

LINEAR GAS DYNAMICS IN THE EXPANDING UNIVERSE

NICKOLAY Y. GNEDIN, EMILY J. BAKER, THOMAS J. BETHELL, MEREDITH M. DROSBACK, A. GAYLER HARFORD,
 AMALIA K. HICKS, ADAM G. JENSEN, BRIAN A. KEENEY, CHRISTOPHER M. KELSO, MARK C. NEYRINCK,
 SCOTT E. POLLACK, AND TIMOTHY P. VAN VLIET

Department of Astrophysical and Planetary Sciences, University of Colorado, Boulder, CO 80309; gnedin@casa.colorado.edu,
 ejbaker@colorado.edu, thomas.bethell@colorado.edu, meredith.drosback@colorado.edu, gayler_harford@post.harvard.edu,
 amalia.hicks@colorado.edu, adam.jensen@colorado.edu, brian.keeney@colorado.edu, kelsoc@colorado.edu,
 neyrinck@glow.colorado.edu, pollacks@colorado.edu, timothy.vanvliet@colorado.edu

Received 2002 June 24; accepted 2002 October 7

ABSTRACT

We investigate the relationship between the dark matter and baryons in the linear regime. This relation is quantified by the so-called “filtering scale.” We show that a simple Gaussian *Ansatz* that uses the filtering scale provides a good approximation to the exact solution.

Subject headings: cosmic microwave background — cosmology: theory — galaxies: formation — intergalactic medium — large-scale structure of universe

1. INTRODUCTION

If humans had dark matter vision, cosmology would be a solved problem by now. However, this is not the case, and in our attempt to understand the distribution of matter in the universe we need to rely on baryons—stars and gas—that trace the underlying distribution of the dark matter.

The baryons, though, are not the perfect tracer, since they are subject to other forces in addition to gravity. In this paper we restrict our attention to the linear regime, but even in this simplest physical situation the relationship between the dark matter and baryons (i.e., cosmic gas) is nontrivial, because gas pressure will erase fluctuations on small scales.

The effect of gas pressure on small fluctuations is canonically described by the Jeans scale, which is defined as the scale at which the gravity force equals the gas pressure force. On large scales gravity wins, and small fluctuations grow exponentially, while on small scales, gas pressure turns all fluctuations into sound waves.

However, in the expanding universe the effect of the Jeans scale becomes less direct, because gravitational instability leads only to slow, power-law growth of fluctuations. Let us consider the following simple thought experiment: in a universe with linear dark matter fluctuations, the gas is instantaneously heated to a high temperature. The Jeans scale also increases by a large factor instantaneously, whereas it takes about a Hubble time for the fluctuations in the gas to respond to the changed pressure force. Thus, the instantaneous value of the Jeans scale does not correspond to the characteristic scale over which the fluctuations are suppressed, but instead the suppression scale—which we call the “filtering” scale following Gnedin & Hui (1998)—depends on the whole previous thermal history. Only in the unphysical case of temperature evolving as an exact power law of the scale factor at all times does the Jeans scale become proportional to the filtering scale (Bi, Borner, & Chu 1992; Fang et al. 1993). However, incorrect expressions for the pressure force filtering have been used even until quite recently (Choudhury, Padmanabhan, & Srianand 2001a; Choudhury, Srianand, & Padmanabhan 2001b).

In this paper we investigate the role of the filtering scale further. Specifically, it has been suggested (Gnedin & Hui

1998; Gnedin 1998) that the relationship between the gas density fluctuation δ_B and the dark matter fluctuation δ_X in the Fourier domain can be approximated by the following simple expression:

$$\frac{\delta_B}{\delta_X} = e^{-k^2/k_F^2}, \quad (1)$$

where k is a wavenumber, and k_F is the wavenumber corresponding to the filtering scale. Our goal will be to investigate the range of validity of this approximation.

