions for weak boson decays, W and Z.

Following the book or in-class derivation for 2-particle decays (1→2,3):

$$\Gamma = \frac{S\,|p|}{8\pi\hbar\, m_1^2 c}|\mathcal{M}|^2$$

The branching fraction is the ratio of the partial decay rate for a given final state to the total decay rate for the particle.

$$BR(X \to ab) = \frac{\Gamma_{ab}}{\sum \Gamma_i}$$

In the limit that differences in kinematics don't matter (ie, all decay products have the same mass, etc) we have:

$$BR(X \to ab) = \frac{|\mathcal{M}_{ab}|^2}{\sum |\mathcal{M}_i|^2}$$

An Aside….

In the limit that differences in kinematics don't matter (ie, all decay products have the same mass, etc) we have:

$$BR(X \to ab) = \frac{|\mathcal{M}_{ab}|^2}{\sum |\mathcal{M}_i|^2}$$

$$|\mathcal{M}(W \to \ell\nu)|^2 \propto g_w^2$$

$$|\mathcal{M}(W \to qq')|^2 \propto g_w^2 |V_{qq'}|^2$$

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

In the approximation that Vab~1, then we just need to count up the ways a W boson can decay: eν, μν, τν, (ud, cs)**x3** = 9 total

x3 for color states!

$$BR(W \to ab) = 1/9$$

An Aside….

In the limit that differences in kinematics don't matter (ie, all decay products have the same mass, etc) we have:

$$BR(X \to ab) = \frac{|\mathcal{M}_{ab}|^2}{\sum |\mathcal{M}_i|^2}$$

$$|\mathcal{M}(Z \to f\bar{f})|^2 \propto g_w^2[(c_V^f)^2 + (c_A^f)^2]$$

$$BR(Z \to f\bar{f}) = \frac{(c_V^f)^2 + (c_A^f)^2}{\sum_i (c_V^i)^2 + (c_A^i)^2}$$

f	c_V	c_A
ν_ℓ	$+\frac{1}{2}$	$+\frac{1}{2}$
ℓ^-	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
q_u	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$+\frac{1}{2}$
q_d	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

An Aside…

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q_u	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$+\frac{1}{2}$
q_d	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

Fermion	N=Flavors x Colors	$F = c_V^2 + c_A^2$	NxF	BR
Neutrino	3	0.5	1.5	0.205
Charged Lepton	3	0.251	0.75	0.103
Up-type Quark	2x3	0.287	1.72	0.236
Down-type Quark	3x3	0.370	3.33	0.456
		Total:	7.30	

Unstable Particles

$$\sigma = \frac{1}{3\pi}\left(\frac{g_z^2 E}{4[(2E)^2 - M_Z^2]}\right)^2 [(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2]$$

The source of the "Z pole" problem is that the kinematics are such that is a physically allowable process even without a subsequent decay to $f\bar{f}$.

To fix this we need to modify the Z-propagator in order to account for the instability of the Z boson. Here's what we do:

1. We recall the familiar configuration-space wavefunction of a stable particle:

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt}$$

Unstable Particles

1. We recall the familiar configuration-space wavefunction of a stable particle:

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iEt}$$

2. Since the particle is stable, the probability of finding the particle somewhere is always equal to 1 since the wavefunction is normalized:

$$P(t) = \int |\Psi|^2 \, d^3\mathbf{r} = 1$$

Unstable Particles

3. If the particle is unstable, we expect the probability of finding the particle to fall off with time according to the decay rate Γ

$$P(t) = \int |\Psi|^2 \, d^3\mathbf{r} = e^{-\Gamma t}$$

4. In the particle rest frame, this means that $\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-iMt - \frac{\Gamma t}{2}}$

5. We then apply the substitution $M \to M - i\Gamma/2$ to the propagator of an unstable particle and assume that Γ is sufficiently small that we can neglect the Γ^2 term:

