QCD and Jets

J. Huston
Michigan State University

...thanks to the Ellis brothers, Keith and Steve, Nikos Varelas, ...
Some references

In this paper, we will develop the perturbative framework for the calculation of hard-scattering processes. We will undertake to provide both a reasonably rigorous development of the formalism of hard-scattering of quarks and gluons as well as an intuitive understanding of the physics behind the scattering. We will emphasize the role of logarithmic corrections as well as power counting in $\alpha_s$ in order to understand the behaviour of hard-scattering processes. We will include ‘rules of thumb’ as well as ‘official recommendations’, and where possible will seek to dispel some myths. We will also discuss the impact of soft processes on the measurements of hard-scattering processes. Experiences that have been gained at the Fermilab Tevatron will be recounted and, where appropriate, extrapolated to the LHC.

(Some figures in this article are in colour only in the electronic version)

goal is to provide a reasonably global picture of LHC calculations (with rules of thumb)
Towards Jetography

Gavin P. Salam

LPTHE, UPMC Univ. Paris 6, CNRS UMR 7589, 75252 Paris 05, France

Abstract

As the LHC prepares to start taking data, this review is intended to provide a QCD theorist’s understanding and views on jet finding at hadron colliders, including recent developments. My hope is that it will serve both as a primer for the newcomer to jets and as a quick reference for those with some experience of the subject. It is devoted to the questions of how one defines jets, how jets relate to partons, and to the emerging subject of how best to use jets at the LHC.

Jets are sprays of particles that fly out from certain high-energy collisions—for instance, from violent collisions of protons and antiprotons at Fermilab’s Tevatron accelerator, or in the similar proton-proton collisions that will take place at CERN’s Large Hadron Collider. These collisions create very energetic quarks and gluons as they travel away from the collision point, they emit more gluons, which can split into even more gluons. This results in a relatively narrow cascade, or jet of particles.

In the last stage of jet creation, quarks and gluons combine to form particles such as protons, pions, and kaons. By measuring these end products, physicists can determine the properties of a jet, and thus the details of the collision that produced it. Scientists expect to see jets in the signatures of almost every interesting collision at the Large Hadron Collider.

The most violent collisions will produce jets with the highest momentum, and these can be used to probe the smallest distances within the colliding protons, less than one-billionth of a billionth of a meter. Physicists hope they can use these most energetic jets to look inside the quarks that make up protons.

Joeign Huston, Michigan State University

“When you’re a jet, you’re a jet all the way, from your first gluon split to your last K decay...”
Caveat

- I’m not a theorist
- I don’t even play one on TV
- So my lectures are not going to be in as much technical detail as a theorist would
  - because I probably wouldn’t get the details right
  - and I like the intuitive “rules-of-thumb” approach better
- But there are references that do go into such detail, as for example the book to the right
  - often termed as the “pink book”
We’ll look back on early LHC trouble in 15 years and laugh

**LHC vs time: a wild guess...**

You are here (even though it’s now 2009)
Understanding cross sections at the LHC

- But before we can laugh, we have to understand the Standard Model at the LHC
- We’re all looking for BSM physics at the LHC
- Before we publish BSM discoveries from the early running of the LHC, we want to make sure that we measure/understand SM cross sections
  - detector and reconstruction algorithms operating properly
  - SM backgrounds to BSM physics correctly taken into account
  - and, in particular, that QCD at the LHC is properly understood
Cross sections at the LHC

- Experience at the Tevatron is very useful, but scattering at the LHC is not necessarily just “rescaled” scattering at the Tevatron
- Small typical momentum fractions $x$ for the quarks and gluons in many key searches
  - dominance of gluon and sea quark scattering
  - large phase space for gluon emission and thus for production of extra jets
  - intensive QCD backgrounds
  - or to summarize,...lots of Standard Model to wade through to find the BSM pony
Cross sections at the LHC

- Note that the data from HERA and fixed target cover only part of kinematic range accessible at the LHC
- We will access pdf’s down to $10^{-6}$ (crucial for the underlying event) and $Q^2$ up to 100 TeV$^2$
- We can use the DGLAP equations to evolve to the relevant $x$ and $Q^2$ range, but…
  - we’re somewhat blind in extrapolating to lower $x$ values than present in the HERA data, so uncertainty may be larger than currently estimated
  - we’re assuming that DGLAP is all there is; at low $x$ BFKL type of logarithms may become important (more later about DGLAP and BFKL)

$$\frac{d\sigma}{dM^2dy} = \frac{\hat{g}_0}{N_s} \left[ \sum_k Q_k^2(q_k(x_1, M^2)\bar{q}_k(x_2, M^2) + [1 \leftrightarrow 2]) \right]$$

LHC parton kinematics

$Q = M$

$M = 10$ TeV

$M = 1$ TeV

$M = 100$ GeV

$M = 10$ GeV

$\chi_{1,2} = (M/14 \text{ TeV}) \exp(\pm y)$

$y = 6, 4, 2, 0, 2, 4, 6$
Understanding cross sections at the LHC

I’ll try to touch on all of these topics in these lectures.

Most experimenters are/will still mostly use parton shower Monte Carlo for all predictions/theoretical comparisons at the LHC. I’ll try to show that there’s more than that.
I’ll try to touch on all of these topics in these lectures, with a concentration on pdf’s and jets, two areas I work in.
Some definitions

- The fundamental challenge to interpret experimentally observed final states is that pQCD is most easily applied to the short-distance degrees of freedom, i.e. to quarks and gluons, while the long-distance degrees of freedom seen in the detectors are color-singlet bound states.

- The overall scattering process evolves from the incoming long-distance hadrons in the beams, to the short-distance scattering process, to the long-distance outgoing final states.