2. LINEAR DYNAMICS IN THE EXPANDING UNIVERSE

The linear evolution of fluctuations in the expanding universe containing dark matter and cosmic gas is described by two second-order differential equations (Peebles 1980),

$$\begin{aligned} \frac{d^2 \delta_X}{dt^2} + 2H \frac{d\delta_X}{dt} &= 4\pi G \bar{\rho} (f_X \delta_X + f_B \delta_B), \\ \frac{d^2 \delta_B}{dt^2} + 2H \frac{d\delta_B}{dt} &= 4\pi G \bar{\rho} (f_X \delta_X + f_B \delta_B) - \frac{c_S^2}{a^2} k^2 \delta_B, \end{aligned} \quad (2)$$

where $\delta_X(t, k)$ and $\delta_B(t, k)$ are Fourier components of density fluctuations in the dark matter and cosmic gas with respective mass fractions f_X and f_B , $H(t)$ is the Hubble constant, $a(t)$ is the cosmological scale factor, $\bar{\rho}(t)$ is the average mass density of the universe, $c_S(t)$ is the sound speed in the cosmic gas (where the sound speed is simply defined by $c_S^2 \equiv dP/d\rho$, assuming an equation of state that relates P and ρ), k is the comoving wavenumber, and t is the proper time.

On large scales ($k \rightarrow 0$) the relationship between the fluctuations in the gas and in the dark matter can be expanded as

$$\frac{\delta_B(t, k)}{\delta_X(t, k)} = 1 - \frac{k^2}{k_F^2} + O(k^4), \quad (3)$$

where k_F is the filtering scale and in general is a function of time.

The filtering scale is related to the Jeans scale k_J ,

$$k_J \equiv \frac{a}{c_S} \sqrt{4\pi G \bar{\rho}}, \quad (4)$$

by the following relation:

$$\frac{1}{k_F^2(t)} = \frac{1}{D_+(t)} \int_0^t dt' \frac{\ddot{D}_+(t') + 2H(t')\dot{D}_+(t')}{k_J^2(t')} \times a^2(t') \int_{t'}^t \frac{dt''}{a^2(t'')}, \quad (5)$$

where $D_+(t)$ is the linear growing mode in a given cosmology (Gnedin & Hui 1998).

Inspection of equation (5) shows that the filtering scale as a function of time is related to the Jeans scale as a function of time, but at a given moment in time those two scales are unrelated and can be very different. Thus, given the Jeans scale at a specific moment in time, nothing can be said about the scale over which the fluctuations in the gas are suppressed. It is only when the whole time evolution of the Jeans scale up to some moment in time is known that the filtering scale at this moment can be uniquely defined.

In order to investigate the properties of equation (2) as a function of the thermal history of the universe, we parameterize the evolution of the sound speed (as a function of cosmological redshift z) in the following way. Before the beginning of reionization at $z = z_{\text{BEG}}$ the sound speed is much smaller than the sound speed in the photoionized intergalactic medium (IGM) and is set to zero. Between z_{BEG} and z_{REI} the temperature in the IGM rises linearly with redshift. The sound speed reaches a maximum at the moment of reionization z_{REI} with the value of $c_{S,\text{REI}}$, after which moment it falls off as $(1+z)^{\beta/2}$. This parameterization is schematically illustrated in Figure 1. Thus, our thermal history is parameterized with four parameters: z_{BEG} , z_{REI} , $c_{S,\text{REI}}$ (which we replace with the mean temperature at reionization in units of 10^4 K, $T_{4,\text{REI}}$), and β . We adopt a flat cosmological model as our background cosmology, which introduces two more parameters: Ω_0 and Ω_B . We adopt the following values as our fiducial model:

$$\begin{aligned} \Omega_0 &= 0.3, \\ \Omega_B &= 0, \\ T_{4,\text{REI}} &= 2.5, \\ z_{\text{REI}} &= 7, \\ z_{\text{BEG}} &= 10, \\ \beta &= 1. \end{aligned} \quad (6)$$

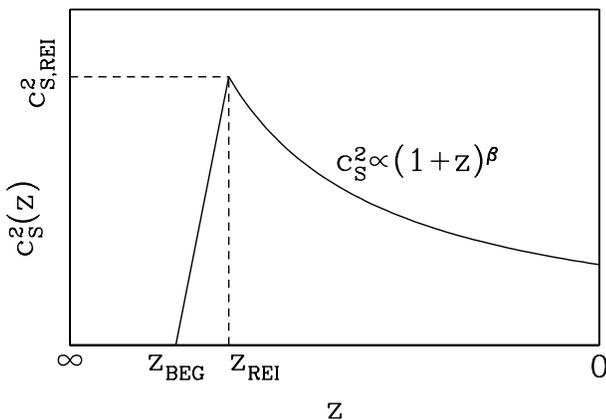


FIG. 1.—Schematic representation of our parameterization of the thermal history.

