# Announcements

- Help room hours (1248 BPS)
  - Ian La Valley(TA)
  - Mon 4-6 PM
  - Tues 12-3 PM
  - Wed 6-9 PM
  - Fri 10 AM-noon
- LON-CAPA #9 due on Thurs Nov 15
- Third hour exam Thursday Dec 6
- Final Exam Tuesday Dec 11 7:45-9:45 AM

# Transformation of velocities

- Addition of velocities doesn’t work the way we’re used to when the velocities approach that of light
  - nothing can go faster than the speed of light
- Suppose I have two velocities u and v and I want to add them together
- The total velocity is not u +v
- It’s $\dfrac{u+v}{1+\dfrac{uv}{c^2}}$

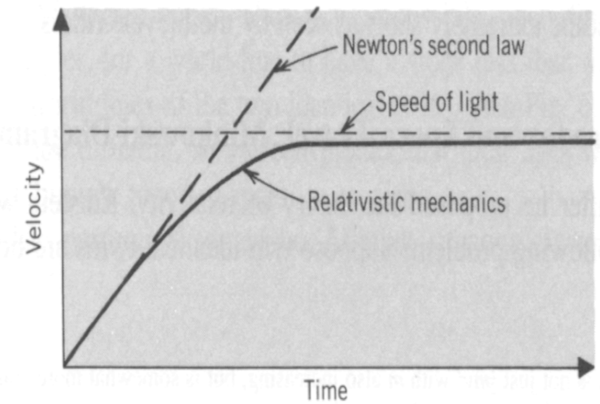


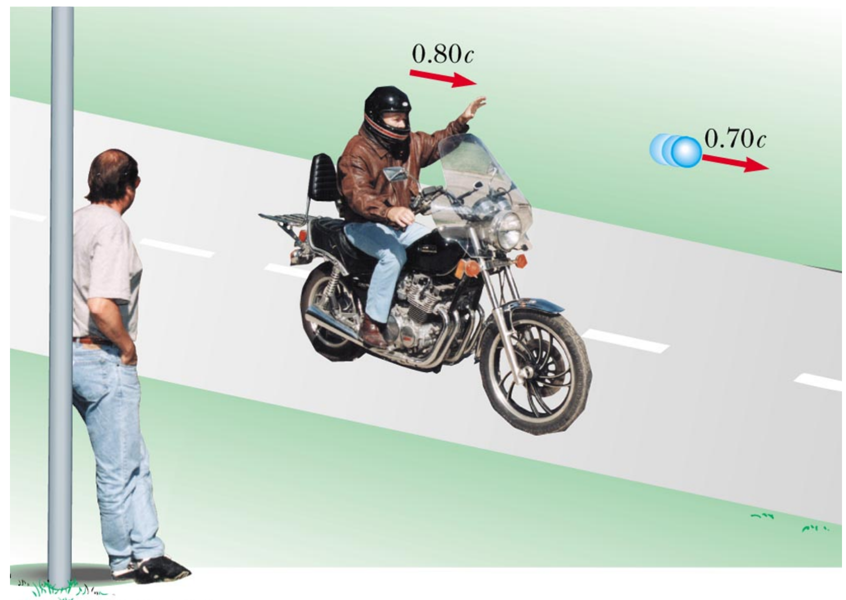

**Figure 6.14. Graph of velocity versus time for constant force.** In relativistic mechanics, the velocity cannot increase indefinitely, but rather is limited by the speed of light.

**Table 6.2.** Comparison of Results for Velocity Addition According to Galilean and Lorentz Transformations

| u | v | Galilean u + v | Lorentz $\left(\dfrac{u+v}{1+uv/c^2}\right)$ |
|---|---|---|---|
| 60 mph | 30 mph | 90 mph | 90 mph |
| 186 mps (0.001$c$) | 18.6 mps (0.0001$c$) | 204.6 mps | 204.59998 mps |
| 0.6$c$ | 0.3$c$ | 0.9$c$ | 0.763$c$ |
| 0.5$c$ | 0.5$c$ | $c$ | 0.800$c$ |
| 0.75$c$ | 0.75$c$ | 1.5$c$ | 0.960$c$ |
| 0.9$c$ | 0.6$c$ | 1.6$c$ | 0.974$c$ |
| $c$ | 0.1$c$ | 1.1$c$ | 1.000$c$ |
| $c$ | $c$ | 2$c$ | $c$ |

# Relativistic example

- A man on a (very fast) motorcycle travelling 0.80 c throws a baseball forward (he has a very good arm) with a speed of 0.70 c (from his persective)
- How fast does the innocent bystander see the ball travelling?





