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EFFECTS OF DEPARTURES FROM IONIZATION EQUILIBRIUM ON COOLING-FLOW X-RAY SPECTRA

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ABSTRACT

X-ray spectra of galaxy cluster cooling cores have traditionally been modeled under the assumption that the cooling plasma is in ionization equilibrium. Departures from ionization equilibrium could affect the expected line-emission spectra from these objects and consequently the physical quantities inferred from such spectra. We have determined the elemental ionization fractions for six astrophysically abundant elements for temperatures from 10^4 to 10^8 K, assuming both equilibrium and nonequilibrium plasma models, and using cooling function values derived for both solar and one-third solar abundance. The magnitude of deviations from ionization equilibrium in a cooling plasma increases with decreasing atomic number: it is noticeable for C and O and becomes negligible for Si and S. The results of our recalculation of ionization fractions were used to derive X-ray emissivities for cluster cooling cores. Comparisons between luminosities produced by helium-like $^2P_1 \rightarrow {}^1S_0$ transitions and hydrogen-like Lyα transitions indicate that nonequilibrium ionization significantly affects (by 10%–20%) only low-Z elements, and that the determination of the cooling rate via spectral analysis remains unchanged.

Subject headings: atomic processes — cooling flows — galaxies: clusters: general — line: formation — X-rays: galaxies

1. INTRODUCTION

A significant fraction of galaxy clusters are observed to possess cooling flow characteristics (Fabian 1994). Properties of these clusters, such as temperature, abundance, and mass cooling rate, are often determined via the observed strengths of various X-ray emission lines. Despite the evidence for cooling, the models used to analyze these lines generally incorporate the assumption that the plasma is in ionization equilibrium. The validity of that assumption has been tested in the case of iron at high temperatures (Canizares, Markert, & Donahue 1998) and for other elements at temperatures at or below 10^6 K (Shapiro & Moore 1976; Edgar & Chevalier 1986). Prior to the advent of the high spectral resolution achievable with X-ray missions such as *Chandra* and *XMM*, relatively small departures from equilibrium at high temperatures would not have been detectable. Since we now possess the capability to resolve such features, a more thorough look at the effects of nonequilibrium ionization on X-ray emission lines at high temperatures seems warranted.

The estimates of the characteristic timescale for radiative cooling are of the same order as the time it takes for the plasma to reach thermal (statistical) equilibrium. The characteristic time that it takes a plasma to cool isobarically through an increment of temperature δT can be written

$$\delta t_{\rm cool} = \frac{5}{2} \frac{k}{n\Lambda(T)}\, \delta T \,, \tag{1}$$

where n is the electron density, $\Lambda(T)$ is the total emissivity of the plasma (the cooling function), and k is Boltzmann's constant. In the case of isochoric cooling, 5/2 is replaced by 3/2. In terms of the parameters that are of interest to us in this model,

$$n\,\delta t_{\rm cool} = 2.2 \times 10^{11} \left(\frac{T}{10^6}\right) \left(\frac{\Delta \log T}{0.1}\right) \times \left(\frac{1.6 \times 10^{-22}}{\Lambda}\right) {\rm s\ cm^{-3}} \,. \tag{2}$$

The characteristic time it takes the plasma to reach thermal equilibrium is inversely proportional to its recombination rate, ($\alpha_{\rm rec}$), as

$$n\,\delta t_{\rm rec} \sim \alpha_{\rm rec}^{-1}\ {\rm s\ cm^{-3}} \,. \tag{3}$$

Using recombination of C V to C IV at 1×10^6 K as an example,

$$\alpha_{\rm rec} = \alpha^{\rm rad} + \alpha^{\rm diel} \sim 1.1 \times 10^{-12}\ {\rm cm^3\ s^{-1}} \,, \tag{4}$$

$$n\,\delta t_{\rm rec} \sim 9.1 \times 10^{11}\ {\rm s\ cm^{-3}} \,, \quad \delta t_{\rm cool}/\delta t_{\rm rec} \sim 0.2 \,, \tag{5}$$

where $\alpha^{\rm rad}$ and $\alpha^{\rm diel}$ are radiative and dielectronic contributions to the recombination rate, calculated using the formulae of Shull & van Steenberg (1982). According to this approximation, cooling proceeds more rapidly than relaxation, and therefore the plasma remains overionized. This results in a set of ionization fractions that are not identical to equilibrium values, and which depend on the history of the plasma.