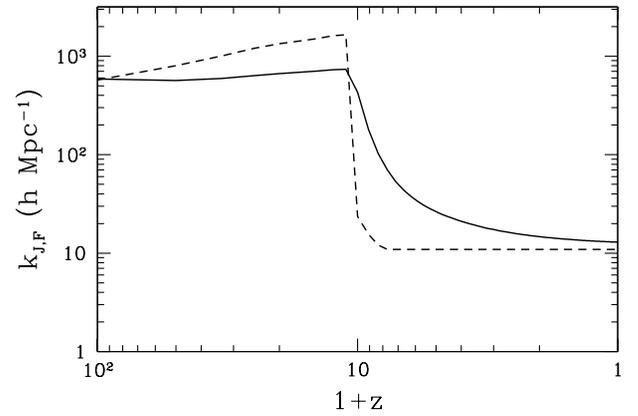


FIG. 2.—Evolution of the filtering scale (solid line) and the Jeans scale (dashed line) for our fiducial thermal history. Because for $\beta = 1$ the Jeans scale is constant after reionization, the filtering scale approaches it at late times.

The choice of $\beta = 1$ is motivated by the asymptotic evolution of the IGM temperature after reionization (Miralda-Escudé & Rees 1994; Hui & Gnedin 1997).

Figure 2 illustrates the evolution of the Jeans and filtering scales in our fiducial model. Note that after reionization the spatial filtering scale is always smaller than the Jeans scale ($k_F > k_J$), but they approach each other at later times if the Jeans scale is constant in time. In a more general case, when the Jeans scale is a function of time, the filtering scale always deviates from the Jeans scale.

Given the thermal history, we can solve equation (2) with identical initial conditions for dark matter and baryons. The specific initial conditions are not important as long as

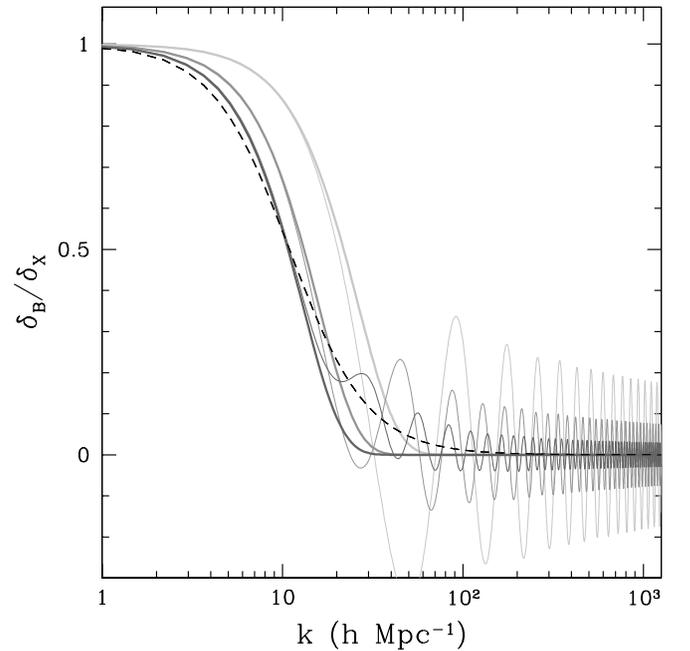


FIG. 3.—Solutions to equation (2) for our fiducial model at $z = 4$ (light grey), $z = 1.5$ (medium grey), and $z = 0$ (dark grey): thin lines show the exact solutions, thick lines give the approximation in eq. (1). The dashed black line gives the shape of filtering parameterized by the Jeans scale, $\delta_B(t, k)/\delta_X(t, k) = 1/[1 + (k/k_J)^2]$, which is constant in time for our fiducial model.

the computation starts at early enough time, because a decaying mode disappears in a few Hubble times and the solution quickly approaches a purely growing mode.