From Galilean perspective: 0.80 c +0.70 c =1.5 c
Using Lorentz transformation of velocities:
$[u+v]/[1+uv/c^2] = [0.8c+0.7c]/[1+(.8c)(.7c)/c^2]$
$=0.96$ c

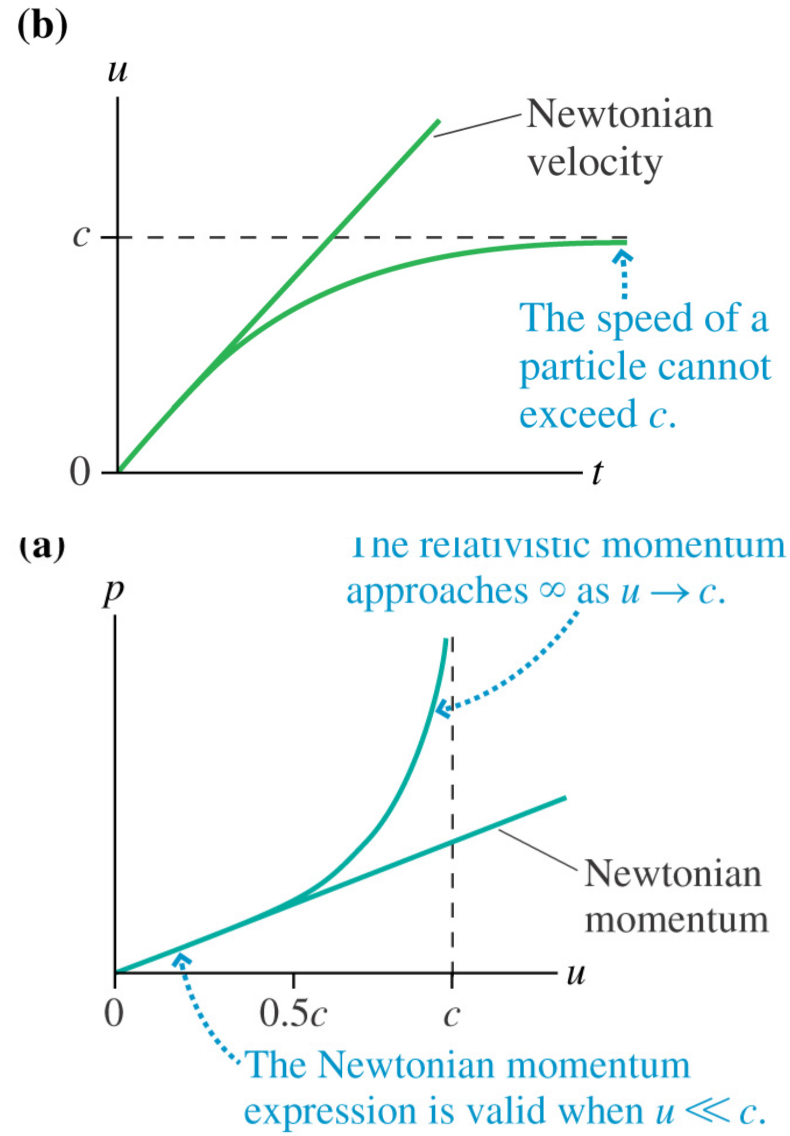
# Relativistic momentum

- No object can travel at the speed of light
  - only electromagnetic waves can travel at the speed of light
- What happens as you keep adding energy and momentum to a particle
  - its speed increases, but only asymptotically towards c
  - but its energy and momentum keep growing
  - formula for momentum (p=mu) not valid for high speeds; it has to be modified