2. CALCULATIONS

2.1. *X-Ray Line Luminosity*

The increment of luminosity given off by a particular line transition in a plasma that is cooling radiatively (and isobarically) from T to $T - \delta T$ can be written (Canizares et al. 1998, their eq. [8]) as

$$\delta L_i = \frac{\dot{M}}{\mu m} \frac{5k\epsilon_i(T)}{2\Lambda(T)} \delta T \ , \tag{7}$$

where μm is the mean mass per particle, $\Lambda(T)$ is the total emissivity of the plasma (the cooling function), $\dot{M}$ represents the rate at which matter is cooling, and $\epsilon_i(T)$ is the emissivity of the i^{th} line.

If we entertain the idea that the plasma cools through the entire temperature range of interest (in regard to our particular line), we find that the total luminosity in the line is given by

$$L_i = \frac{\dot{M}}{\mu m} \frac{5}{2} k \int_{T\min}^{T\max} \frac{\epsilon_i(T)}{\Lambda(T)} dT \equiv \frac{\dot{M}}{\mu m} \xi_i \tag{8}$$

We have calculated values of ξ_i for prominent lines of six elements, using both equilibrium and nonequilibrium plasma models. The ratio $\xi_i(\text{noneq})/\xi_i(\text{eq})$ is a measure of the importance of departures from ionization equilibrium on the spectra of cooling plasma.

2.2. *Line Emissivity and Ionization Equilibrium*

There are a handful of atomic processes that contribute to the emission of a particular spectral line. We have chosen, for simplicity, to neglect all but collisional excitation and radiative recombination, which dominate the emission (see, e.g., Mewe & Gronenschild, their Fig. 7). In this scenario, the emissivity of a line is calculated using excitation rates, recombination rates, and the relative populations of pertinent ions.

In order to test the assumption of ionization equilibrium, we numerically solved a set of coupled differential equations that describe the ionization balance of a plasma (Mewe & Gronenschild 1981, their eq. [1]). These equations rely upon both the instantaneous ionization fractions and temperature-dependent values of ionization and recombination rates. We incorporated cooling by relating a time interval dt to a temperature increment via equation (1):

$$\frac{dt}{dT} = \frac{5}{2} \frac{k}{n\Lambda(T)} \ . \tag{9}$$

Because departures from equilibrium are small, we use values of Λ for an equilibrium plasma—in one case with solar abundance, and in the other with one-third solar abundance. Our initial temperature in each case was one at which the vast majority of atoms of the element in question would be fully stripped (generally near 10^8 K). We then allowed the plasma to cool numerically until it had largely recombined to a lithium-like ionization state (10^4–10^5 K).

2.3. *Rate Coefficients*

2.3.1. *Ionization/Recombination*

We calculated the temperature-dependent values of ionization rates using formulae supplied by Arnaud & Rothenflug (1985), taking into account both collisional ionization and autoionization. Recombination rates came from Shull & van Steenberg (1982), Savin (1999), and Nahar, Pradhan, & Zhang (2000), and all include contributions from both radiative and dielectric recombination. To verify our code, we calculated equilibrium values of ionization fractions and checked them against Arnaud & Rothenflug's (1985) tabulated values.

In this temperature range, the ionization fractions of the heavier elements we studied (Si, S, and Mg) did not depart significantly from equilibrium values. The lighter elements (C, O, and Ne) showed enough discrepancy between equilibrium and nonequilibrium values to warrant the further calculation and comparison of line emissivities.