$$\frac{1}{q^2 - M^2} \quad \to \quad \frac{1}{q^2 - (M - i\Gamma/2)^2}$$

$$\simeq \quad \frac{1}{q^2 - M^2 + iM\Gamma}$$

Back to the Z Peak

With the modification to the Z propagator,

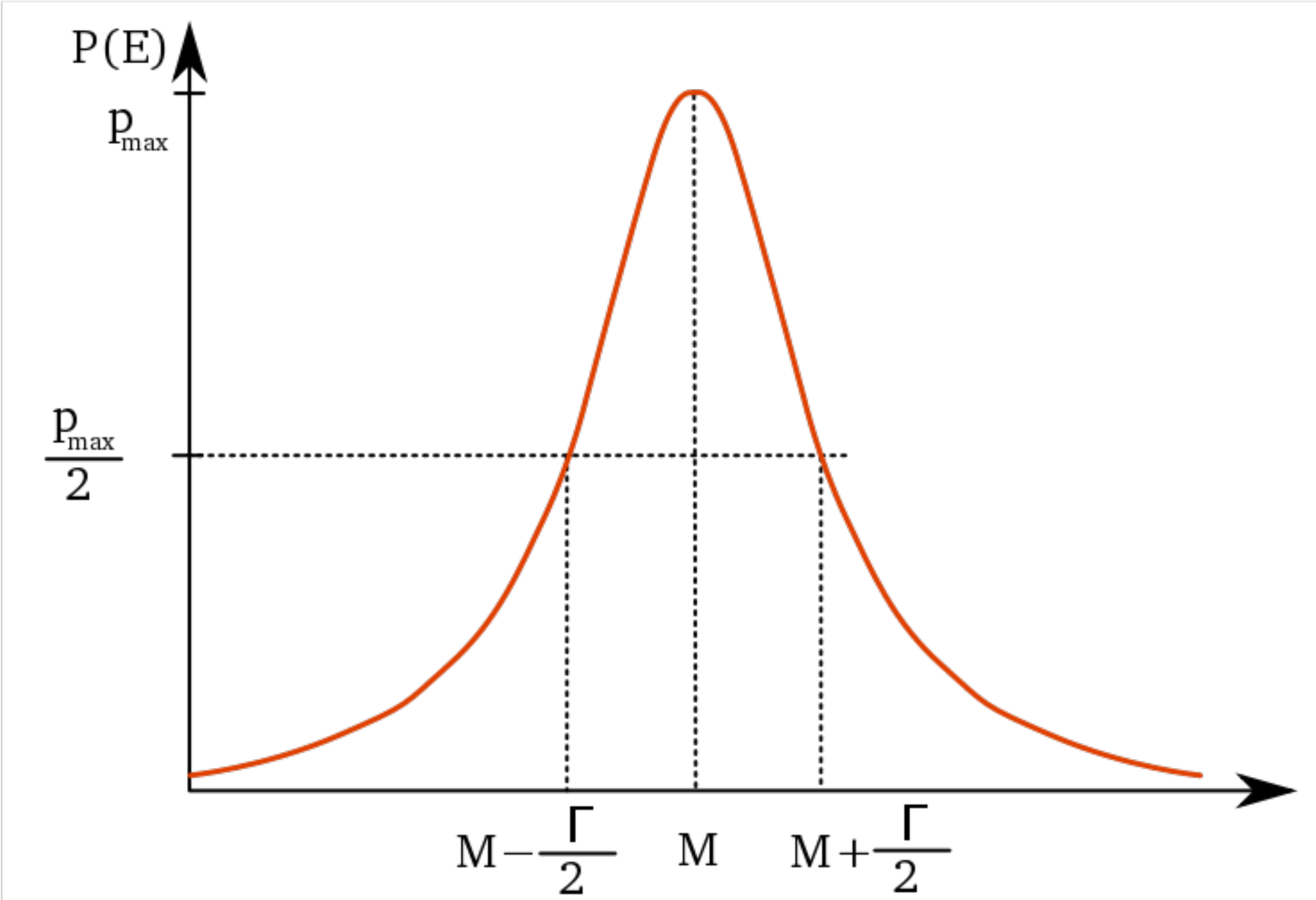
$$\frac{1}{q^2 - M_Z^2} \quad \rightarrow \quad \frac{1}{q^2 - M_Z^2 + iM_Z\Gamma_Z}$$

the cross section takes the form

$$\sigma \quad \sim \quad \frac{1}{[(2E)^2 - M_Z^2]^2 + (M_Z\Gamma_Z)^2}$$

This is known as a Breit-Wigner resonance. Both the height and width of the resonance peak are determined by the decay width Γ_Z.

Breit-Wigner Resonance



Z Boson Peak Measurement

Measurement of the $e^+e^- \rightarrow \mu^+\mu^-$ cross section with center-of-momentum energies near the Z mass.