u here used for velocity instead of v

$$p = \frac{mu}{\sqrt{1 - \frac{u^2}{c^2}}} = \gamma_p mu$$

again depending on γ

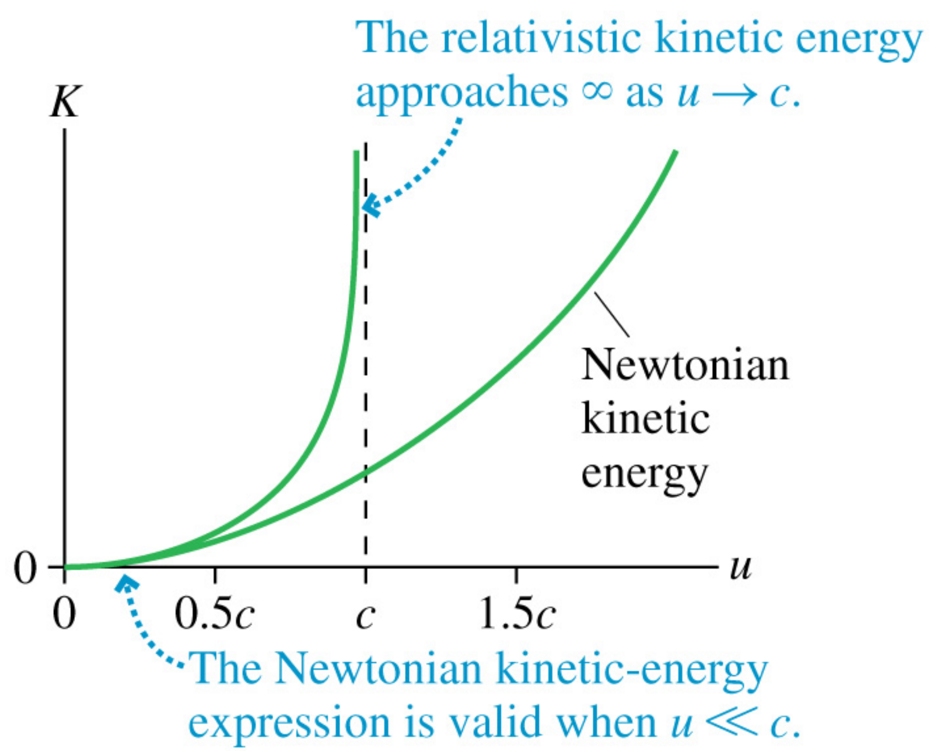


# Relativistic energy

$$\gamma_p mc^2 = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \approx \left(1+\frac{1}{2}\frac{u^2}{c^2}\right)mc^2 = mc^2 + \frac{1}{2}mu^2$$

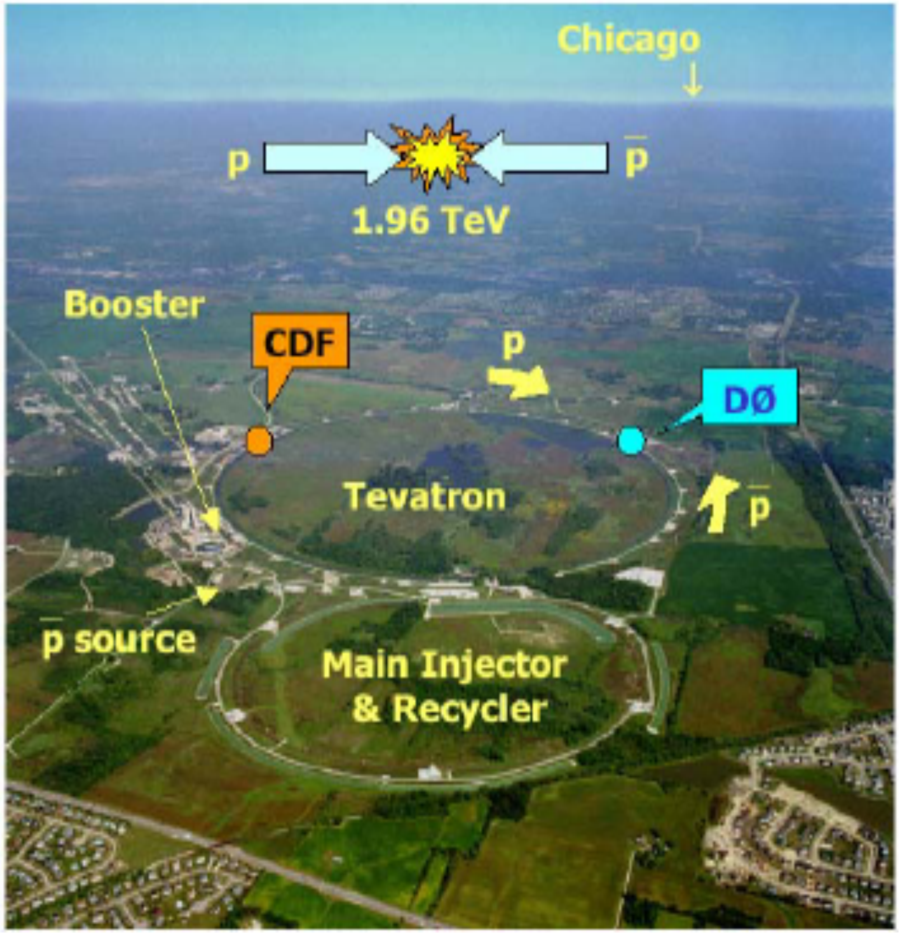
- $E=\gamma_p mc^2=E_o+K$
  =rest energy + kinetic energy