2.3.2. *Excitation*

The spectral emission lines that we focused on are those produced by a helium-like ion undergoing a $^2P_1 \rightarrow {}^1S_0$ transition, or a hydrogen-like ion undergoing a Lyα transition. The (temperature dependent) collisional excitation rates that we used to calculate the emission for the helium-like line came largely from Zhang & Sampson (1987), except in the case of carbon, for which we referred to Pradhan, Norcross, & Hummer (1981). The excitation rates used to calculate Lyα emissivities came from Hutcheon & McWhirter (1973). Radiative recombination rates were determined using formulae from Shull & van Steenberg (1982) and Savin (1999).

There exists a finite temperature range, specific to individual ions, for which collisional excitation rates are appreciable. This range begins at a high enough temperature that there are basically no ions of that type in existence, and ends where the order of magnitude of the collisional ionization rate has fallen by a factor of about 10. This temperature cutoff is reflected in the available data for collisional excitation rates. Just to be thorough, we extended this range to higher temperatures. Our results remained the same (to within 1%) in both scenarios.

We ran the line emissivity code using both equilibrium and nonequilibrium ionization fractions as input. The emissivities computed using the equilibrium model are close to the values published by Mewe, Gronenschild, & van den Oord (1985).

3. SUMMARY AND DISCUSSION

The ionization fractions of a radiatively cooling plasma deviate from equilibrium fractions in the sense that the plasma remains more highly ionized at a given temperature (Figs. 1 and 2). The magnitude of this deviation increases

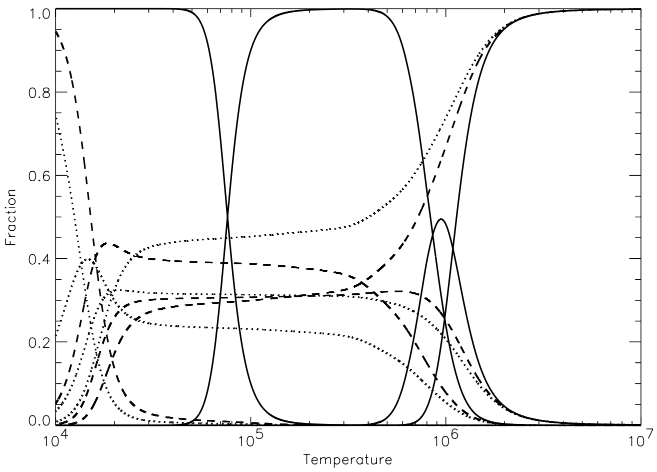

FIG. 1.—Carbon ionization fractions. *Solid lines*, equilibrium ionization fractions; *dashed lines*, ionization fractions calculated using an isobaric cooling model for a plasma with solar abundance; *dotted lines*, fractions derived from isochoric cooling. The progression of lines from right to left corresponds to fully stripped, hydrogen-like, helium-like, and lithium-like fractions.