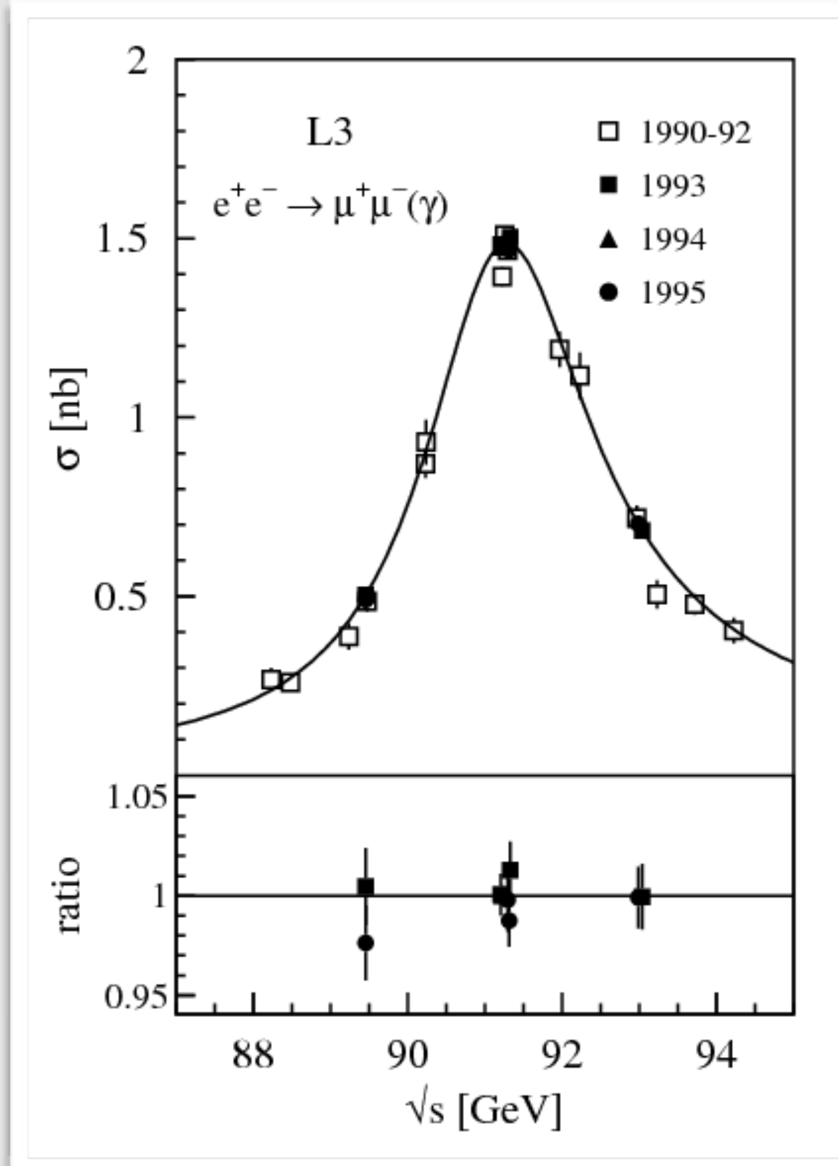
Why is this measurement not ideal to determine the Z decay width (Γ_Z)?