- The separation of these steps is essential both conceptually and calculationally.
...and a word about jets

- Most of the interesting physics signatures at the Tevatron and LHC involve final states with jets of hadrons.
- A jet is reconstructed from energy depositions in calorimeter cells and/or from charged particle track momenta, and ideally is corrected for detector response and resolution effects so that the resultant 4-vector corresponds to that of the sum of the original hadrons.
- The jets can be further corrected, for hadronization effects, back to the parton(s) from which the jet originated.
- The resultant measurements can be compared back to parton shower predictions, or to the short-distance partons described by fixed-order perturbative calculations.
...another word about jets

- We pick out from the incident beam particles, the short-distance partons that participate in the hard collision
- The partons selected can emit radiation prior to the short distance scattering leading to initial state radiation
- The remnants of the original hadrons, with one parton removed, will interact with each other, producing an underlying event
- Next comes the short-distance, large momentum transfer scattering process that may change the character of the scattering partons, and/or produce more partons
  - the cross section for this step is calculated to fixed order in pQCD
…still another word about jets

- Then comes another color radiation step, when many new gluons and quark pairs are added to the final state.
- The final step in the evolution to the long distance states involves a nonperturbative hadronization process that organizes the colored degrees of freedom.
- This non-perturbative hadronization step is accomplished in a model-dependent fashion.
The Standard Model has been extremely successful, although admittedly incomplete.

In these lectures, we’re most interested in QCD and thus the force carrier of the strong force (the gluon) and its interaction with quarks (and with itself).

Start with the QCD Lagrangian…
The (Classical) QCD Lagrangian

\[ L_{QCD} = -\frac{1}{4} F^B_{\alpha \beta} F^\alpha_{\beta} + \sum_f \bar{q}_{f,a} \left( i D_\mu \gamma^\mu - m_f \right)_{ab} q_{f,b} \]

\[ F^B_{\alpha \beta} = \left[ \partial_\alpha A^B_\beta - \partial_\beta A^B_\alpha - g f^{BCD} A^C_\alpha A^D_\beta \right] \]

Acting on the triplet and octet, respectively, the covariant derivative is

\[ (D_\mu)_{ab} = \partial_\mu \delta_{ab} + ig \left( t^C A^C_\mu \right)_{ab} ; (D_\mu)_{CD} = \partial_\mu \delta_{CD} + ig \left( T^B A^B_\mu \right)_{CD} \]

The matrices for the fundamental \( (t^B_{ab}) \) and adjoint \( (T^{CD}_B) \) representations carry the information about the Lie algebra

\[ [t^B, t^C] = if^{BCD} t^D ; [T^B, T^C] = if^{BCD} T^D ; (T^B)_{CD} = -if^{BCD} ; (f^{BCD} \text{ is the structure constant of the group}) \]

\[ Tr[t^B t^C] = \frac{\delta^{BC}}{2} \equiv T_R \delta^{BC} ; t^B_{ab} t^B_{bc} = \frac{4}{3} \delta_{ac} \equiv C_F \delta_{ac} ; \]

\[ Tr[T^B T^C] = 3 \delta^{BC} \equiv C_A \delta^{BC} \]

...thanks to Steve Ellis for the next few slides

\[ C_A = N_{\text{colors}} = 3 \]

\[ C_F = \frac{N_{\text{colors}}^2 - 1}{2 N_{\text{colors}}} = \frac{4}{3} \]
Feynman Rules:

Propagators – (in a general gauge represented by the parameter $\lambda$, Feynman gauge is $\lambda = 1$; this form does not include axial gauges)

Vertices –
Quark – gluon

Quark $\alpha_a$ $\rightarrow$ $\beta_b$

$\frac{i\delta_{ab}}{(\gamma^\mu q_\mu - m)_{\alpha\beta}} = \frac{i(\gamma^\mu q_\mu + m)_{\alpha\beta}}{q^2 - m^2}$

Gluon

$A^{\mu}_{\mu} \rightarrow B^{\nu}_{\nu}$

$\frac{-i}{q^2} \delta_{AB} [g^{\mu\nu} - (1 - \lambda) \frac{q^\mu q^\nu}{q^2}]$

Vertices –
Quark – gluon
3 gluons

$A, \ q_{1}, \ e_{\mu}$

$B, \ q_{2}, \ e_{\nu}$

$C, \ q_{3}, \ e_{\lambda}$

$-gf^{ABC}[(q_{1} - q_{2})^{\mu} g^{\nu\lambda} + (q_{2} - q_{3})^{\mu} g^{\nu\lambda} + (q_{3} - q_{1})^{\mu} g^{\nu\lambda}]$
Feynman Rules II:

4 gluons

\[ -ig^2 f_{EAD} f_{EBC} [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma}] \]

\[ -ig^2 f_{EAC} f_{EBD} [g^{\alpha\beta} g^{\gamma\delta} - g^{\alpha\delta} g^{\beta\gamma}] \]

\[ -ig^2 f_{EAB} f_{ECD} [g^{\alpha\gamma} g^{\beta\delta} - g^{\alpha\delta} g^{\beta\gamma}] \]
Naïve QCD Feynman diagrams exhibit infinities at nearly every turn, as they must in a conformal theory with no “bare” dimensionful scales (ignore quark masses for now).***

First consider life in the Ultra-Violet – short distance/times or large momenta (the Renormalization Group at work):

- The UV singularities mean that the theory
  - does not specify the strength of the coupling in terms of the “bare” coupling in the Lagrangian
  - does specify how the coupling varies with scale [$\alpha_s(\mu)$ measures the “charge inside” a sphere of radius $1/\mu$]

*** Typical of any renormalizable gauge field theory. This is one reason why String theorists want to study something else! We will not discuss the issue of choice of gauge. Typically axial gauges ($\hat{n} \cdot \hat{A} = 0$) yield diagrams that are more parton-model-like, so-called physical gauges.
Consider a range of distance/time scales – $1/\mu$

- use the renormalization group below some (distance) scale $1/m$ (perhaps down to a GUT scale $1/M$ where theory changes?) to sum large logarithms $\ln[M/\mu]$

