Some Jet Studies Using SpartyJet

M. Campanelli
University College London

K. Geerlings, J. Huston
Michigan State University

Abstract

In this note, we compare the behavior of several different jet algorithms (and parameters) for analyses in ATLAS, using the SpartyJet framework. Our goal is to motivate both the use of the most modern jet clustering algorithms, as well as the use of multiple algorithms, in ATLAS physics analyses.
1 Introduction

Almost all physics channels at the LHC will contain jets in the final state. For this reason, it is important to give careful consideration to the jet clustering algorithms used both in physics analyses and in event triggering. Jets have been reconstructed in high energy physics experiments for over 30 years, using a variety of algorithms, but there have recently been advances both in the understanding of the physics involved in jet clustering as well as efficient coding that reduces the amount of computing power needed for practical analyses [1–4]. In addition, a great deal of experience has been gained at the Tevatron regarding the use of modern jet algorithms in a hadron-hadron collider environment. Our aim in this paper is to provide a systematic study of some characteristics of representative jet clustering algorithms and parameters in the ATLAS environment, using as an input one of the closest analogues an experiment can provide to four-vectors, i.e. the ATLAS topological clusters. Topological clusters are clusters of calorimeter cells already calibrated for detector measurement effects, to effectively the hadron level. We use as a framework for jet clustering the SpartyJet framework [5], an interface to the major jet clustering algorithms that allows easy change and control over relevant parameters. Our goal is to motivate both the use of the most modern jet clustering algorithms, as well as the use of multiple algorithms, in ATLAS physics analyses.

2 Algorithms considered

Jet clustering algorithms can be divided into two main classes: cones and iterative recombination (as, for example, the $k_T$ algorithm). Historically, cone algorithms have been used in hadron colliders, being the only algorithm fast enough to be implemented at the trigger level, and for fear of large systematic effects in busy multi-jet environments from recombination algorithms. Fast implementations of the $k_T$ clustering algorithm [4], as well as studies from the Tevatron performing precision measurements with it [6, 7], call for a detailed comparison of the $k_T$ algorithm with cone-based ones. Many implementations of cone algorithms have been developed over the years (and the experiments). Many of the algorithms have been shown to suffer from infrared safety issues, i.e. the results of the algorithm can change if very soft particles, that do not affect the overall topology of the event, are added or subtracted. Unfortunately, algorithms that have long been the default for large experiments, like JetClu for CDF and the Atlas cone for Atlas, belong to this category. Other algorithms, like Midpoint [8, 9] are stable under infrared corrections for most (but still not all) cases. But, since they start clustering jets around energy depositions larger than a given value (seed thresholds), the outcome will depend, in principle, on the value of this threshold. The manner in which this will affect clustering under real experimental conditions is one of the questions we will attempt to address in this study.

A seedless infrared-safe cone algorithm has recently emerged [3], providing most of the desirable features a cone algorithm needs from the theoretical point of view and a similar ease of use as previous cone algorithms. Its adoption by the experimental community has been slow due to the lack of a comprehensive comparison with more traditional approaches. Most of the studies presented in the following sections will involve comparisons between the $k_T$ algorithm (for the two different jet sizes (D parameter) of 0.4 and 0.6), the legacy Atlas cone, the Midpoint cone algorithm, the Cambridge/Aachen algorithm (similar to the $k_T$ algorithm but using only the distance between clusters and not their energy) and the seedless infrared cone algorithm (SISCone). Throughout the whole paper these algorithms will be identified by the same color, i.e. black for the $k_T$ algorithm (D=0.4; hereafter named kT04), red for the $k_T$ algorithm (D=0.6; hereafter named kT06), green for the Atlas cone (R=0.4), dark blue for SISCone (R=0.4), pink for MidPoint (R=0.4) and light blue for Cam-
3 Datasets

To perform our studies, we have used the Monte Carlo datasets produced in the context of the Atlas CSC notes exercise. In particular, we are interested in the behavior of jet algorithms in a multi-jet environment and in the forward region where small changes in cluster position can result in large rapidity differences. It was therefore natural to use samples involving the production of $W +$ jets and of a Higgs boson (through the weak boson fusion sub-process). The former were generated with ALPGEN [10], for the case of a $W$ boson decaying into a muon and a neutrino, produced in association with a number of partons ranging from 0 to 5; the latter are Herwig [11] samples, with a Higgs ($M_H = 120$ GeV) decaying into tau pairs, with each of the taus decaying into an electron, or a muon, and neutrinos.

