K-factors

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1. Introduction

Although lowest order calculations can in general describe broad features of a particular process and provide the first estimate of its cross section, in many cases this approximation is insufficient. The inherent uncertainty in a lowest order calculation derives from its dependence on the unphysical renormalization and factorization scales, which is often large. In addition, some processes may contain large logarithms that need to be resummed or extra partonic processes may contribute only when going beyond the first approximation. Thus, in order to compare with predictions that have smaller theoretical uncertainties, next-to-leading order calculations are imperative for experimental analyses at the LHC.

A next-to-leading order QCD calculation requires the consideration of all diagrams that contribute an additional strong coupling factor, α_s. These diagrams are obtained from the lowest order ones by adding additional quarks and gluons and they can be divided into two categories, virtual (or loop) contributions and the real radiation component. Next-to-leading order (NLO) is the first order at which the normalization, and in some cases the shape, of perturbative cross sections can be considered reliable [1]. A great deal of effort has recently been devoted towards the calculation of complex cross-sections at NLO. A prioritized list of NLO cross sections was assembled at Les Houches in 2005 [2] and added to in 2007 [3]and 2009 [4]. This list includes cross sections which are experimentally important, and which are theoretically feasible (if difficult) to calculate. Basically all $2 \to 3$ cross sections of interest have been calculated, with the frontier now extending to $2 \to 4$ calculations. Often these calculations exist only as private codes. To reach full utility, the codes should be made public and/or the authors should generate ROOT ntuples providing the parton level event information from which experimentalists can assemble any cross sections of interest. A format for such ntuples has been developed [4]. Of course the ultimate goal will be the ability to link any NLO calculation to a parton shower Monte Carlo [5].

2. K-factors

Experimentalists typically deal with leading order (LO) calculations, especially in the context of parton shower Monte Carlos. Some of the information from a NLO calculation can be encapsulated in the K-factor, the ratio of the NLO to LO cross section for a given process, with the caveat that the value of the K-factor depends upon a number of variables, including the values of the renormalization and factorization scales, as well as the parton distribution functions (PDFs) used at LO and NLO. In addition, the NLO corrections often result in a shape change, so that one K-factor is not sufficient to describe the impact of the NLO corrections on the LO cross section. A further complication is caused by the fact that the K-factor can depend quite strongly on the region of phase space that is being studied. The K-factor which is appropriate for the total cross section of a given process may be quite different from the one when stringent analysis cuts are applied. For processes in which basic cuts must be applied in order to obtain a finite cross section, the K-factor again depends upon the values of those cuts. A still further complication is that there can be significant differences in K-factors for multi-parton final states depending on whether one is dealing with a (fixed) partonic final state, or the final state from a parton shower.

Even with these caveats, it is still useful to calculate the K-factors for interesting processes at the Tevatron and LHC. A K-factor table, originally shown in the CHS review article [1] and then later expanded in the Les Houches 2007 and 2009 proceedings [3], [4] is presented below. The K-factors are shown for several different choices of scale and with the use of either LO or NLO PDFs for the LO calculation. Also shown are the K-factors when the CTEQ modified LO PDFs are used [6].

Process	Fact. scales μ_0	μ_1	Tevatron K-factor $\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	LHC K-factor $\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	$\mathcal{K}''(\mu_0)$
W	m_W	$2m_W$	1.33	1.31	1.21	1.15	1.05	1.15	0.95
W+1 jet	m_W	$p_T^{\rm jet}$	1.42	1.20	1.43	1.21	1.32	1.42	0.99
W+2 jets	m_W	$p_T^{\rm jet}$	1.16	0.91	1.29	0.89	0.88	1.10	0.90
WW+1 jet	m_W	$2m_W$	1.19	1.37	1.26	1.33	1.40	1.42	1.10
$t\bar{t}$	m_t	$2m_t$	1.08	1.31	1.24	1.40	1.59	1.19	1.09
$t\bar{t}$+1 jet	m_t	$2m_t$	1.13	1.43	1.37	0.97	1.29	1.10	0.85
$b\bar{b}$	m_b	$2m_b$	1.20	1.21	2.10	0.98	0.84	2.51	–
Higgs	m_H	$p_T^{\rm jet}$	2.33	–	2.33	1.72	–	2.32	1.43
Higgs via VBF	m_H	$p_T^{\rm jet}$	1.07	0.97	1.07	1.23	1.34	0.85	0.83
Higgs+1 jet	m_H	$p_T^{\rm jet}$	2.02	–	2.13	1.47	–	1.90	1.33
Higgs+2 jets	m_H	$p_T^{\rm jet}$	–	–	–	1.15	–	–	1.13