While QED dominates at low energies

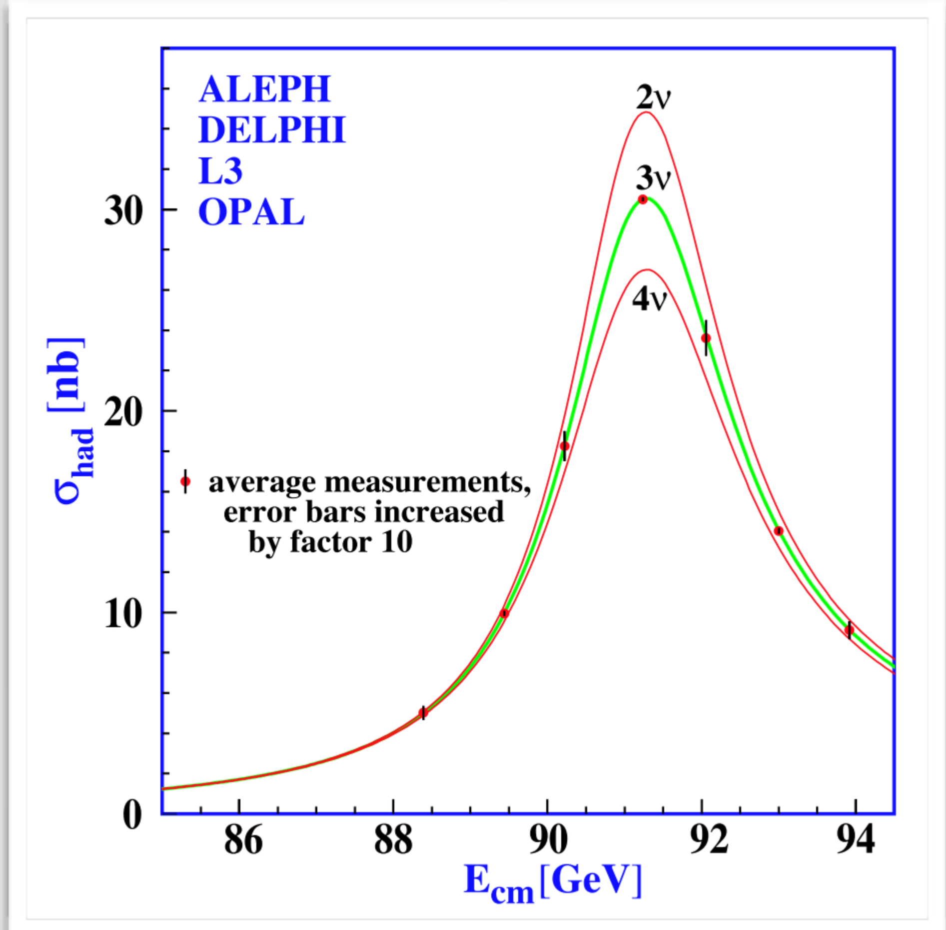
$$\frac{\sigma_Z}{\sigma_\gamma} \simeq 2\left(\frac{E}{M_Z}\right)^4$$

The Z-mediated process dominates at the peak, but there is still a small photon contamination.

$$\frac{\sigma_Z}{\sigma_\gamma} \simeq \frac{1}{8}\left(\frac{M_Z}{\Gamma_Z}\right)^2 \simeq 200$$



Z Peak Using Neutrinos



The Z can decay into neutrinos with each neutrino species contributing to the total width.

Thus a measurement of the Z width to neutrinos tells us about the number of neutrino generations.

The Z Peak at LEP

Precise measurements of electroweak parameters (M_W, M_Z, and $\sin^2\theta_w$) also shed light on other Standard Model parameters such as m_t and m_H.

In the early days at LEP (started in 1989), a number of unusual systematic effects needed to be accounted for in order to measure these parameters accurately:

1. Tidal distortions of the ring

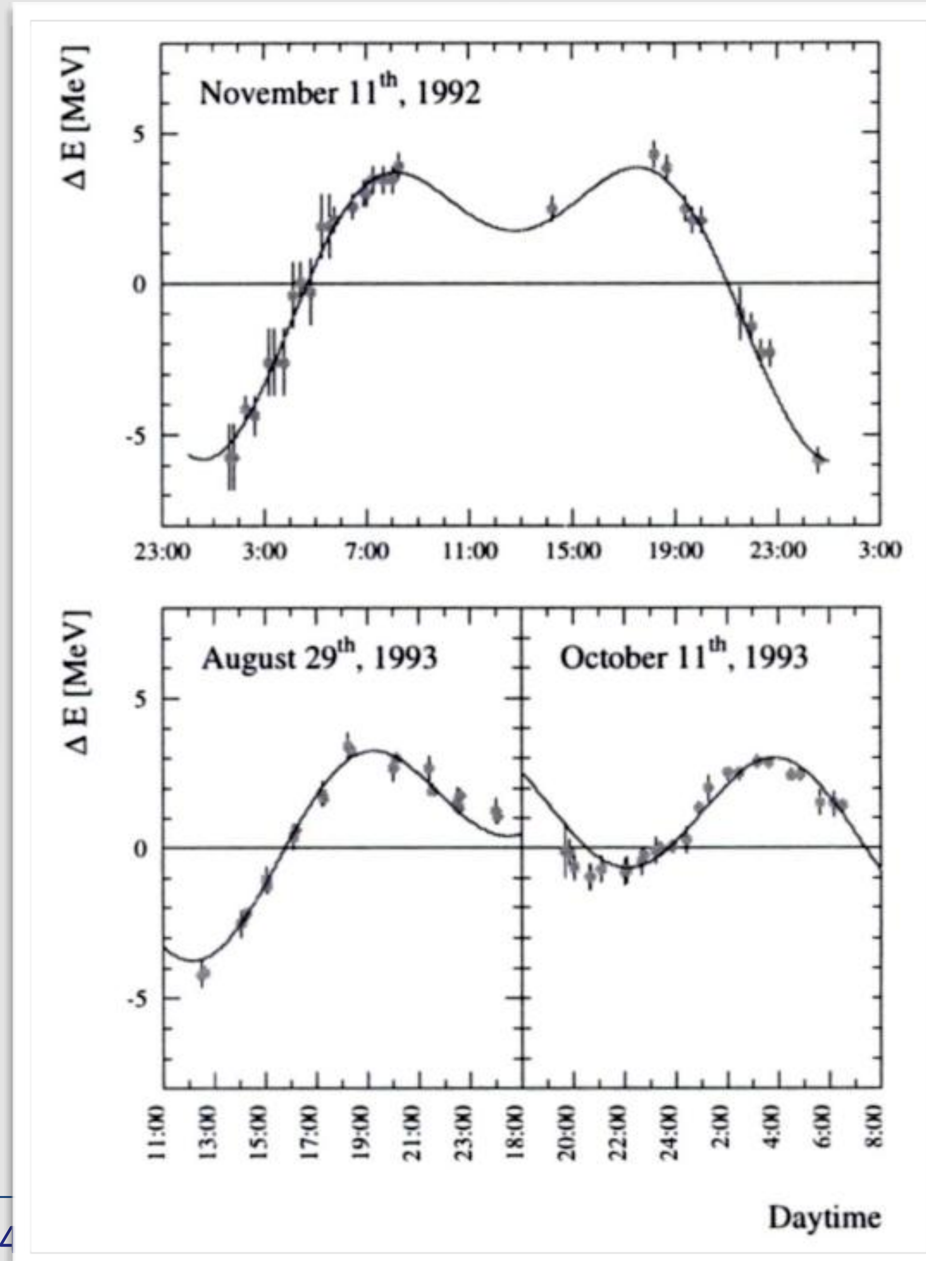
2. Water levels in nearby Lake Geneva

3. Correlations with the TGV

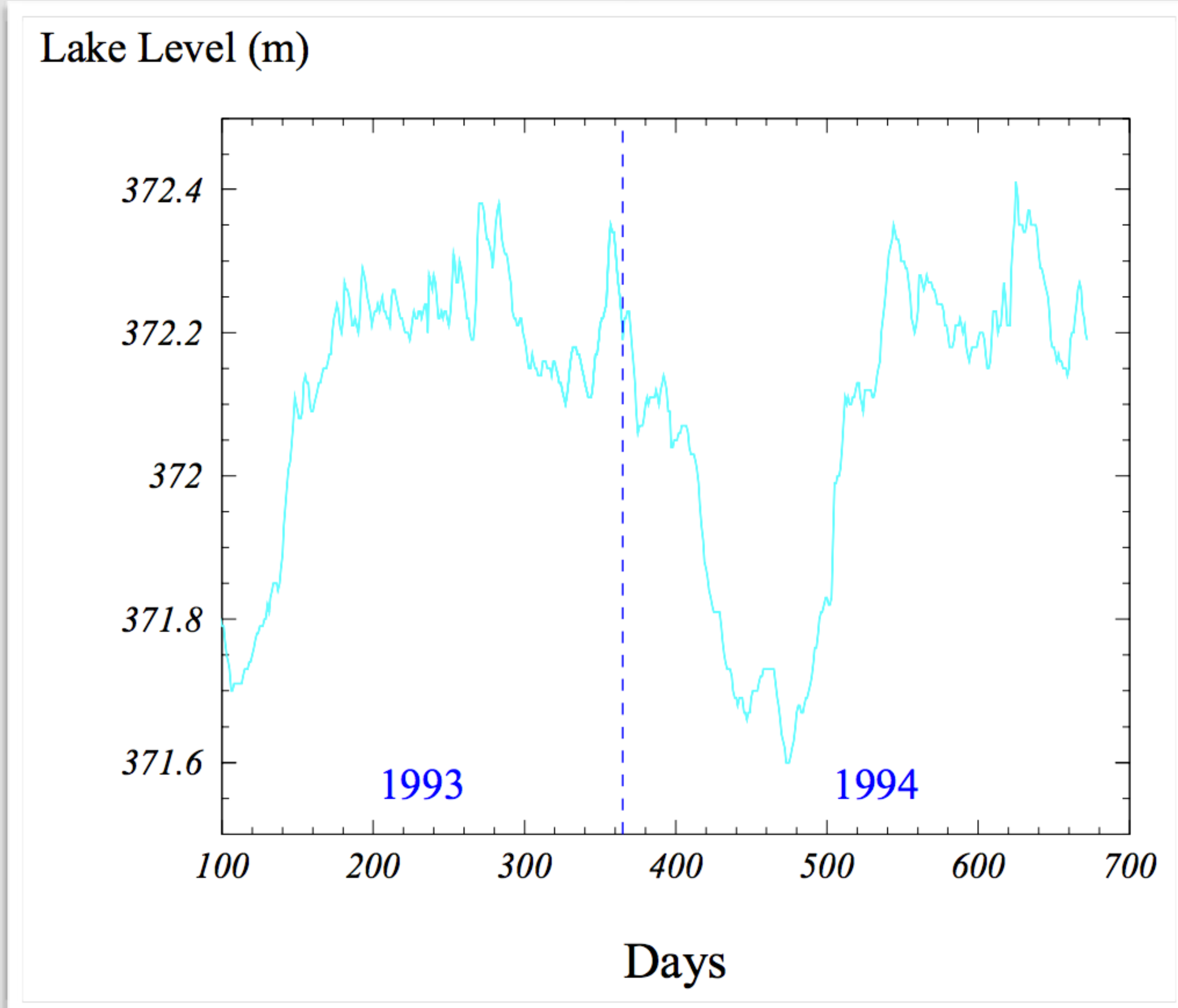
LEP Ring Tidal Distortions

Change in total beam energy (MeV) due to the moon's tidal force acting on the LEP collider ring.

Total beam energy is ~100 GeV (100,000 MeV), making this a 0.005% effect.