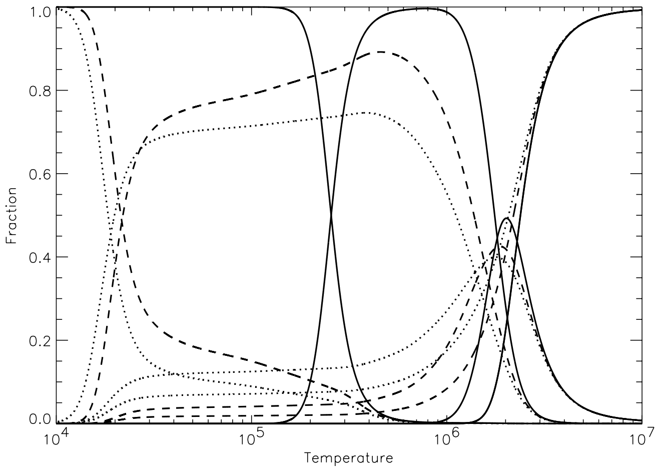

FIG. 2.—Similar to Fig. 1, but for oxygen ionization fractions

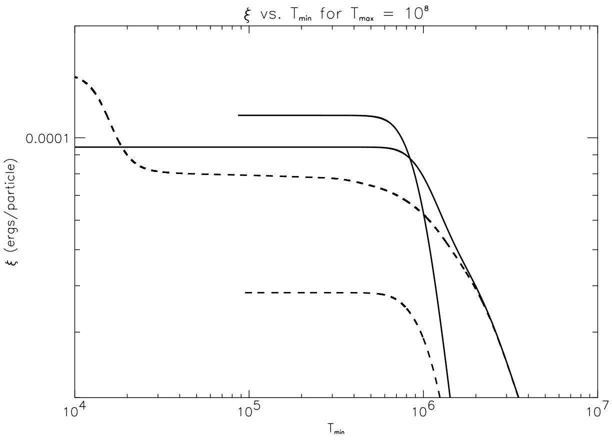

FIG. 3.—Carbon ξ vs. $T_{\rm min}$ for $T_{\rm max} = 10^8$ K helium resonance. The solid line is indicative of the energy emitted per particle between $T_{\rm max}$ and T by a plasma in ionization equilibrium. The dotted line portrays this value for an isochorically cooling plasma with solar abundance. Lines that begin just above 10^6 K are contributions to ξ due to collisional excitation, and the other two lines represent energy emitted via radiative recombination. Note the 50% increase in ξ in the rapidly cooling plasma over the temperature range that is dominated by radiative recombination.

with decreasing atomic number: it is sizable for C and O and becomes negligible for Si and S.

Similarly, line luminosities for low-Z ions produced by a rapidly cooling plasma with nonequilibrium ionization fractions fall below those produced by a plasma in ionization equilibrium by up to 20%. This effect rapidly becomes insignificant as Z increases (see Table 1).

It is interesting that the line luminosities for nonequilibrium, cooling plasmas are so similar to those of equilibrium plasmas, even for ions with considerably larger differences in ionization fractions, such as carbon. This results from the approximate cancellation of differences in the contributions to line emission from collisional excitation and radiative recombination. The contribution to line emission from collisional excitation, which dominates in equilibrium plasmas, is exponentially cut off when kT is below the excitation energy for the line. In cooling plasmas, the persistence of a given ionization fraction to lower temperatures suppresses this component. In contrast, the contribution from radiative recombination is enhanced in cooling plasmas because recombination occurs at all temperatures at a rate that increases in cooler plasmas (see Fig. 3.)

TABLE 1

LUMINOSITY RATIOS $\xi_{\rm cool}/\xi_{\rm eq}$

		SOLAR ABUNDANCE		0.3 SOLAR ABUNDANCE	
ELEMENT	LINE	Isobaric	Isochoric	Isobaric	Isochoric
C........	Lyα	0.99	0.99	0.99	0.98
	He res.	0.80	0.88	0.81	0.78
O........	Lyα	0.99	0.99	1.00	0.99
	He res.	0.85	0.83	0.92	0.88
Ne	Lyα	1.00	1.00	1.00	1.00
	He res.	0.96	0.93	0.98	0.97
Mg......	Lyα	1.00	1.00	1.00	1.00
	He res.	0.98	0.97	0.99	0.99
Si........	Lyα	1.00	1.00	1.00	1.00
	He res.	0.99	0.98	0.99	0.98
S	Lyα	1.00	1.00	1.00	1.00
	He res.	0.99	0.99	1.00	0.99

The discrepancies between rapidly cooling and ionization equilibrium plasma line luminosities are generally smaller for one-third solar abundance plasmas than for those with solar abundances. This trend is easily explained, as a one-third solar abundance plasma cools more slowly, and therefore its ionization populations lie closer to equilibrium values. Carbon is an exception to this trend because of unusually large contributions from recombination at low temperatures in the solar abundance case. Of course, once one gets down to such low temperatures, there are almost certainly other factors that influence the behavior of the plasma, but these are beyond the scope of this study.

In conclusion, mass cooling rates derived from X-ray emission-line luminosities are not significantly affected by the departures from equilibrium in the cooling plasma. If cooling proceeds steadily, then the assumption of quasi-equilibrium conditions would cause at most a 10%–20% error in the derived cooling rates for lines of lower Z elements. Some future observations may actually show this trend, although it is likely that other physical effects also contribute at that level.

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