Table 1: K-factors for various processes at the LHC (at 14 TeV) calculated using a selection of input parameters. In all cases, for NLO calculations, the CTEQ6M PDF set is used. For LO calculations, $\mathcal{K}$ uses the CTEQ6L1 set, whilst $\mathcal{K}'$ uses the same PDF set, CTEQ6M, as at NLO, and $\mathcal{K}''$ uses the LO-MC (2-loop) PDF set CT09MC2. For Higgs+1 or 2 jets, a jet cut of $40~{\rm GeV}/c$ and $|\eta| < 4.5$ has been applied. A cut of $p_T^{\rm jet} > 20~{\rm GeV}/c$ has been applied to the $t\bar{t}$+jet process, and a cut of $p_T^{\rm jet} > 50~{\rm GeV}/c$ to the WW+jet process. In the W(Higgs)+2 jets process, the jets are separated by $\Delta R > 0.4$ (with $R_{sep} = 1.3$), whilst the vector boson fusion (VBF) calculations are performed for a Higgs boson of mass 120 GeV. In each case the value of the K-factor is compared at two often-used scale choices, μ_0 and μ_1.

Several patterns can be observed in the K-factor table. NLO corrections appear to be larger for processes in which there is a great deal of color annihilation, such as $gg- > Higgs$ in which two color octet gluons produce a color singlet Higgs boson. NLO corrections also tend to decrease as more final-state legs are added [1]. The K-factors at the LHC are similar to the K-factors for the same processes at the Tevatron, but have a tendency to be smaller. The K-factors tend to be closer to unity if either the same (NLO) PDF is used for both the LO and NLO calculations, or equivalently, if the CTEQ modified LO PDFs (intended to mimic the NLO behavior) are used. The modified LO PDFs will also mimic (by construction) the shape distributions (rapidity, mass,) of many key benchmark cross sections.

While explicit loop corrections often do increase the NLO cross sections with respect to the LO ones, it can be observed from the table above that the K-factors do not have to be greater than unity.

3. Some Selected Processes

3.1 Inclusive jet production

It is informative to plot the K-factors (the ratio of the NLO to LO cross sections) for three different rapidity intervals for inclusive jet production. As discussed previously, the value of the K-factor is a scale-dependent quantity; the K-factors shown in Figure 1 are calculated with the nominal scale of $p_T^{jet}/2$. The K-factors have a somewhat complicated shape due to the interplay between the different subprocesses comprising inclusive jet production and the behaviours of the relevant pdfs in the different regions of parton momentum fraction x. In the central region, the K-factor is within 10% of unity for the observable range. There are no new parton-parton subprocesses that contribute at NLO but not at LO.

[1] A rule-of-thumb derived by Lance Dixon is that the K-factor is often proportional to the factor $C_{i1} + C_{i2}? - C_{f,max}$, where C_{i1} and C_{i2} are the Casimir color factors for the initial state and $C_{f,max}$ is the Casimir factor for the biggest color representation that the final state can be in. Of course, this is not intended to be a rigorous rule, just an illustrative one.

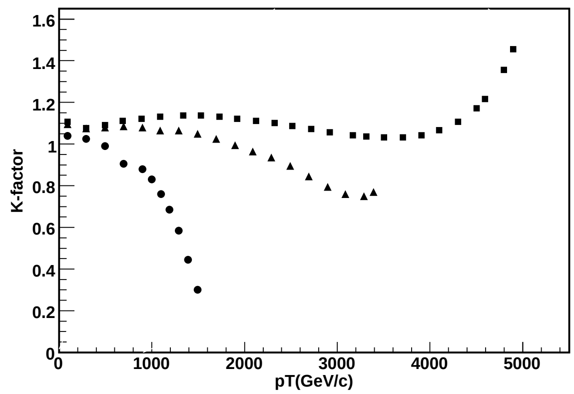

Fig. 1: The ratios of the NLO to LO jet cross section predictions for the LHC (14 TeV) using the CTEQ6.1 pdfs for the three different rapidity regions (0–1 (squares), 1–2 (triangles), 2–3 (circles)).