- Kinetic energy goes to $1/2mu^2$ when $u<<c$
- $E_o=mc^2$ (rest energy)



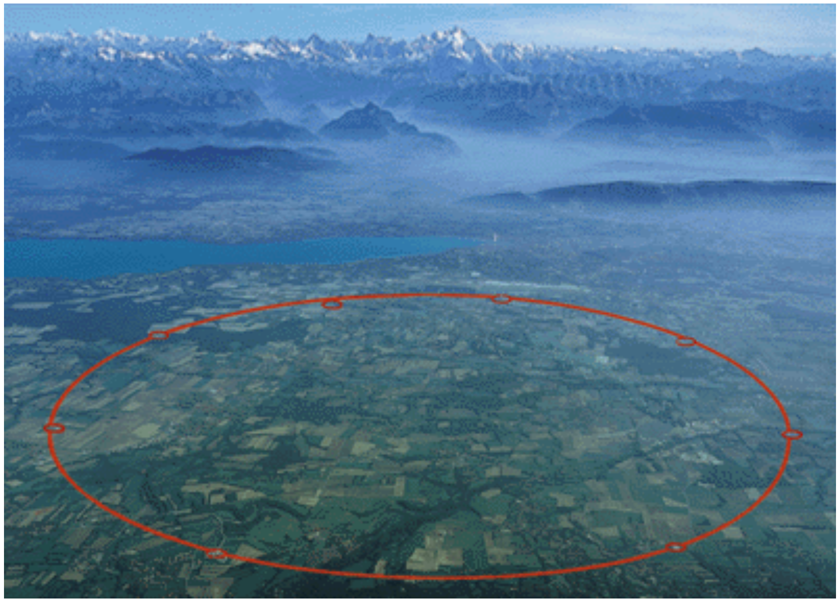
# Fermilab accelerator

- The protons and anti-protons are accelerated to an energy of 980 GeV (billion electron-volts)
  - i.e. their total energy is about 1000 times their rest mass energy
  - or $\gamma$ is about 1000



# CERN accelerator

- The proton beams are accelerated to 7 TeV (trillion electron-volts)
  - i.e. their total energy is about 7000 times their rest mass
  - $\gamma$ is ~7000



# Cockroft and Walton

What does $\Delta E = \Delta Mc^2$ mean?

The Δ means "change in". When a particle has no energy its mass is its "rest mass" $M_0$. Adding energy increases the mass. So simply put the energy-mass relation means that adding energy (for example kinetic) increases the mass.

However because c is so huge, the ΔM is usually very small. That is why the relation was not verified until 1932, and yes it was in nuclear physics.

$$^{1}H + {}^{7}Li \rightarrow {}^{4}He + {}^{4}He + Q$$

The Q was found the be just what you would calculate from the masses.

Ernest Rutherford

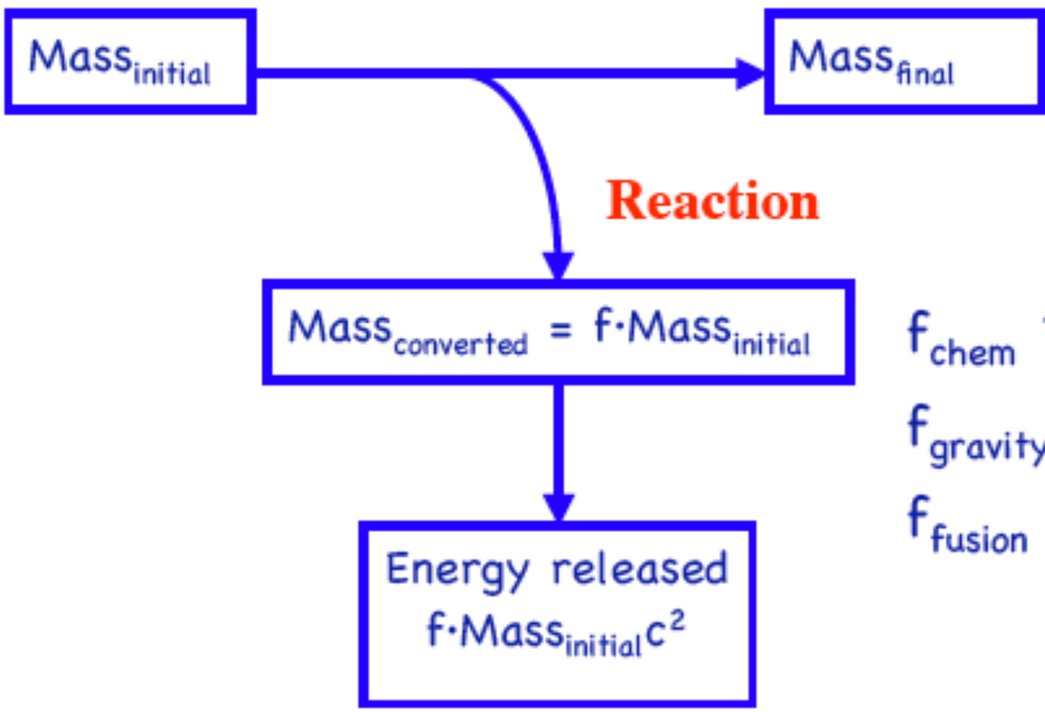
Let's hear it directly from Einstein.