Solutions to equation (2) at three different redshifts are shown in Figure 3 with thin lines. As can be expected, on large scales ($k \rightarrow 0$) the gas follows the dark matter, whereas on small scales oscillations in the gas turn into slowly decaying sound waves. We also show with thick lines the Gaussian approximation (eq. [1]) to illustrate its level of accuracy for our fiducial model. Clearly, the Gaussian approximation does not reproduce the small scale oscillations, but it appears to do a decent job in reproducing the transitional region between large and small scales. In comparison, we also show the filtering form for the constant Jeans scale (Peebles 1980),

$$\frac{\delta_B(t, k)}{\delta_X(t, k)} = \frac{1}{1 + (k/k_J)^2} .$$

As one can see, this form does not reproduce numerical results as well as the Gaussian approximation (eq. [1]) even at late times, when the Jeans scale is close to the filtering scale.

3. RESULTS

In this section we present our investigation of the accuracy of the Gaussian approximation (eq. [1]). Our approach to testing the approximation is motivated by its applicability. We can envision using the approximation as a simpler way of computing the rms density fluctuations in the gas:

$$\begin{aligned} \sigma_B^2(t) &= \frac{1}{2\pi^2} \int_0^\infty P_X(k) \left[\frac{\delta_B(t, k)}{\delta_X(t, k)} \right]^2 k^2 dk \\ &\approx \frac{1}{2\pi^2} \int_0^\infty P_X(k) e^{-2k^2/k_F^2(t)} k^2 dk , \end{aligned} \quad (7)$$

where $P_X(k)$ is the dark matter power spectrum. Another use of the approximation could be in giving the shape of the

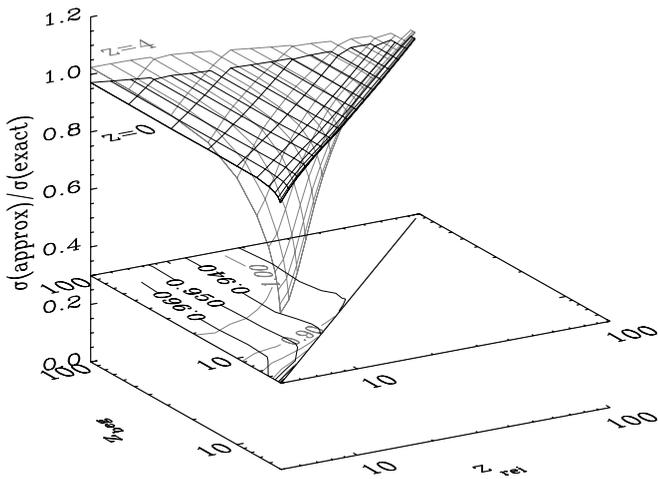


FIG. 4.—Accuracy of the Gaussian approximation in computing the rms gas fluctuation as a function of z_{REI} and z_{BEG} at $z = 4$ (grey) and $z = 0$ (black). The horizontal plane gives a contour representation of the surface plot. Note that the accuracy of the approximation at $z = 4$ deteriorates as reionization becomes faster and more recent, i.e., when z_{BEG} is close to z_{REI} and $z_{\text{REI}} \lesssim 10$

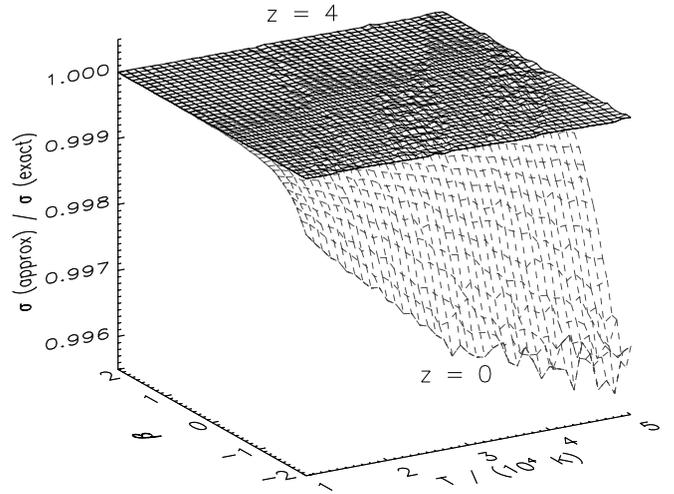


FIG. 5.—Same as Fig. 4, except as a function of $T_{4,\text{REI}}$ and β . For all reasonable and even not very reasonable values (large negative β implies a large amount of heating after reionization) of these two parameters the approximation holds exceptionally well.