- use fixed order perturbation theory around the physical scale $1/\mu \sim 1/Q$ (at hadronic scale $1/m$ things become non-perturbative, above the scale $M$ the theory may change)

\begin{tikzpicture}
  \draw[->,thick] (0,0) -- (6,0);
  \draw (0,0) node[anchor=north west] {renormalization group} -- node[anchor=south west] {?} (2,0) node[anchor=north west] {fixed order} -- node[anchor=south west] {renormalization group} (4,0) node[anchor=north west] {log($1/M$)} -- (6,0) node[anchor=north west] {log($1/m$)};
  \draw[->,purple] (-0.5,0.5) -- (6.5,0.5) node[anchor=south] {Short distance} -- (6.5,-0.5) -- (-0.5,-0.5) node[anchor=north] {Long distance};
  \draw[<->,purple] (-0.5,0) -- (6.5,0) node[anchor=south] {} node[anchor=north] {};\end{tikzpicture}

new physics?? perturbative non-perturbative
Strong coupling constant $\alpha_s$

An important component of all QCD cross sections

QED:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \frac{\beta_0}{2\pi} \alpha(\mu^2) \log \frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{2}{3}$$

QCD:

$$\beta_0 = \frac{2}{3} T_F N_F - \frac{11}{3} C_A = \frac{2}{3} N_F - \frac{11}{3} N_C$$

Low resolution: charge is screened by $ee$-pairs
High resolution: charge is big

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0 \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2}}$$

positive term, since $b_0 < 1$

where $b_0 = -\frac{\beta_0}{4\pi}$

Note there can be different conventions, i.e. $\beta_0$ can be defined so it’s positive, but then $b_0 = +\frac{\beta_0}{4\pi}$
It’s important that the \( \beta \) function is negative.

An important component of all QCD cross sections is

\[
\beta_0 = \frac{2}{3} T_F N_F - \frac{11}{6} C_A = \frac{2}{3} N_F - \frac{11}{3} N_C
\]

\( \alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0 \alpha_s(\mu^2) \log \frac{Q^2}{\mu^2}} \)

The positive term, since \( b_0 < 1 \)

where \( b_0 = \frac{-\beta_0}{4\pi} \)

\( \beta_0 < 0 \) for \( N_F \leq 16 \)

\( \rightarrow \) anti-screening

Charge is spread-out by gluons, i.e., at infinite resolution charge is very small.

\( N_F \) number of fermions
\( N_c \) number of colours
\( T_F = \frac{1}{2}, C_A = N_c \) colour factors
\( \alpha_s \) and \( \Lambda \)

At 1-loop:
\[
\alpha(Q^2) = \alpha(\mu^2) \frac{1}{1 + b_0 \alpha(\mu^2) \log \frac{Q^2}{\mu^2}}
\]

with \( b_0 = \frac{33 - 2N_f}{12 \pi} \)

\( \mu \) is arbitrary parameter (left-over from renormalisation)

Choose \( \mu = \Lambda \): point where effective coupling becomes large

\[
\Lambda^2 = \mu^2 \exp(\frac{1}{b_0} \alpha_s(\mu^2)) \text{ or } \alpha_s(\mu^2) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2}}
\]

Therefore:

\[
\alpha_s(Q^2) = \frac{1}{b_0 \log \frac{\mu^2}{\Lambda^2} + b_0 \log \frac{Q^2}{\mu^2}} = \frac{1}{b_0 \log \frac{Q^2}{\Lambda^2}}
\]

\( Q^2 \gg \Lambda^2 \): \( \alpha_s(Q^2) \) small \( \rightarrow \) perturbative QCD applicable

\( Q^2 \approx \Lambda^2 \): quark and gluons form bound states

\( \Lambda \) is free parameter of theory, has to be determined by experiment \( \rightarrow \) expected to be of order of hadron mass

QCD explains confinement of colour and allows calculations of hard hadronic processes via perturbative expansion of coupling!
see www-theory.lbl.gov/~ianh/alpha/alpha.html

usually working in this range, so $\alpha_s < 0.2$

@ scale of $m_Z$, world average for $\alpha_s$ is 0.118 (NLO) and 0.130 (LO)

$\Lambda$ is a free parameter of the theory, has to be determined by experiment; expected to be of order of hadron mass

QCD explains confinement of colour and allows calculations of hard hadronic processes via perturbative expansion of coupling!
Factorization

- Factorization is the key to perturbative QCD
  - the ability to separate the short-distance physics and the long-distance physics
- In the pp collisions at the LHC, the hard scattering cross sections are the result of collisions between a quark or gluon in one proton with a quark or gluon in the other proton
- The remnants of the two protons also undergo collisions, but of a softer nature, described by semi-perturbative or non-perturbative physics

The calculation of hard scattering processes at the LHC requires:
(1) knowledge of the distributions of the quarks and gluons inside the proton, i.e. what fraction of the momentum of the parent proton do they have
-> parton distribution functions (pdf’s)
(2) knowledge of the hard scattering cross sections of the quarks and gluons, at LO, NLO, or NNLO in the strong coupling constant $\alpha_s$
Parton distributions

- The momentum of the proton is distributed among the quarks and gluons that comprise it:
  - about 40% of the momentum is with gluons, the rest with the quarks
- We’ll get back to pdf’s for more detail later, but for now notice that the gluon distribution dominates at small momentum fractions (x), while the (valence) quarks dominate at high x

Figure 27. The CTEQ6.1 parton distribution functions evaluated at a $Q$ of 10 GeV.
Factorisation Theorem:
PDF is universal
Once extracted can\_p calculate any
cross-section within
same theoretical
scheme

Mandelstamm variables:
\( s = (p_1 + p_4)^2 \)
\( t = (p_1 + p_3)^2 \)
\( u = (p_1 + p_4)^2 \)

\[ \sigma = \sum_{ij} \int dx_1 \, dx_2 \, dt \, f_i(x_1, \mu^2) \, f_j(x_2, \mu^2) \, d\hat{\sigma}_{ij}/d\hat{t} \]