Thus a LO prediction, using the NLO CTEQ6.1 pdfs, will reproduce fairly closely the NLO calculation. For rapidities between 1 and 2, the K-factor is within 20% of unity, dropping below one at higher transverse momentum. For forward rapidities, the K-factor drops almost immediately below one, due to the behaviour of the high-x pdfs that contribute to the cross section in this region. There is nothing wrong with the NLO prediction in this region; its relationship to the LO cross section has just changed due to the kinematics. LO predictions in this region will provide an overestimate of the NLO cross section. Similar behavior is observed at the Tevatron, with suitably scaled-down transverse momentum values.

3.2 W + 3 jets

The cross section for the production a W boson and 3 jets has recently been calculated at NLO [7], [8]; this represents one of the most complex NLO calculations to be performed to date and illustrates the complexities that may result from multi-jet final states. The scale dependence for this cross section is shown in Figure 2 for the Tevatron and for the LHC(14 TeV) [7]. It can be observed that, using a scale of m_W, the K-factor at the Tevatron is approximately unity, while at the LHC it less than 0.6.

The K-factors for W + 1, 2 or 3 jets, at a renormalization/factorization scale of m_W, are plotted in Figure 3 (along with similar K-factors for Higgs + 1 or 2 jets) [2]. In this plot, a pattern becomes obvious. The K-factors appear to decrease linearly as the number of final state jets increases, with a similar slope at the Tevatron as at the LHC (but with an offset). A similar slope is observed for Higgs boson+ jets at the LHC. To further understand this pattern (in addition to the color flow argument discussed in the previous section), we first have to review jet algorithms at LO and NLO. At LO, one parton equals one jet. By choosing a jet algorithm with size parameter D, we are requiring any two partons to be a distance D or greater apart. The matrix elements have $1/\Delta R$ poles, so a larger value of D means smaller cross sections. At NLO, there can be two partons in a jet, and jets for the first time can have some structure. No ΔR cut is needed since the virtual corrections cancel the collinear singularity from the gluon emission (but there are residual logs that can become important if the value of D is too small). Increasing the size parameter D increases the phase space for including an extra gluon in the jet, and thus increases the cross section at NLO (in most cases). The larger the number of final state partons, the greater the differences will be between the LO and NLO dependence on jet size.

In Figure 4, the cross sections for W + 1, 2 and 3 jets are plotted as a function of the jet size and

[2]For these plots, the NLO CTEQ6.6 PDFs [9] have been used with both the LO and NLO matrix elements, in order to separate any PDF effects from matrix element effects. If a LO PDF such as CTEQ6L1 were used instead, the LO cross sections would shift upwards, but the trends would be the same.

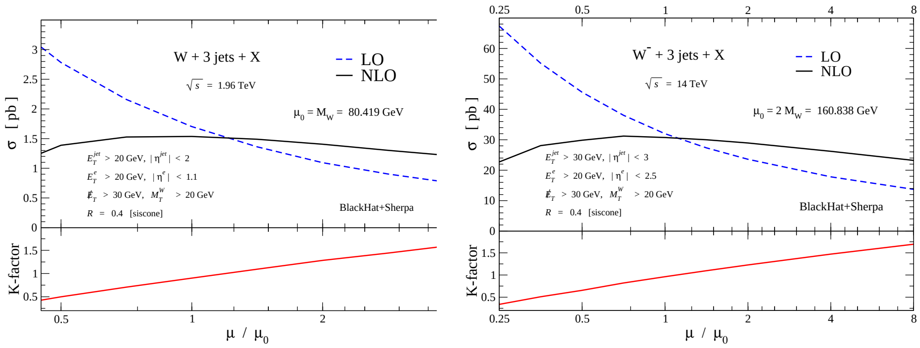

Fig. 2: The scale dependence of the cross sections for W + 3 jet production at the Tevatron and LHC (14 TeV) [7].

of the jet transverse momentum, at LO and NLO (for the 1 and 2 jet case). The NLO cross sections are observed to increase with increasing jet sizes, while the LO cross sections decrease (except for the trivial behavior for W + 1 jet, where there is only 1 parton in the final state. The slope for the LO cross sections becomes steeper as the number of partons increases. NLO predictions for W + 3 jets are not available, but would be very interesting to plot for comparison.