gas power spectrum on small scales:

$$\begin{aligned} P_B(k, t) &= P_X(k) \left[\frac{\delta_B(t, k)}{\delta_X(t, k)} \right]^2 \\ &\approx P_X(k) e^{-2k^2/k_F^2(k)} . \end{aligned} \quad (8)$$

Thus, our goal is to estimate the accuracy of the Gaussian approximation both in equation (7) and in equation (8) in our six-dimensional parameter space.

A full sampling of this parameter space would be unrealistic, so we focus on varying two of the parameters at a time, keeping the rest of the parameters fixed to our fiducial values. For the dark matter power spectrum $P_X(k)$, we adopt a canonical cold dark matter + cosmological constant power spectrum with $\Omega_0 h = 0.25$, Harrison-Zeldovich scale-free primordial spectrum, and the BBKS transfer function (Bardeen et al. 1986).

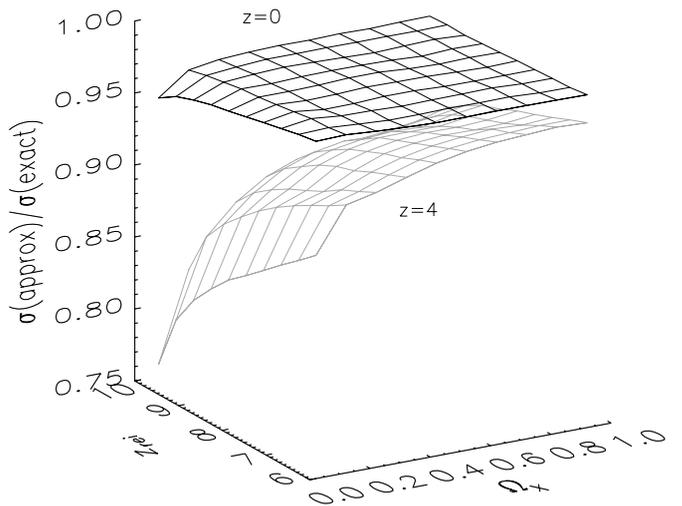


FIG. 6.—Same as Fig. 4, except as a function of Ω_X and z_{REI} (with z_{BEG} being kept fixed at 10). Again, the approximation becomes worse when reionization is too fast.

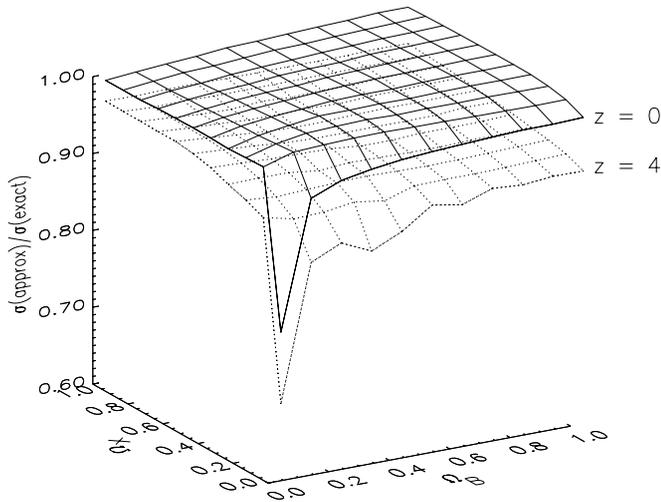


FIG. 7.—Same as Fig. 4, except as a function of Ω_X and Ω_B . These two parameters do not affect the accuracy of the Gaussian approximation much except when they are both ridiculously small.