\( e.g.: \) qq - scattering in LO: \[ d\hat{\sigma}_{ij}/d\hat{t} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \alpha \frac{1}{p_T^2} \]

\( \rightarrow \) Diverges for low \( P_T \to 0 \)
Factorization theorem

Factorisation Theorem: PDF is universal
Once extracted can calculate any cross-section within same theoretical scheme

Mandelstamm variables:
\( \hat{s} = (p_1 + p_1)^2 \)
\( \hat{t} = (p_i + p_3)^2 \)
\( \hat{u} = (p_1 + p_4)^2 \)

\[
\sigma = \sum_{ij} \int dx_1 dx_2 d\hat{t} f_i(x_1, \mu^2) f_j(x_2, \mu^2) d\hat{\sigma}_{ij} / d\hat{t}
\]

e.g.: \( qq \)-scattering in LO:
\[
d\hat{\sigma}_{ij} / d\hat{t} = \frac{\pi}{\hat{s}^2} \left( \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \right) \propto \frac{1}{p_T^2}
\]

→ Diverges for low \( P_T \to 0 \)
Factorization theorem

Factorisation Theorem: PDF is universal. Once extracted can\( p \) calculate any cross-section within same theoretical scheme.

Mandelstamm variables:
\[
\begin{align*}
\bar{s} &= (p_1 + p_1)^2 \\
\bar{t} &= (p_1 + p_3)^2 \\
\bar{u} &= (p_1 + p_4)^2
\end{align*}
\]

\[
\sigma = \sum_{ij} \int dx_1 dx_2 d\hat{t} f_i(x_1, \mu^2) f_j(x_2, \mu^2) d\hat{\sigma}_{ij} / d\hat{t}
\]

\( e.g.: qq - scattering \) in \( LO \):
\[
d\hat{\sigma}_{ij} / d\hat{t} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2} \propto \frac{1}{p_T^2}
\]

\( \Rightarrow \) Diverges for low \( p_T \rightarrow 0 \)
Go back to some SM basics: Drell Yan

- Consider Drell-Yan production
  - write cross section as
    \[
    \sigma_{AB} = \int dx_a dx_b \ f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}_{ab \to X}
    \]
  - where \( X = l^+l^- \)
- Potential problems appeared to arise from when perturbative corrections from real and virtual gluon emissions were calculated
  - but these logarithms were the same as those in structure function calculations and thus can be absorbed, via DGLAP equations in definition of parton distributions, giving rise to logarithmic violations of scaling
  - can now write the cross section as
    \[
    \sigma_{AB} = \int dx_a dx_b \ f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab \to X}
    \]

*Figure 1. Diagrammatic structure of a generic hard-scattering process.*
...but

- Key point is that all logarithms appearing in Drell-Yan corrections can be factored into renormalized (universal) parton distributions
  - factorization
- But finite corrections left behind after the logarithms are not universal and have to be calculated separately for each process, giving rise to order $\alpha_s^n$ perturbative corrections
- So now we can write the cross section as

$$\sigma_{AB} = \int dx_a dx_b \ f_{a/A}(x_a, \mu_F^2) \ f_{b/B}(x_b, \mu_F^2) \times \left[ \hat{\sigma}_0 + \alpha_S(\mu_R^2) \hat{\sigma}_1 + \cdots \right]_{ab \to X}.$$  

- where $\mu_F$ is the factorization scale (separates long and short-distance physics) and $\mu_R$ is the renormalization scale for $\alpha_s$
- choose $\mu_R = \mu_F \sim Q$

An all-orders cross section has no dependence on $\mu_F$ and $\mu_R$; a residual dependence remains (to order $\alpha_s^{n+1}$) for a finite order ($\alpha_s^n$) calculation.
Parton distributions used in hard-scattering calculations are solutions of DGLAP equations (or in Italy the AP equations)

- the DGLAP equations determine the $Q^2$ dependence of the pdf's
- the splitting functions have the perturbative expansions

\[
\frac{\partial q_i(x, \mu^2)}{\partial \log \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{q_i q_j}(z, \alpha_s) q_j \left( \frac{x}{z}, \mu^2 \right) + P_{q_i g}(z, \alpha_s) g \left( \frac{x}{z}, \mu^2 \right) \right\},
\]

\[
\frac{\partial g(x, \mu^2)}{\partial \log \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left\{ P_{g q_j}(z, \alpha_s) q_j \left( \frac{x}{z}, \mu^2 \right) + P_{g g}(z, \alpha_s) g \left( \frac{x}{z}, \mu^2 \right) \right\},
\]

Thus, a full NLO calculation will contain both $\hat{\alpha}_1$ (previous slide) and $P_{ab}^{(1)}$
Altarelli-Parisi splitting functions

Note that the emitted gluon likes to be soft

Here the emitted gluon can be soft or hard

We’ll also encounter the A-P splitting functions later, when we discuss parton showering and Sudakov form factors.
Kinematics

- Double differential cross section for production of a Drell-Yan pair of mass $M$ and rapidity $y$ is given by

$$\frac{d\sigma}{dM^2dy} = \frac{\hat{\sigma}_0}{N_s}\left[ \sum_k Q_k^2(q_k(x_1, M^2)\bar{q}_k(x_2, M^2) + [1 \leftrightarrow 2]) \right]$$

- where

$$\hat{\sigma}_0 = \frac{4\pi\alpha^2}{3M^2}$$

- and

$$x_1 = \frac{M}{\sqrt{s}} e^y, \quad x_2 = \frac{M}{\sqrt{s}} e^{-y}.$$  

- Thus, different values of $M$ and $y$ probe different values of $x$ and $Q^2$.
Cross sections for on-shell $W'/Z$ production (in narrow width limit) given by

$$\hat{\sigma}_{q\bar{q}' \rightarrow W} = \frac{\pi}{3} \sqrt{2} G_F M_W^2 |V_{qq'}|^2 \delta(\hat{s} - M_W^2),$$

$$\hat{\sigma}_{q\bar{q} \rightarrow Z} = \frac{\pi}{3} \sqrt{2} G_F M_Z^2 (v_q^2 + a_q^2) \delta(\hat{s} - M_Z^2),$$

Where $V_{qq'}$ is appropriate CKM matrix element and $v_q$ and $a_q$ are the vector and axial coupling of the Z to quarks.