Unless otherwise specified, the various algorithms are run on the same datasets of events; therefore the results obtained are not statistically independent, and even small differences can be significant. Jets reconstructed with an axis closer than $\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2} = 0.4$ with respect to the closest lepton (either from a $W$ decay or a $\tau$ (from $H \rightarrow \tau\tau$)) are discarded, to avoid biasing the jet reconstruction performances either by inclusion of those leptons in the jet or by mistaking a jet for a lepton or a tau decay product.

4 Jet Multiplicity

The first variable we examined is the jet multiplicity for events with a leptonically decaying $W$ and a number of partons from 0 to 5. The reconstructed number of jets with $p_T > 20$ GeV for the various algorithms is shown in Figure 1, where each plot represents a different number of generated partons. To understand the trends somewhat better, Figure 2 shows the difference between the mean number of reconstructed jets and the original number of partons, while Figure 3 shows the RMS of this distribution. As expected, the distribution of reconstructed number of jets broadens as the the number of partons increases, both at the reconstructed and at the generator level. Since only jets passing the 20 GeV $p_T$ cut are included, it is understandable that the multiplicity is higher for the kT06 than for the kT04 algorithm. This is true as well for large jet multiplicities, where the effect of the smaller available phase space for the larger jet size is not relevant for the multiplicities considered. On the other hand, SISCone tends to reconstruct a smaller number of jets than the other algorithms.

5 Matching efficiency

An important figure of merit of a jet algorithm is the ability to correctly find, after detector effects, jets with an axis as close as possible to the original. All comparisons between generated and reconstructed quantities are performed with jets reconstructed from stable particles at the hadron level using the same algorithm as at detector level. The matching efficiencies are defined as the number of hadron level jets in a given $p_T$ or $\eta$ bin that have a reconstructed jet within a given $\Delta R$ cut.

The $\Delta R$ distributions between the generated and the closest reconstructed jet are shown in Figure 4 for the SISCone, kT04, kT06 and Atlas cone jet algorithms, for the $W + 2$ partons Monte Carlo dataset. We note that the kT06 algorithm has the largest mean value of $\Delta R$, and therefore the worst matching, most probably because of fluctuations far from the core of the jet. Figure 5 shows the efficiency...
Figure 1: The number of reconstructed jets for the $W + n$ partons Monte Carlo datasets, with the number of partons increasing (from 0 to 5) with the plot order. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

Figure 2: The difference between the number of reconstructed jets versus the number of original partons plotted versus the number of original partons, for the $W + n$ partons Monte Carlo datasets. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

Figure 3: The RMS values for the distribution of the number of reconstructed jets, as a function of the number of generated partons, for the $W + n$ partons Monte Carlo datasets. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.
Figure 4: The distribution of $\Delta R$ values between generated and reconstructed jets, for the $W + 2$ partons Monte Carlo dataset. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

for various $p_T$ bins and for a range of $\Delta R$ cuts, for the $W + 2$ partons Monte Carlo dataset. For all algorithms, an efficiency higher than 95% (in red) is reached at high jet momenta even for quite tight $\Delta R$ cuts, while small differences among algorithms emerge at lower jet momenta. If we take the slices of this 2d plot corresponding to cut $\Delta R < 0.3$ and $\Delta R < 0.4$, respectively, we obtain the results in Figure 6.

These plots were produced from a $W + 2$ partons dataset, but all other datasets exhibit a similar behaviour, even for large parton multiplicities (see, for example, Figure 7 for distributions for $W + 5$ partons). SISCone does a very good job in these difficult situations, and fears of the kT algorithm picking up too much underlying event energy seem justified only in case of large jet size. The matching efficiency as a function of the jet $\eta$ for VBF Higgs events is shown in Figure 8. The different $\eta$ distribution, as well as the harder spectrum, may explains why jets from VBF Higgs events have a better matching efficiency than those from $W + n$ parton events.

6 Seed threshold and split/merge parameter

An obvious argument in favour of a seedless clustering algorithm is that the seed threshold is, in principle, an arbitrary parameter, and the dependence of jet reconstruction on arbitrary parameters should be avoided as much as possible. On the other hand, from the experimental point of view, any seed below the calorimeter noise-suppression cut should be equivalent, and no dependence on seed threshold should be observed for reasonable values of this parameter. To test this hypothesis, we looked at $W + 5$ parton events, with very low jet $p_T$ threshold (10 GeV), in order to enhance any differences. The number of jets reconstructed with the MidPoint algorithm with seed thresholds of 0.1, 1 and 2 GeV is almost the same, as shown in Figure 9. We see that no significant difference is
found for the different seed values, so the claim that reasonable seed values lead to similar results seems justified, at least for inclusive distributions of the type examined here. This does not reduce the theoretical merits of the use of a seedless algorithm, however.