The choice of the power spectrum does matter. For example, if the power spectrum contains a lot of power on very small scales (below the filtering scale), then oscillations shown in Figure 3 will be amplified, which, in turn, will compromise the approximation (1). However, we believe that the dark matter power spectrum is known sufficiently well up to scales of interest, so our choice is justified.

Figures 4, 5, 6, 7, and 8 show the accuracy of the Gaussian approximation as a function of our parameters. One can see that in general the approximation holds very well, with the approximate σ_B being within 10% of the exact value. The approximation breaks down at $z \gtrsim 4$ in the limit when reionization is quite fast ($z_{\text{BEG}} \lesssim 1.2z_{\text{REI}}$) and the redshift of reionization is low ($z_{\text{REI}} < 6$). It also gets somewhat worse for very low values of Ω_0 . This is not surprising, since in those two regimes the oscillations at large k are substantial and make a non-negligible contribution to the integral in equation (7). On the other hand, it is virtually insensitive to the thermal evolution of the IGM after reionization (parameters $T_{4,\text{REI}}$ and β).

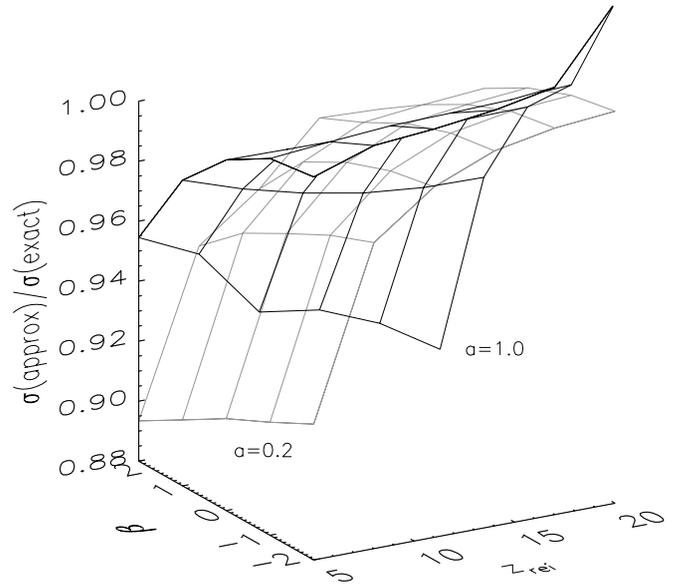


FIG. 8.—Same as Fig. 4, except as a function of z_{REI} and β . The accuracy of the approximation is almost independent of the thermal evolution of the IGM after reionization.

We have also found that in essentially all cases the Gaussian approximation gives a better than 20% (10% in amplitude) fit to the gas power spectrum (eq. [8]) for $k < 0.9k_F$.

4. CONCLUSIONS

We have shown that the Gaussian approximation (eq. [1]) provides a reasonably good fit (better than 10%) to the rms density fluctuations in the gas, and for the shape of the gas power spectrum for $k < 0.9k_F$. It is important to underscore that there is no known physical reason that this approximation works so well, so it should be considered as a mathematical coincidence. Notwithstanding, the Gaussian approximation can be used in semianalytical models of the Ly α forest and early universe, when a full solution to equation (2) is not practical.

REFERENCES

- Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, *ApJ*, 304, 15
 Bi, H. G., Borner, G., & Chu, Y. 1992, *A&A*, 266, 1
 Choudhury, T. R., Padmanabhan, T., & Srianand, R. 2001a, *MNRAS*, 322, 561
 Choudhury, T. R., Srianand, R., & Padmanabhan, T. 2001b, *ApJ*, 559, 29
 Fang, L. Z., Bi, H., Xiang, S., & Boerner, G. 1993, *ApJ*, 413, 477
 Gnedin, N. Y. 1998, *MNRAS*, 299, 392
 Gnedin, N. Y., & Hui, L. 1998, *MNRAS*, 296, 44
 Hui, L., & Gnedin, N. Y. 1997, *MNRAS*, 292, 27
 Miralda-Escudé, J., & Rees, M. J. 1994, *MNRAS*, 266, 343
 Peebles, P. J. E. 1980, *The Large-Scale Structure of the Universe* (Princeton: Princeton Univ. Press)