# Reminder

## What Powers the Sun?

Nuclear reactions power the stars (and the sun)

- All energy, gravitional, chemical, or nuclear comes from mass

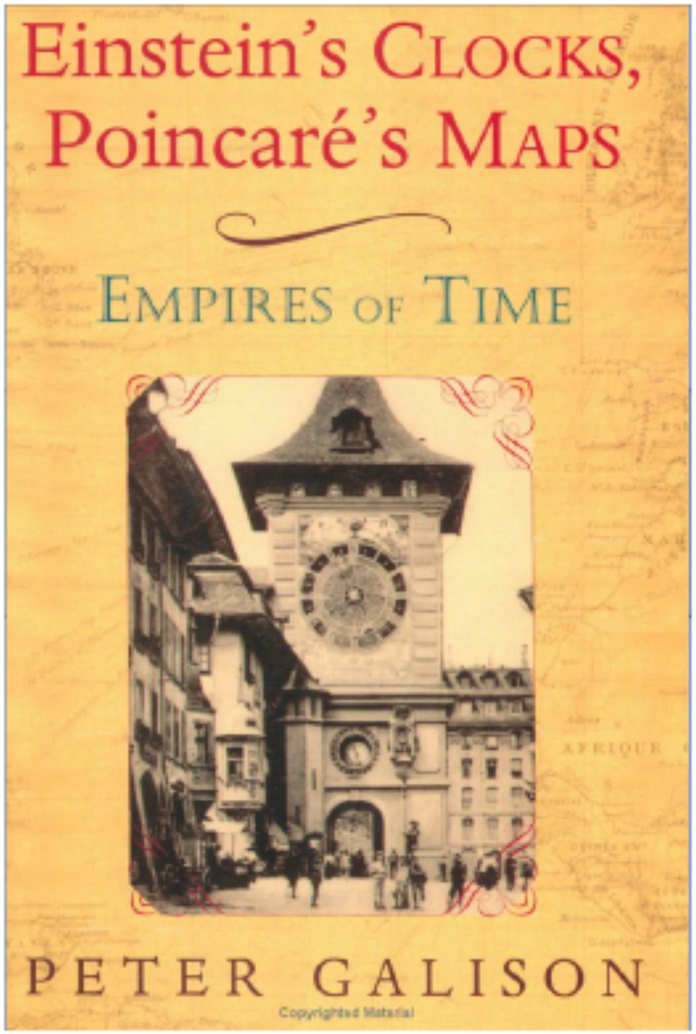



Note that this is why Lavoisier concluded that mass was conserved in chemical reactions; $f_{chem}$ is so small

| | | |
|---|---|---|
| $f_{chem}$ | $\sim 1.5 \times 10^{-10}$ | 2200 y |
| $f_{gravity}$ | $\sim 7.5 \times 10^{-6}$ | $10^7$ y |
| $f_{fusion}$ | $\sim 1.0 \times 10^{-3}$ | $10^{11}$ y |

No other source lasts lifetime of sun ($4.5 \times 10^9$ yr)

# Interesting book




Year of Physics, March 10, 2005, slide 24

Very interesting section on Einstein's work in the patent office.