In this context, the K-factor for W + 3 jets, at a scale of m_W, can be at least partially understood. The problem does not lie with the NLO cross section. That is well-behaved. The problem is that the LO cross section sits $too - high$, due at least partially to the collinear enhancement that comes from a small jet size (0.4). For soft gluons (on the order of 20 GeV/c), there is in addition a residual impact from a soft singularity. The K-factor for W + 3 jets at the LHC would be smaller if (a) a larger jet size were used (b) a larger jet transverse momentum were used, or (c) a larger scale were used. In Ref.[7], it has been shown that a scale such as H_T results (at the LHC) not only in a K-factor closer to unity for W + 3 jets, but in similar shapes for kinematic distributions at LO and NLO. Scales that are typically used at the Tevatron, such as m_W or $m_W^2 + p_{T,W}^2$ lead to low normalizations and kinematic shapes that can be significantly different at LO than at NLO. Another study (Ref. [10]) has found that the kinematic shapes at LO and NLO can also be made similar if a $local$ scale, such as that obtained with the CKKW [11] procedure is used. The connection between these two observations is not obvious and deserves further investigation. More details are discussed in Ref. [4].

In Figure 5, we show the cross section for W^- + 3 jets at LO and NLO, as a function of jet size, uaing a renormalization and factorization scale of $H_T/2$, and two different jet algorithms (SISCone and antikT). Note that from the previous discussion, the LO and NLO cross sections should be more equal for this (large) scale. That is indeed found to be the case. Both the SISCone and antikT algorithm cross sections decrease linearly with increasing jet size at LO, while the cross section is relatively constant with jet size at NLO. The cross sections at LO have to decrease with increasing jet size, as a larger jet size excludes other partons from a greater region of phase space; SISCone algorithm cross sections are smaller than the antikT algorithm cross sections for the same jet size, as it excludes an effectively larger area. But at NLO, the algorithms from the two cross sections are much more similar (in fact exactly equivalent for a jet size of 0.4). At NLO, there is the possibility of combining two partons into a single jet, and this can effectively increase the cross section, partially compensating the exclusion effect discussed earlier.

From the above plot, it is apparent that the antikT cross section is larger than the SISCone cross section at LO, independent of jet size. Some theorists have drawn conclusions from this fact. In fact, it is fairly meaningless to compare the effects of jet algorithms on multi-parton final states at LO. We can see that at NLO the cross sections for the two algorithms are (1) different than at LO and (2) very similar to each other (exactly equal for R=0.4). In ATLAS, we dont measure LO partonic level cross sections; nature produces all order cross sections. This can be mimicked not only by parton shower Monte Carlos, but also to a large exent by NLO calculations. Also shown in the figure above are the results of running the antikT and SISCone algorithms on truth level information for W + 3 jet final states generated using

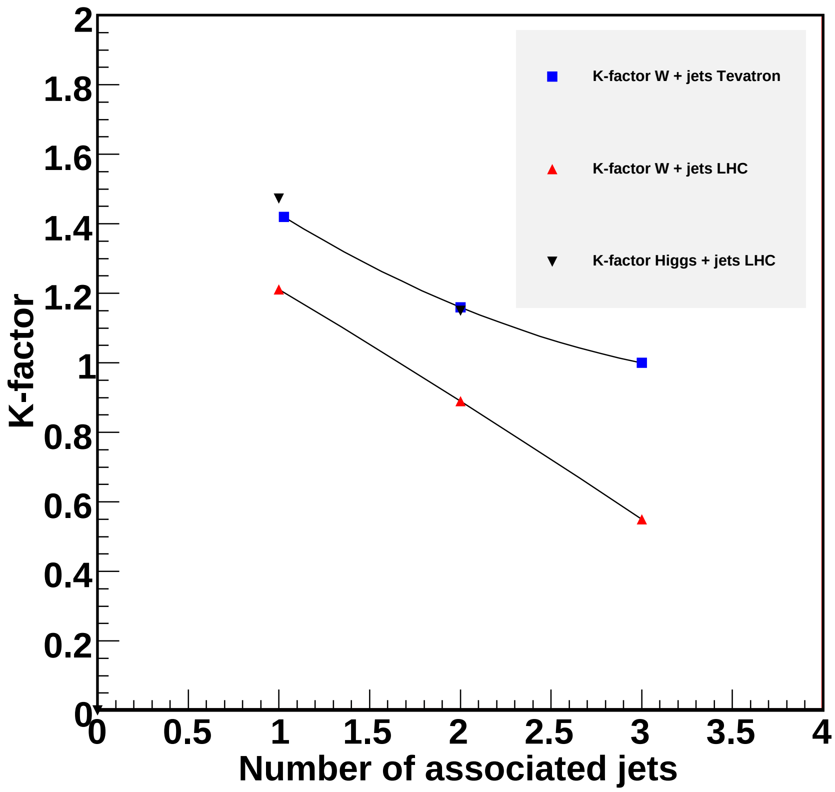

Fig. 3: The K-factors (NLO/LO) are plotted for W production at the Tevatron and LHC and for Higgs production at the LHC as a function of the number of accompanying jets. The k_T jet algorithm with a D parameter of 0.4 has been used.