To address the issue of the dependence of jet clustering on the split/merge parameter, we clustered
Figure 7: The matching efficiency for $W + 5$ parton events. The efficiency is smaller for all algorithms with respect to the $W + 1$ parton case, but recombination-based algorithms show no worse behavior than the cone-based ones. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone and dark blue for SISCone.

Figure 8: The matching efficiency for a fixed $\Delta R$ cut as a function of the jet $\eta$ for VBF Higgs Monte Carlo events. The matching requirement is $\Delta R < 0.3$ (top) and $\Delta R < 0.4$ (bottom). The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone and dark blue for SISCone.

$W + 2$ parton events using the Atlas cone and SISCone algorithms with the split/merge parameter set to 0.5, 0.625 and 0.75. Large differences are observed, as seen for instance for the SISCone algorithm in Figure 10, and perhaps a systematic study to fine tune this parameter could be useful. We
Figure 9: The number of jets reconstructed using the MidPoint algorithm with seed thresholds of 0.1 (red), 1 (black) and 2 GeV (green).

Figure 10: The matching efficiency for the requirement $\Delta R < 0.1$ for SISCones, for values of the split/merge parameter of 0.5 (black), 0.625 (green) and 0.75 (red).

have noticed that, out of the three options considered here, the best value of this parameter may be algorithm-dependent, and is in fact 0.5 for the Atlas cone and 0.75 for SISCones, which are presently the default values for these algorithms.

7 Energy reconstruction

Even after compensation for the different calorimeter response to electromagnetic and hadronic showers, Atlas topological clusters underestimate the total visible energy by about 5% due to noise-suppression threshold, particle losses, inefficiencies etc. This effect results in a systematically higher hadron-level energy with respect to the detector-level one, and is visible as a function of jet $p_T$ and $\eta$ for $W + 2$ parton events in Figures 11 and 12. As expected, this bias is larger for low-energy jets, where the relative importance of low-energy clusters (more prone to losses etc.) is higher. Also, the behavior in regions where there is substantial variation in calorimeter coverage, due to provision for services, differs considerably between the various algorithms.
Figure 11: The difference between the hadron and detector level jet $p_T$, divided by the hadron level transverse momentum, as a function of the jet $p_T$. The observed bias is due to a small residual correction needed for topoclusters, especially at low energy. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

Figure 12: The difference between the hadron and detector level jet $p_T$, divided by the hadron level transverse momentum, as a function of the jet $\eta$, integrating over the jet $p_T$ spectrum. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.
8 Cross sections

The measurement of $W + n$ jet cross sections allows a study of the effect of jet clustering on energy distributions as well as jet multiplicities. To select events with $W$ boson decays into a muon and a neutrino, we require the presence in the event of a muon of at least 25 GeV in the acceptance region $|\eta| < 2.4$ and missing transverse energy of at least 25 GeV. We accept jets if they have transverse momentum larger than 15 GeV, $|\eta| < 5$ and $\Delta R > 0.4$ with respect to the muon direction. Events are classified according to the number of reconstructed jets, and we studied the distribution of the $p_T$ of the leading jet for $W + n$ parton events. For space reasons, we show here only those results obtained with the $W + 2$ parton sample, but all other distributions show similar characteristics. The reconstructed $p_T$ spectra of the leading jet of each event passing the $W +$ jet selection are shown in Figure 13. We see that the different behavior observed for the larger jets in the kT06 algorithm is mainly due to the very soft transverse momentum region. With this jet size, there is the tendency to reconstruct a larger average number of jets. Thus, there are fewer events in the $W + 1$ jet category (the red histogram is always below the others for the first plot), and more in the cases where the reconstructed multiplicity is higher than the generated one (all plots from the third one on). By examining the $p_T$ spectra, we realize that this effect is mainly present for events with a soft leading jet, while for hard events (i.e. for higher $p_T$ of the leading jets) all distributions tend to converge.

![Figure 13: The reconstructed cross sections for the $W + 2$ partons sample, as a function of the $p_T$ of the leading jet, for six jet multiplicities (number of reconstructed jets = 1-6 from top to bottom) (arbitrary units). The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.](image)

9 Pileup

We know that in the first phases of LHC operation, the proton density in the bunches will be already high enough for the events to exhibit non-negligible pileup. No study of clustering algorithms would be complete without an assessment on the behaviour under realistic running conditions. Assuming
that pileup can be added linearly to the event (an approximation), we overlapped three minimum-bias events to the $W + n$ partons and Higgs VBF events considered in the previous sections, and examined how the quantities considered above get modified for the various algorithms by the presence of pileup energy. Note that we are considering only pileup in the same bunch crossing as the interaction of interest.