Note that at LO, there is no $\alpha_s$ dependence; EW vertex only.

NLO contribution to the cross section is proportional to $\alpha_s$; NNLO to $\alpha_s^2$...

LO->NLO is a large correction at the Tevatron
NLO->NNLO is a fairly small (+) Correction

W/Z cross sections have small experimental systematic errors with theory errors (pdf’s/higher orders) also under reasonable(?) control.

Figure 4. Predictions for the $W$ and $Z$ total cross sections at the Tevatron and LHC, using MRST2004 [10] and CTEQ6.1 pdfs [11], compared with recent data from CDF and D0. The MRST predictions are shown at LO, NLO and NNLO. The CTEQ6.1 NLO predictions and the accompanying pdf error bands are also shown.
W/Z $p_T$ distributions

- Most W/Z produced at low $p_T$, but can be produced at non-zero $p_T$ due to diagrams such as shown on the right; note the presence of the QCD vertex, where the gluon couples (so one order higher)

$$
\sum |M^{q\bar{q}' \rightarrow Wg}|^2 = \pi \alpha_s \sqrt{2} G_F M_W^2 |V_{qq'}|^2 \frac{8}{9} \frac{\hat{t}^2 + \hat{u}^2 + 2M_W^2 \hat{s}}{\hat{t}\hat{u}} ,
$$

$$
\sum |M^{gq \rightarrow Wq'}|^2 = \pi \alpha_s \sqrt{2} G_F M_W^2 |V_{qq'}|^2 \frac{1}{3} \frac{\hat{s}^2 + \hat{u}^2 + 2\hat{t}M_W^2}{-\hat{s}\hat{u}} ,
$$

- Sum is over colors and spins in initial state, averaged over same in final state
- Transverse momentum distribution is obtained by convoluting these matrix elements with pdf's in usual way

Note that 2->2 matrix elements are singular when final state partons are soft or collinear with initial state partons (soft and collinear->double logarithms)

Related to poles at $t=0$ and $u=0$

But singularities from real and virtual emissions cancel when all contributions are included, so NLO is finite
W/Z $p_T$ distributions

- Back to the 2->2 subprocess

\[ |M_{ud \rightarrow Wg}|^2 \sim \left( \frac{\hat{t}^2 + \hat{u}^2 + 2Q^2 \hat{s}}{\hat{t}\hat{u}} \right) \]

- where $Q^2$ is the virtuality of the W boson

- Convolute with pdf's

\[ \sigma = \int dx_1 dx_2 f_u(x_1, Q^2) f_d(x_2, Q^2) \frac{|M|^2}{32\pi^2 \hat{s}} \frac{d^3p_W}{E_W} \frac{d^3p_g}{E_g} \delta(p_u + p_d - p_g - p_W) \]
W/Z $p_T$ distributions

- Transform into differential cross section

$$\frac{d\sigma}{dQ^2 dy dp_T^2} \sim \frac{1}{s} \int dy_q f_u(x_1, Q^2) f_{\bar d}(x_2, Q^2) \frac{|M|^2}{\hat{s}}$$

- where we have one integral left over, the gluon rapidity

- Note that $p_T^2 = t\bar{u}/s^2$
  - thus, leading divergence can be written as $1/p_T^2$ (Brems)

- In this limit, behavior of cross section becomes

$$\frac{d\sigma}{dQ^2 dy dp_T^2} \sim \frac{2}{s} \frac{1}{p_T^2} \int dy_q f_u(x_1, Q^2) f_{\bar d}(x_2, Q^2) + \text{(sub-leading in } p_T^2)$$

- As $p_T$ of $W$ becomes small, limits of $y_g$ integration are given by $+/\log(s^{1/2}/p_T)$

- The result then is

...diverges unless we apply a $p_T^{\text{min}}$ cut; so we end up with a distribution that depends not only on $\alpha_s$ but on $\alpha_s \times \log(s/p_T^2)$

universal theme
Now look at rapidity distributions for jet for two different choices of $p_T^{\text{min}}$

- Top plot implies that gluon is radiated off initial state parton at an early time (ISR)
- With collinear pole, this would imply that these gluons would be emitted primarily at forward rapidities
- But the distributions look central
- The reason is that we are binning in $p_T$ and not in energy, and the most effective place to convert from $E$ to $p_T$ is at central rapidities

Suppose I re-draw the Feynman diagrams as shown to the right
- is there a difference from what is shown at the top of the page?
Now on to $W + 2$ jets

- For sake of simplicity, consider $Wgg$
- Let $p_1$ be soft
- Then can write

\[
\mathcal{M}^{q\bar{q}\to Wgg} = t^A t^B (D_2 + D_3) + t^B t^A (D_1 - D_3)
\]

- where $t^A$ and $t^B$ are color labels of $p_1$ and $p_2$
- Square the matrix amplitude to get

\[
|\mathcal{M}^{q\bar{q}\to Wgg}|^2 = N C_F^2 \left[ |D_2 + D_3|^2 + |D_1 - D_3|^2 \right] - C_F \text{ Re} \left[ (D_2 + D_3)(D_1 - D_3)^* \right]
\]

\[
= \frac{C_F N^2}{2} \left[ |D_2 + D_3|^2 + |D_1 - D_3|^2 - \frac{1}{N^2} |D_1 + D_2|^2 \right].
\]
Since $p_1$ is soft, can write D’s (color-ordered amplitudes) as product of an eikonal term and the matrix elements containing only 1 gluon:

$$D_2 + D_3 \rightarrow \epsilon_\mu \left( \frac{q^\mu}{p_1.\bar{q}} - \frac{p_2^\mu}{p_1.p_2} \right) \mathcal{M}_{q\bar{q}\rightarrow Wg},$$

$$D_1 - D_3 \rightarrow \epsilon_\mu \left( \frac{p_2^\mu}{p_1.p_2} - \frac{\bar{q}^\mu}{p_1.\bar{q}} \right) \mathcal{M}_{q\bar{q}\rightarrow Wg},$$

where $\epsilon_\mu$ is the polarization vector for gluon $p_1$.