ALPGEN+PYTHIA (to be added). The behavior, as a function of jet size, is observed to be much closer to the NLO predictions than the LO predictions.

Below we reproduce some of the cross sections from the first table, but now we add the K-factor obtained for the cross sections (with experimental cuts) as implemented in ATLAS Monte Carlo production, i.e. K-factor = NLO/(LO=ALPGEN+HERWIG/PYTHIA). The PDFs used for the ATLAS Monte Carlo production are specified, while the NLO cross sections uses the CTEQ6.6 PDFs.

4. Jet Sizes

From the experimental perspective, in complex final states such as W + n jets, it is useful to have smaller jet sizes so as to be able to resolve the n jet structure. Smaller jet sizes can also reduce the impact of pileup and underlying event [12]. From the theoretical perspective:

- hadronization effects become larger as R decreases
- for small R, the ln R perturbative terms referred to previously can become noticeable
- the restriction in phase space for small R can affect the scale dependence, i.e the scale uncertainty for an n-jet final state can depend on the jet size

The jet sizes to be used at the LHC should depend primarily on the needs of the experimental analyses. However, it will still be important to understand the impact that any choice of a jet size may have on the LO and NLO predictions, and the relation between the two predictions. This is another motivation for

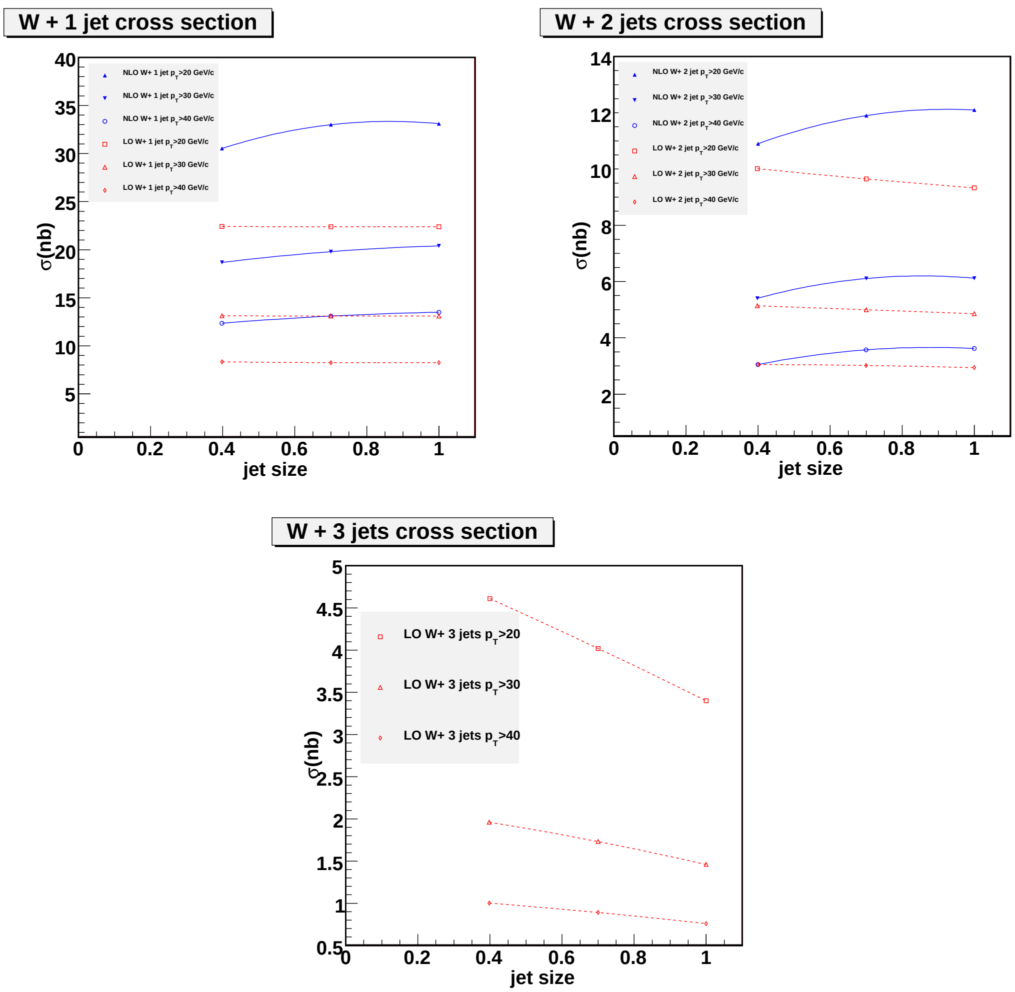

Fig. 4: The cross sections for W^+ + 1, 2 and 3 jets at the LHC at LO and NLO (for the 1 and 2 jet case) as a function of the jet size (using the k_T algorithm) and of the jet transverse momentum.