Figure 14: The number of reconstructed jets for the various $W + n$ partons samples (0,1,2,3,4,5 from top to bottom, left-to-right) in the presence of three pileup events. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

The first property studied here is the jet multiplicity. We see that the distribution of the number of jets for the $W + n$ partons sample (Figure 14) is modified. The behavior of the various algorithms can be seen in the mean value of the reconstructed multiplicity as a function of the number of original partons (Figures 15). A direct comparison between the no-pileup and pileup case is made in Figure 16, where we show the average number of reconstructed jets for Higgs VBF events without and with pileup. kT04 and SISCone are the two algorithms less sensitive to the presence of pileup.

In order to study the influence of pileup on the kinematic distributions of the reconstructed jets, Figure 17 shows the ratio of the $p_T$ distributions with and without pileup for each reconstructed jet multiplicity, for the $W + 2$ parton event sample.

The presence of pileup, leading to a modification of the jet axis direction, also influences the matching efficiency between hadron level and detector level jets. The efficiency as a function of jet $p_T$ and $\eta$, computed using the same definition as in the previous sections, is shown in Figures 18 and 19. The scale of robustness of the various algorithms to the presence of pileup obtained from the other tests is confirmed.

Finally, we tested the effect of using different algorithms on a simple forward jet selection, aiming at discriminating VBF Higgs events from the background. The following cuts were applied to the VBF Higgs and to the $W + 2$ partons and the $W + 3$ partons Monte Carlo event samples:

- two jets with $p_T^1 > 40$ GeV and $p_T^2 > 20$ GeV
Figure 15: The difference between the number of reconstructed jets and the number of generated partons vs the number of partons (with 3 pileup events). The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

Figure 16: The number of reconstructed jets for VBF Higgs events with/out pileup. The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

- both jets have $\Delta R > 0.4$ with respect to tau decay products
- $\Delta \eta_{1,2} > 4.4$
Figure 17: The ratio between cross section, with and without pileup, for all algorithms ($W + 2$ parton sample). The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

Figure 18: The efficiency vs jet $p_T$ with pileup ($\Delta R < 0.3$ and 0.4). The color code is: black for the kT04 algorithm, red for the kT06, green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

- invariant mass between the two jets $> 700$ GeV
- no third jet with $|\eta| < 3.2$ and $p_T > 30$

The efficiencies obtained by the three samples for three of the jet algorithms under study here are
Figure 19: The efficiency vs jet \( \eta \) with pileup \((\Delta R < 0.3 \text{ and } 0.4)\). The color code is: black for the \( kT04 \) algorithm, red for the \( kT06 \), green for the Atlas cone, dark blue for SISCone, pink for MidPoint and light blue for the Cambridge/Aachen algorithm.

summarized in Table 9.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>VBF Higgs</th>
<th>( W + 2p )</th>
<th>( W + 3p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cone 04</td>
<td>15.9±0.4</td>
<td>0.37±0.03</td>
<td>1.17±0.05</td>
</tr>
<tr>
<td>KT 04</td>
<td>15.1±0.4</td>
<td>0.17±0.02</td>
<td>0.85±0.04</td>
</tr>
<tr>
<td>SISCOne 04</td>
<td>14.2±0.4</td>
<td>0.17±0.02</td>
<td>0.76±0.04</td>
</tr>
</tbody>
</table>

Table 1: The selection efficiency (in percent) for the forward jet cuts described in the text, for the various algorithms applied to the three Monte Carlo samples of VBF Higgs, \( W + 2 \) and \( W + 3 \) partons.

While the change in efficiency on the Higgs signal is quite marginal, the same cannot be said for the difference in background rejection. Here the algorithms that have proven to be more robust under the influence of pileup exhibit a much better background rejection, and can improve the power of the analysis.

10 Conclusions

In this note we have systematically explored the behavior of several jet algorithms, \( k_T \) (with different jet sizes, corresponding to the choice of D parameters of 0.4 and 0.6), the Atlas Cone, SISCone, MidPoint and Cambridge/Aachen (all with cone radius of 0.4), on several benchmarks with and without the presence of pileup. The comparison of smaller and larger jet size in the \( k_T \) algorithm has shown that the use of larger jets deteriorates the resolution in jet direction, and is more vulnerable to the presence of pileup, so should be avoided for the purpose of jet finding, even if it may be more accurate in determining jet energy. The comparison of the different algorithms with approximately the same jet
size corresponding to a radius of 0.4 indicates that the $k_T$ and SISCone algorithms have proven to be at least equivalent to those algorithms previously used in hadron colliders. Thus, we encourage the use of the recent available implementations of those algorithms. More detailed studies are still required, however, especially with real data.

References