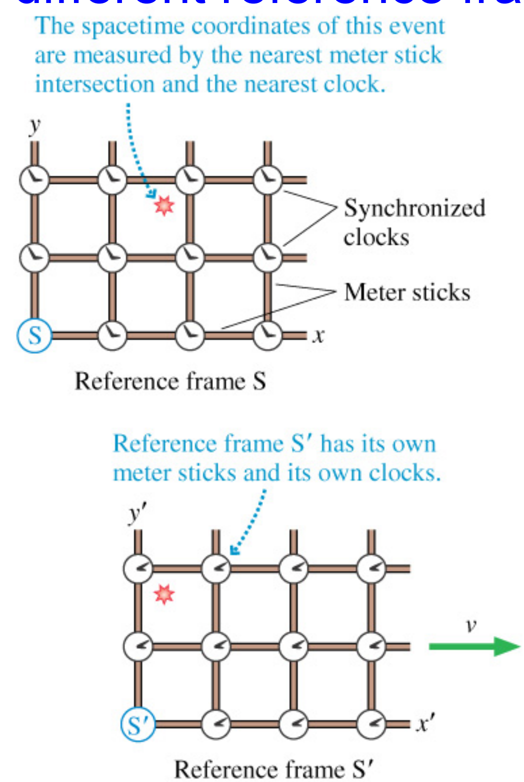
It was the synchronization of clocks, a key technological problem of the late 19th century.

Also describes how Poincaré's essays on important open questions in physics led to Einstein's great discoveries of 1905.

# Why is special relativity *special*?

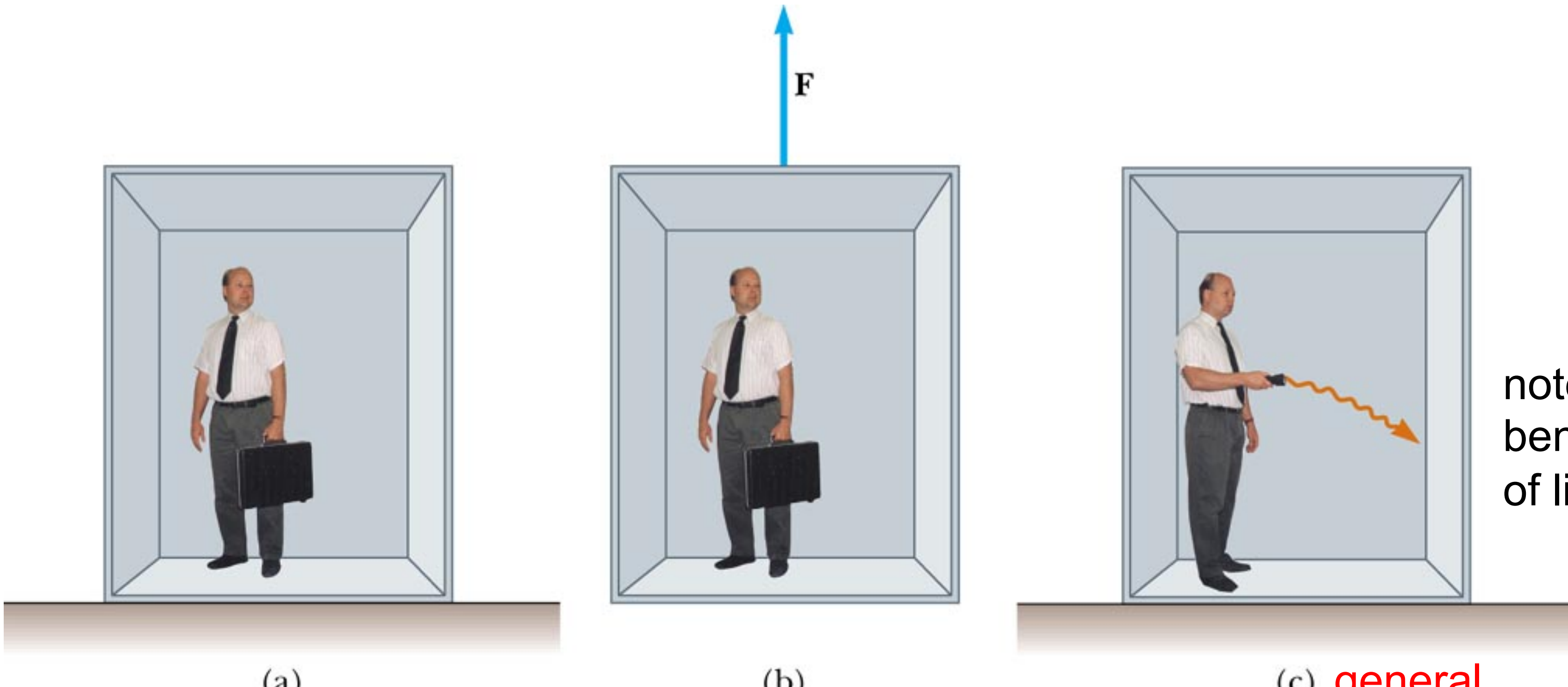
- Because it works with inertial (non-accelerating) frames of reference
- After the special theory of relativity, Einstein went to work on a more general theory of relativity, one that could describe accelerating frames of reference as well
- He continued his use of the idea of space-time
- And he found a connection with gravity

- We identify an event with space-time coordinates (x,y,z,t)
- The same event will have different coordinates in different reference frames



# Equivalence theorem

We said that special relativity applies for inertial frames of reference. What about for non-inertial (accelerating) frames?





note bending of light

general relativity predicts the bending of light by gravity

That's the realm of general relativity (also by A. Einstein). His equivalence principle stated that one can not tell the difference between gravity and an accelerating frame of reference. Being in an accelerating elevator is the same as being in gravity.