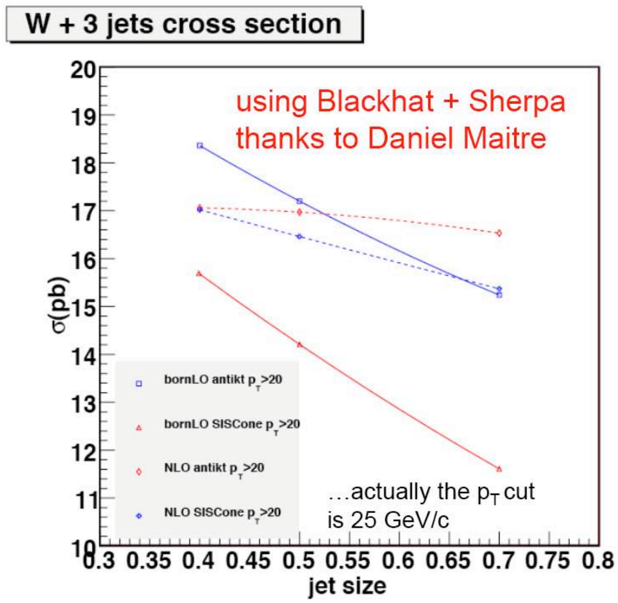

Fig. 5: The partonic level cross sections for W + 3 jets at LO and NLO as a function of the jet size, for the SISCone and antikT jet algorithms.

the use of multiple jet algorithms (and parameters) at the LHC in order to fully understand/explore the wide range of QCD dynamics for both standard model and beyond the standard model physics [13].

References

[1] J. M. Campbell, J. W. Huston and W. J. Stirling, Rept. Prog. Phys. **70**, 89 (2007) [arXiv:hep-ph/0611148].

[2] C. Buttar *et al.*, arXiv:hep-ph/0604120.

[3] Z. Bern *et al.* [NLO Multileg Working Group], arXiv:0803.0494 [hep-ph].

[4] J. R. Andersen *et al.* [SM and NLO Multileg Working Group], arXiv:1003.1241 [hep-ph].

[5] T. Binoth et al., A proposal for a standard interface between Monte Carlo tools and one-loop programs; to appear in the Les Houches 2009 proceedings.

[6] H. L. Lai, J. Huston, S. Mrenna, P. Nadolsky, D. Stump, W. K. Tung and C. P. Yuan, arXiv:0910.4183 [hep-ph].

[7] C. F. Berger *et al.*, Phys. Rev. D **80**, 074036 (2009) [arXiv:0907.1984 [hep-ph]].

[8] R. Keith Ellis, K. Melnikov and G. Zanderighi, Phys. Rev. D **80**, 094002 (2009) [arXiv:0906.1445 [hep-ph]].

[9] P. M. Nadolsky *et al.*, Phys. Rev. D **78**, 013004 (2008) [arXiv:0802.0007 [hep-ph]].

[10] K. Melnikov and G. Zanderighi, arXiv:0910.3671 [hep-ph].

[11] S. Catani, F. Krauss, R. Kuhn and B. R. Webber, JHEP **0111**, 063 (2001) [arXiv:hep-ph/0109231].

[12] M. Dasgupta, L. Magnea and G. P. Salam, JHEP **0802** (2008) 055 [arXiv:0712.3014 [hep-ph]].

[13] S. D. Ellis, J. Huston, K. Hatakeyama, P. Loch and M. Tonnesmann, Prog. Part. Nucl. Phys. **60**, 484 (2008) [arXiv:0712.2447 [hep-ph]].