Summing over gluon polarizations, we get

$$|\mathcal{M}_{q\bar{q}\rightarrow W_{gg}}|^2 \xrightarrow{\text{soft}} \frac{C_F N^2}{2} \left[ [q, p_2] + [p_2, \bar{q}] \right] - \frac{1}{N^2} [q, \bar{q}] \mathcal{M}_{q\bar{q}\rightarrow Wg}$$

where

$$\frac{a.b}{p_1.a \ p_1.b} \equiv [a \ b].$$
The leading term (in number of colors) contains singularities along two lines of color flow—one connecting gluon $p_2$ to the quark and the other connecting it to the anti-quark.

Sub-leading term has singularities along the line connecting the quark and anti-quark.

It is these lines of color that indicate preferred direction for emission of additional gluons.

- Needed by programs like Pythia/Herwig for example.

Figure 12. Two examples of colour flow in a $W + 2$ jet event, shown in red. In the left-hand diagram, a leading colour flow is shown. The right-hand diagram depics the sub-leading colour flow resulting from interference.
Eikonal factors

- Re-write

$$\frac{a.b}{p_1.a \ p_1.b} \equiv [a \ b].$$

- As

$$[a \ b] \ dP S_{\text{gluon}} = \frac{1}{E^2} \frac{1}{1 - \cos \theta_a} \ E \ dE \ \ d \cos \theta_a$$

- It is clear that the cross section diverges either as $\cos \theta_a \to 1$ (gluon is collinear to parton a) or as $E \to 0$
  - similar for parton b
- Each divergence is logarithmic and regulating the divergence by providing a fixed cutoff (in angle or energy) will produce a single logarithm from collinear configurations and another from soft ones
  - so again the double logs

Figure 12. Two examples of colour flow in a $W + 2$ jet event, shown in red. In the left-hand diagram, a leading colour flow is shown. The right-hand diagram depicts the sub-leading colour flow resulting from interference.
Logarithms

- You can keep applying this argument at higher orders of perturbation theory.
- Each gluon that is added yields an additional power of $\alpha_s$, and via the eikonal factorization outlined, can produce an additional two logarithms.
- So can write the $W + 1$ jet cross section as:

$$d\sigma = \sigma_0(W + 1 \text{ jet}) \left[ 1 + \alpha_s \left( c_{12} L^2 + c_{11} L + c_{10} \right) + \alpha_s^2 \left( c_{24} L^4 + c_{23} L^3 + c_{22} L^2 + c_{21} L + c_{20} \right) + \cdots \right]$$

- where $L$ represents the logarithm controlling the divergence, either soft or collinear.
- note that $\alpha_s$ and $L$ appear together as $\alpha_s L$.

- Size of $L$ depends on criteria used to define the jets (min $E_T$, cone size).
- Coefficients $c_{ij}$ depend on color factors.
- Thus, addition of each gluon results in additional factor of $\alpha_s$ times logarithms.
- In many (typically exclusive) cases, the logs can be large, leading to an enhanced probability for gluon emission to occur.
- For most inclusive cases, logs are small and $\alpha_s$ counting may be valid estimator for production of additional jets.
Re-shuffling

\[ d\sigma = \sigma_0(W + 1 \text{ jet}) \left[ 1 + \alpha_s(c_{12}L^2 + c_{11}L + c_{10}) \\
+ \alpha_s^2(c_{24}L^4 + c_{23}L^3 + c_{22}L^2 + c_{21}L + c_{20}) + \cdots \right] \]

- re-write the term in brackets as

\[
\cdots = 1 + \alpha_sL^2c_{12} + (\alpha_sL^2)^2c_{24} + \alpha_sLc_{11}(1 + \alpha_sL^2c_{23}/c_{11} + \cdots) + \cdots \\
= \exp\left[c_{12}\alpha_sL^2 + c_{11}\alpha_sL\right],
\]

- Where the infinite series has been resummed into an exponential form
  - first term in expansion is called leading logarithm term, 2nd next-to-leading logarithm, etc

\[
\sigma_W = \sigma_{W+0j} + \sigma_{W+1j} + \sigma_{W+2j} + \sigma_{W+3j} + \cdots \\
\sigma_{W+0j} = a_0 + 2\alpha_s(a_{12}L^2 + a_{11}L + a_{10}) \\
+ \quad \alpha_s^2(a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots \\
\sigma_{W+1j} = \alpha_s(b_{12}L^2 + b_{11}L + b_{10}) \\
+ \quad \alpha_s^2(b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots \\
\sigma_{W+2j} = \cdots.
\]

- Now can write out each contribution as a contribution in powers of \( \alpha_s \) and logarithms
Re-shuffling

• Configuration shown to the right can be reconstructed as an event containing up to 2 jets, depending on jet definition and momenta of the partons.
• The matrix elements for this process contain terms proportional to $\alpha_s \log(p_{T3}/p_{T4})$ and as $\log(1/\Delta R_{34})$, so min values for transverse momentum and separation must be imposed.