# Nov 10, 1919 New York Times headline

# LIGHTS ALL ASKEW IN THE HEAVENS

**Men of Science More or Less Agog Over Results of Eclipse Observations.**

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**EINSTEIN THEORY TRIUMPHS**

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**Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.**

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**A BOOK FOR 12 WISE MEN**

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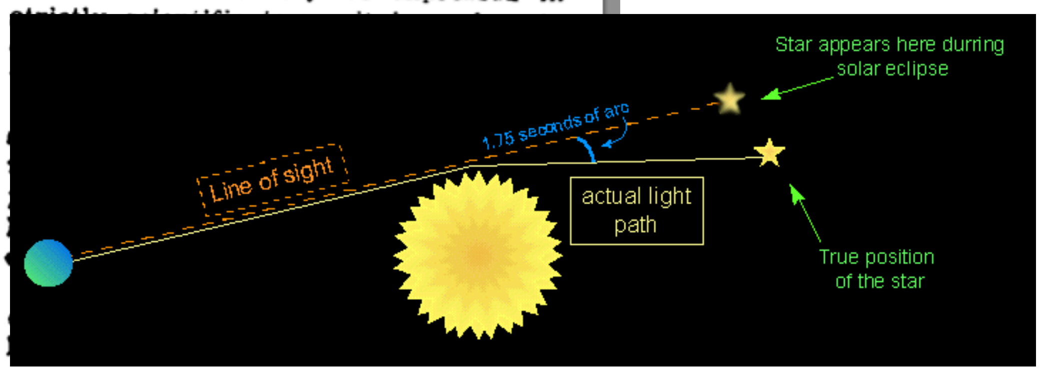
**No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.**

Special Cable to THE NEW YORK TIMES.

LONDON, Nov. 9.—Efforts made to put in words intelligible to the non-scientific public the Einstein theory of light proved by the eclipse expedition so far have not been very successful. The new theory was discussed at a recent meeting of the Royal Society and Royal Astronomical Society. Sir Joseph Thomson, President of the Royal Society, declares it is not possible to put Einstein's theory into really intelligible words, yet at the same time Thomson adds:

"The results of the eclipse expedition demonstrating that the rays of light from the stars are bent or deflected from their normal course by other aerial bodies acting upon them and consequently the inference that light has weight form a most important contribution to the laws of gravity given us since Newton laid down his principles."

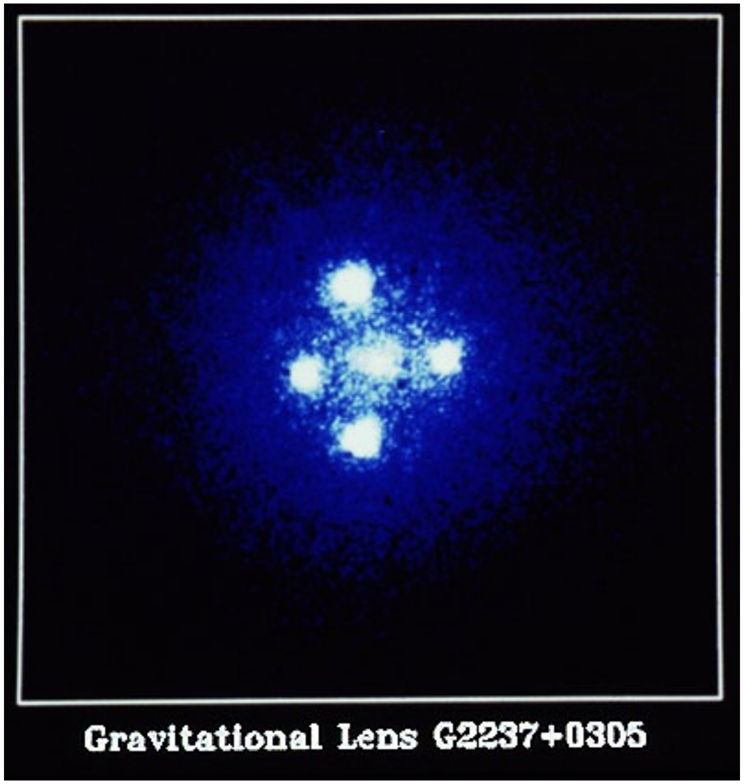
Thompson states that the difference between theories of Newton and those of Einstein are infinitesimal in a popular sense, and as they are purely mathematical and can only be expressed in