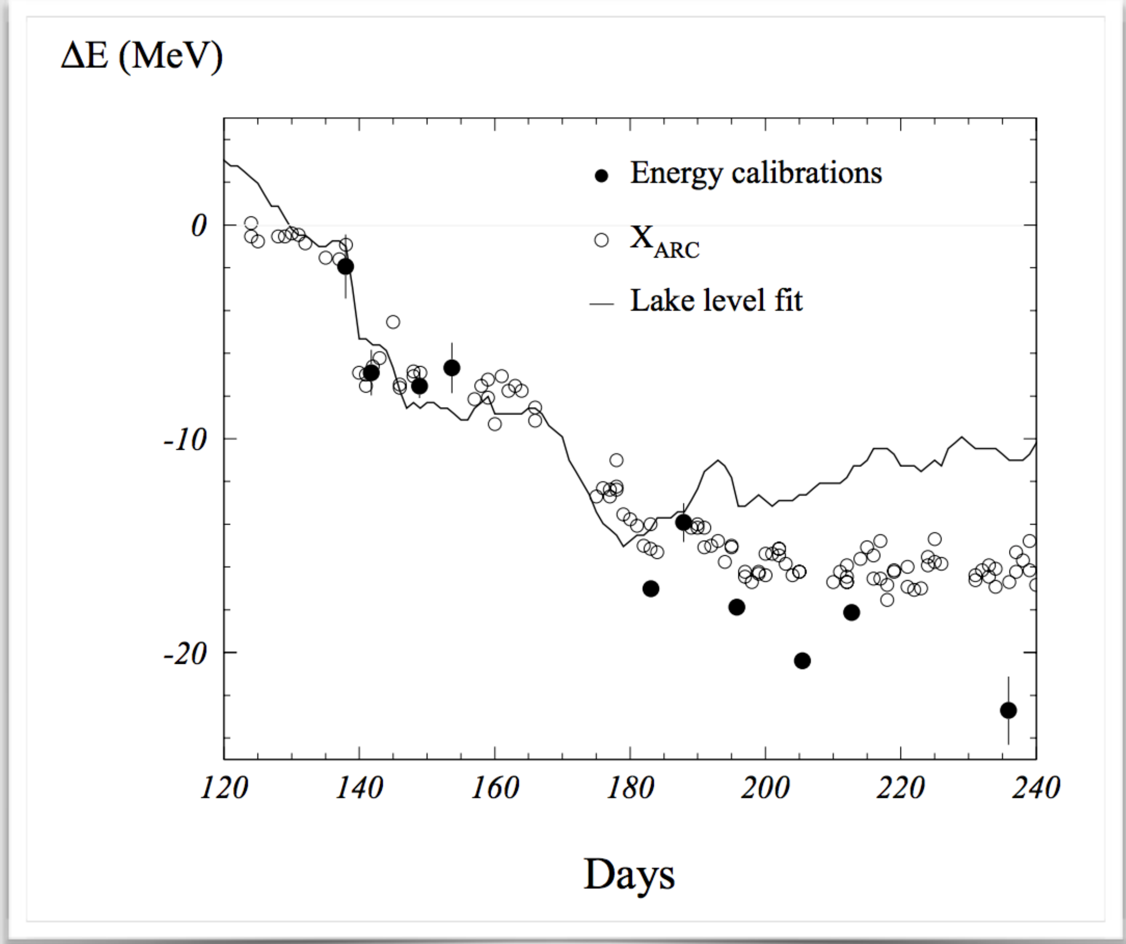
Water Level in Lake Geneva



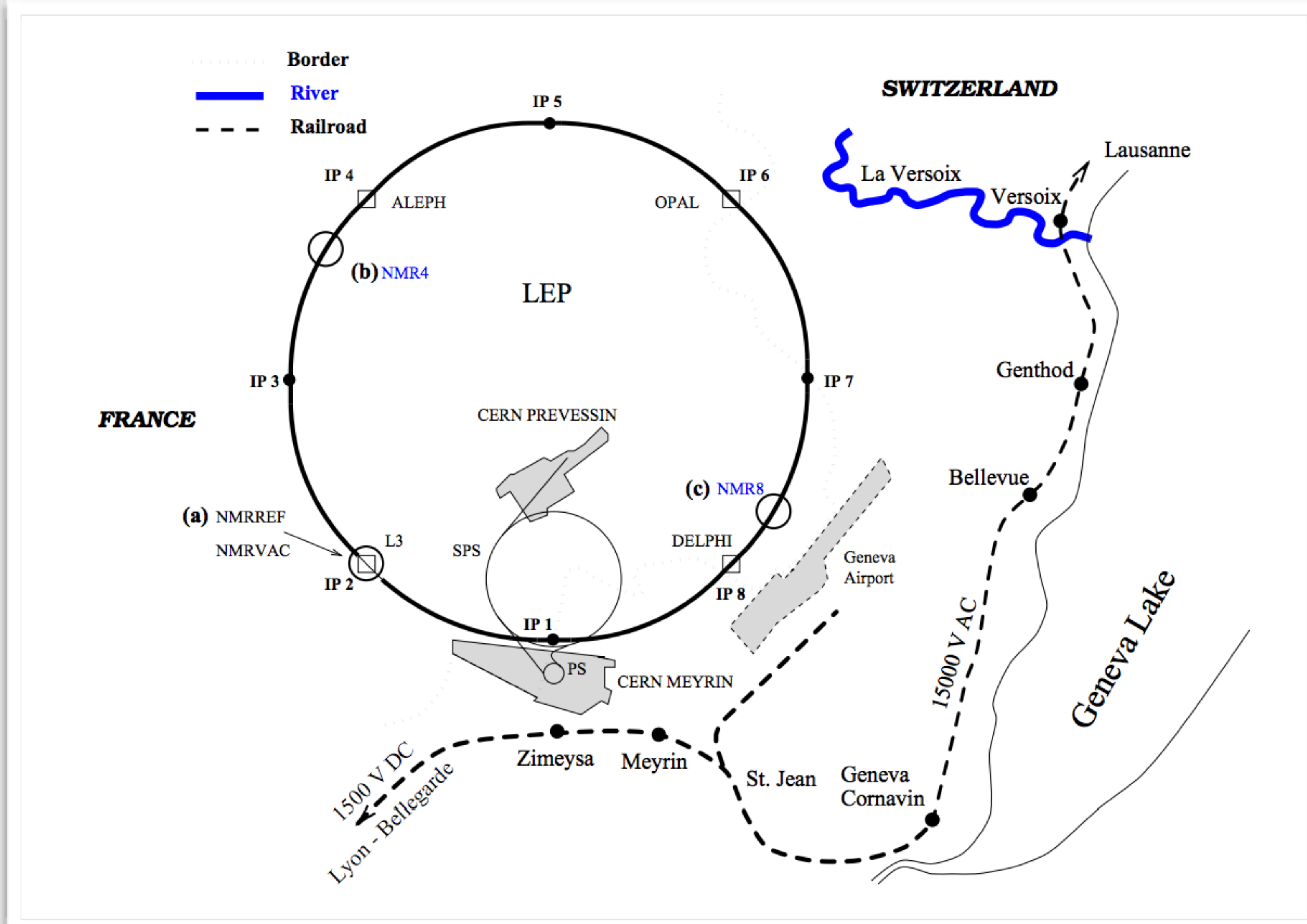
LEP Ring Distortions due to Lake Levels

Change in total beam energy (MeV) due to the upper crust position shift as a function of lake mass.