$\sigma_{W+0j} = a_0 + \alpha_s (a_{12}L^2 + a_{11}L + a_{10}) + \alpha_s^2 (a_{24}L^4 + a_{23}L^3 + a_{22}L^2 + a_{21}L + a_{20}) + \cdots$

$\sigma_{W+1j} = \alpha_s (b_{12}L^2 + b_{11}L + b_{10}) + \alpha_s^2 (b_{24}L^4 + b_{23}L^3 + b_{22}L^2 + b_{21}L + b_{20}) + \cdots$

$\sigma_{W+2j} = \cdots$

At given order, in perturbation theory, sum of logarithms vanishes and we have

$\sigma^{LO}_W = a_0$,
$\sigma^{NLO}_W = \alpha_s (a_{10} + b_{10})$
NLO calculations

- NLO calculation requires consideration of all diagrams that have an extra factor of $\alpha_s$
  - real radiation, as we have just discussed
  - virtual

- For virtual diagram, have to integrate over loop momentum
  - but result contains IR singularities (soft and collinear), just as found for tree-level diagrams

$O(\alpha_s)$ virtual corrections arise from interference between tree level and one-loop virtual amplitudes.
Advantages of NLO

- Less sensitivity to unphysical input scales, i.e. renormalization and factorization scales
- First level of prediction where normalization can be taken seriously
- More physics
  - parton merging gives structure in jets
  - initial state radiation
  - more species of incoming partons
- Suppose I have a cross section $\sigma$ calculated to NLO ($O(\alpha_s^n)$)
- Any remaining scale dependence is of one order higher ($O(\alpha_s^{n+1})$)
  - in fact, we know the scale dependent part of the $O(\alpha_s^{n+1})$ cross section before we perform the complete calculation, since the scale-dependent terms are explicit at the previous order

\[ \frac{d\sigma}{dE_T} = \alpha_s(\mu_R)^2 A + \alpha_s(\mu_R)^3 (B + 2b_0LA) + \alpha_s(\mu_R)^4 (C + 3b_0LB + (3b_0^2L^2 + 2b_1L)A) \]

with $L = \log(\mu_R/E_T)$ and $b_i$ the known beta function coefficients.

Figure 11: Single jet inclusive distribution at $E_T = 100$ GeV and $0.1 < |\eta| < 0.7$ at $\sqrt{s} = 1800$

The NNLO coefficient $C$ is unknown. The curves show the guesses $C = 0$ (solid) and $C = \pm B^2/A$ (dashed).
Predictions tend to be more reliable at higher $E_T$.
Jet algorithms at LO

- At (fixed) LO, 1 parton = 1 jet
  - why not more than 1? I have to put a $\Delta R$ cut on the separation between two partons; otherwise, there’s a collinear divergence. LO parton shower programs effectively put in such a cutoff
  - remember

- But at NLO, I have to deal with more than 1 parton in a jet, and so now I have to talk about how to cluster those partons
  - i.e. jet algorithms
Jet algorithms at NLO

- At (fixed) LO, 1 parton = 1 jet
  - why not more than 1? I have to put a $\Delta R$ cut on the separation between two partons; otherwise, there’s a collinear divergence. LO parton shower programs effectively put in such a cutoff.

- At NLO, there can be two partons in a jet, life becomes more interesting and we have to start talking about jet algorithms to define jets
  - the addition of the real and virtual terms at NLO cancels the divergence.

- A jet algorithm is based on some measure of localization of the expected collinear spray of particles

- Start with an inclusive list of particles/partons/calorimeter towers/topoclusters

- End with lists of same for each jet

- …and a list of particles… not in any jet; for example, remnants of the initial hadrons

- Two broad classes of jet algorithms
  - cluster according to proximity in space: cone algorithms
  - cluster according to proximity in momenta: $k_T$ algorithms
What do I want out of a jet algorithm?

- It should be fully specified, including defining in detail any pre-clustering, merging and splitting issues.
- It should be simple to implement in an experimental analysis, and should be independent of the structure of the detector.
- It should be boost-invariant.
- It should be simple to implement in a theoretical calculation:
  - it should be defined at any order in perturbation theory.
  - it should yield a finite cross section at any order in perturbation theory.
  - it should yield a cross section that is relatively insensitive to hadronization effects.
- It should be IR safe, i.e. adding a soft gluon should not change the results of the jet clustering.
- It should be collinear safe, i.e. splitting one parton into two collinear partons should not change the results of the jet clustering.
Jet algorithms

- The algorithm should behave in a similar manner (as much as possible) at the parton, particle and detector levels.

Projection to jets should be resilient to QCD effects
Some kinematic definitions

Rapidity ($y$) and Pseudo-rapidity ($\eta$)

\[
y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \frac{1}{2} \ln \frac{1 + \beta \cos \theta}{1 - \beta \cos \theta}
\]

\[
\beta \cos \theta = \tanh y \quad \text{where} \quad \beta = \frac{p}{E}
\]

In the limit $\beta \to 1$ (or $m << p_T$) then

\[
\eta \equiv y \bigg|_{m=0} = \frac{1}{2} \ln \frac{1 + \cos \theta}{1 - \cos \theta} = -\ln \tan \frac{\theta}{2}
\]

LAB System ≠ parton-parton CM system

$\Delta \eta$ and $p_T$ are invariant under longitudinal boosts
Some kinematic definitions

To satisfy listed requirements for jet algorithms, use $p_T, y$ and $\phi$ to characterize jets.

**Transverse Energy/Momentum**

$$E_T^2 = p_x^2 + p_y^2 + m^2 = p_T^2 + m^2 = E^2 - p_z^2$$

**Invariant Mass**

$$M_{12}^2 = (p_1^\mu + p_2^\mu)(p_1^\mu + p_2^\mu) = m_1^2 + m_2^2 + 2(E_1E_2 - p_1 \cdot p_2) \xrightarrow{m_1,m_2 \to 0} 2E_{T1}E_{T2}(\cosh \Delta \eta - \cos \Delta \phi)$$

**Partonic Momentum Fractions**

$$x_1 = (e^{\eta_1} + e^{\eta_2})E_T / \sqrt{s}$$
$$x_2 = (e^{-\eta_1} + e^{-\eta_2})E_T / \sqrt{s}$$

Parton CM (energy)$^2$ → $s = x_a x_b s$

$$p_z = E \tanh y$$
$$E = E_T \cosh y$$
$$p_z = E_T \sinh y$$

$$p_T = p \sin \theta \xrightarrow{m \to 0} E_T$$

$$x_T = 2E_T / \sqrt{s} = x_{1,2} (\eta_{1,2} = 0)$$

$$0 < x_1, x_2 < 1$$
$$x_T^2 < x_1 x_2 < 1$$
(Legacy) cone algorithms

- Primary algorithm used in hadron-hadron colliders
  - perhaps most intuitive
  - draw a cone of radius R in $\eta-\phi$ space
    \[ R_{\text{cone}} = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \]

- But where to start the cone?
  - use ‘seeds’ (towers, particles, partons…) of energy $\sim$1 GeV to save computing time

- Typically use
  - R~0.7 for inclusive measurements;
  - R~0.4 for complex measurements

- But you may end up with overlapping jet cones (starting from different jet seeds)

- So need to come up with a provision for splitting/merging
  - merge 2 jets if overlap energy is $> f \cdot p_T$ (smaller jet)
  - f=0.50-0.75

- Note: partons (at NLO) don’t know nothing about splitting/merging
  - experience says f=0.75 is best

streetlight approach

- combine seed towers with other towers within a radius R of the seed tower
- re-calculate jet centroid using new list of towers… inside cone
- lather, rinse, iterate until a stable solution is found
Midpoint cone algorithm

- But this type of cone algorithm is not infra-red safe, since the two partons in the figure on the right will/will not be clustered into a single jet depending on whether or not a soft gluon is present at the midpoint.
  - also (in Run 1 at the Tevatron) used $E_T$ and $\eta$, rather than $p_T$ and $y$.
- Fundamental difference between data and fixed order pert QCD
  - data has "seeds" everywhere.
- So the Midpoint algorithm was devised
  - seeds were placed at the midpoints between nearby protojets.
  - used in Run 2 at the Tevatron.
- this works for 2->3 final states (NLO inclusive), but not for 2->4 (NNLO inclusive) where I may cluster 3 partons in 1 jet.
Seedless cone algorithm

- Put seeds everywhere
- Can be time-consuming
- Enter the SISCon algorithm
  - Seedless Infrared Safe Cone jet algorithm
  - G. Salam, G. Soyez, arXiv: 0704.0292
- ...uses a geometric approach to find all distinct cones
- ...with a speed similar to that of the Midpoint algorithm
- Still have the split/merge issue
- ...and the issue of dark towers
- Differences with the midpoint algorithm typically of the order of 1 percent or so in practice
  - see later discussion, however
**k_T (recombination) algorithms**

- Cluster particles nearby in momentum space first
- The $k_T$ algorithm is IR and collinear safe
- No overlapping of jets
- No biases from seed towers
- But the jets are sensitive to soft particles and the area can depend on pileup
The $k_T$ family of jet algorithms

- $p=1$
  - the regular $k_T$ jet algorithm

- $p=0$
  - Cambridge-Aachen algorithm

- $p=-1$
  - anti-$k_T$ jet algorithm
  - Cacciari, Salam, Soyez ’08
  - also P-A Delsart ’07
  - soft particles will first cluster with hard particles before clustering among themselves
  - no split/merge
  - leads mostly to constant area hard jets

$$d_{ij} = \min\left(p_{T,i}^{2p}, p_{T,j}^{2p}\right) \frac{\Delta R_{ij}^2}{D^2}$$

$$d_{ii} = p_{T,i}^{2p}$$

#1 algorithm for ATLAS, maybe for CMS as well
Cone and $k_T$ jet algorithms at NLO

- Let's set the $p_T$ of the second parton = $z$ that of the first parton and let them be separated by a distance $d$ (=ΔR)
- Then in regions I and II (on the left), the two partons will be within $R_{cone}$ of the jet centroid and so will be contained in the same jet
  - ~10% of the jet cross section is in Region II; this will decrease as the jet $p_T$ increases (and $\alpha_s$ decreases)
  - at NLO the $k_T$ algorithm corresponds to Region I (for D=R); thus at parton level, the cone algorithm is always larger than the $k_T$ algorithm
  - not necessarily true at the hadron level

**Figure 22.** The parameter space ($d,Z$) for which two partons will be merged into a single jet.
Construct what is called a Snowmass potential shown in Figure 50, where the towers unclustered into any jet are shaded black. A simple way of understanding these dark towers begins by defining a “Snowmass potential” in terms of the 2-dimensional vector $\mathbf{r} = (y, \phi)$ via

$$V(\mathbf{r}) = -\frac{1}{2} \sum_j pr_j \left( R^2_{cone} - (r_j - r)^2 \right) \Theta \left( R^2_{cone} - (r_j - r)^2 \right).$$

(30)

The flow is then driven by the “force” $\mathbf{F}(\mathbf{r}) = -\nabla V(\mathbf{r})$ which is thus given by,

$$\mathbf{F}(\mathbf{r}) = \sum_j pr_j \left( r_j - r \right) \Theta \left( R^2_{cone} - (r_j - r)^2 \right)$$

$$= \left( \mathbf{r}_{C(\mathbf{r})} - \mathbf{r} \right) \sum_{j \in C(\mathbf{r})} pr_j,$nabla V(\mathbf{r}),$$

where $\mathbf{r}_{C(\mathbf{r})} = (\bar{y}_{C(\mathbf{r})}, \bar{\phi}_{C(\mathbf{r})})$ and the sum runs over $j \in C(\mathbf{r})$ such that $\sqrt{(y_j - y)^2 + (\phi_j - \phi)^2} \leq R_{cone}$. As desired, this force pushes the cone to the stable cone position.

The minima of the potential function indicates the positions of the stable cone solutions:

- the derivative of the potential function is the force that shows the direction of flow of the iterated cone
- The midpoint solution contains both partons

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure22.png}
\caption{The parameter space $(d, Z)$ for which two partons will be merged into a single jet.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure51.png}
\caption{A schematic depiction of a specific parton configuration and the results of applying the midpoint cone jet clustering algorithm. The potential discussed in the text and the resulting energy in the jet are plotted.}
\end{figure}