Total beam energy is ~100 GeV (100,000 MeV), making this a 0.02% effect.



LEP/CERN/TGV Train Map



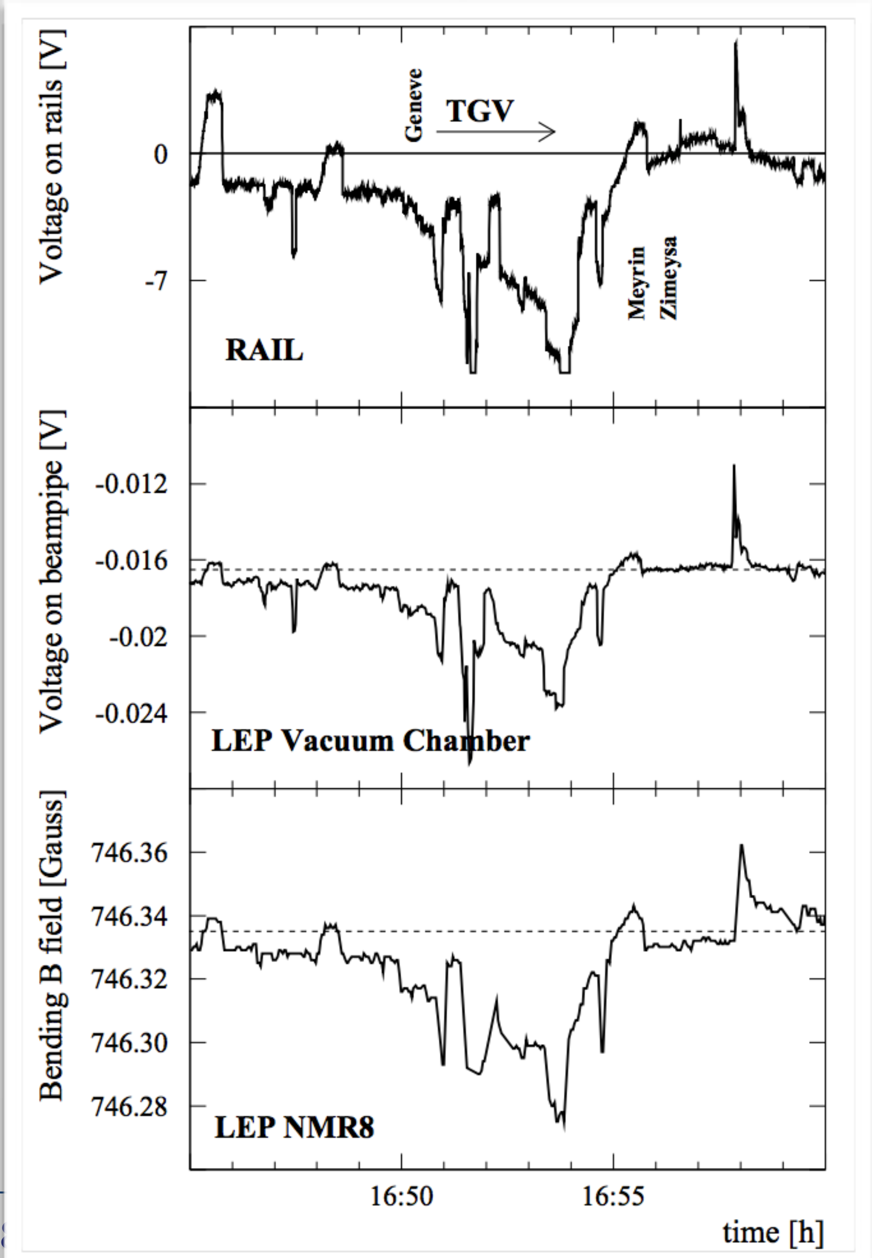
TGV Train Current Effect

Top: TGV rail voltage drop as current is sourced to ground.

Middle: Change in LEP beam pipe voltage.

Bottom: NMR measurement of LEP ring dipole magnet field.

Change in field leads to change in radius & thus beam energy. A ~0.007% effect.



Recap / Up Next

This time:

Quantum Flavor Dynamics

- Charged Currents
- QFD Feynman Rules
- Weak Decays
- Neutral Currents

Next time:

EW Unification

- Solving the chiral problem
- EW fields
- The Higgs boson