…this came after the end of WWI, and made Einstein a 'rock star'

# Other tests of general relativity: gravitational lensing

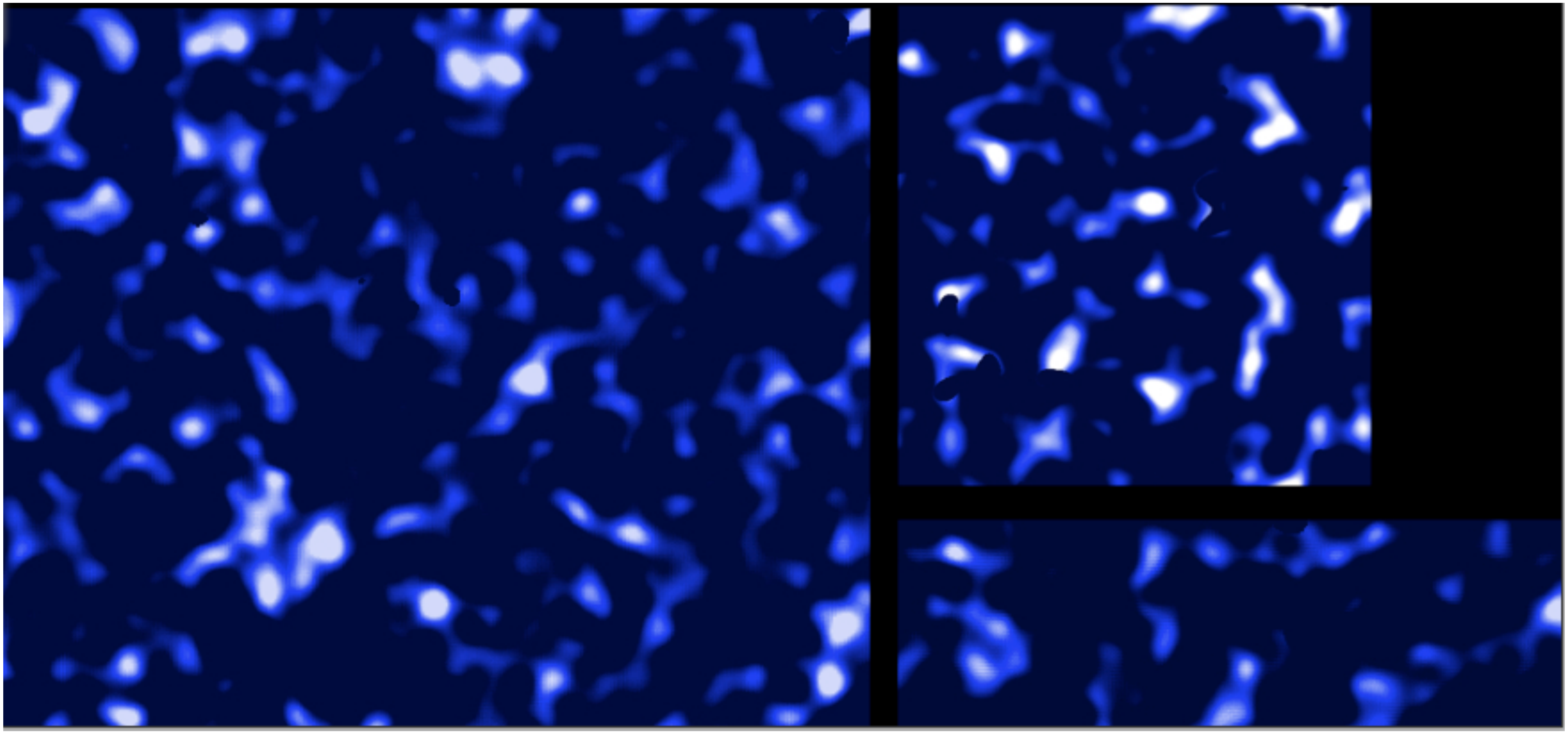
- Quasars are extremely bright objects found mostly in the early universe
- They are bright enough to be seen across the entire universe; the brightest objects in the universe
  - now believed to be powered by giant black holes at the centers of galaxies
- Four images of the same distant quasar
- The light images are bent by the gravitational effects of an intermediate galaxy
- Now used as a method to magnify distant images




video

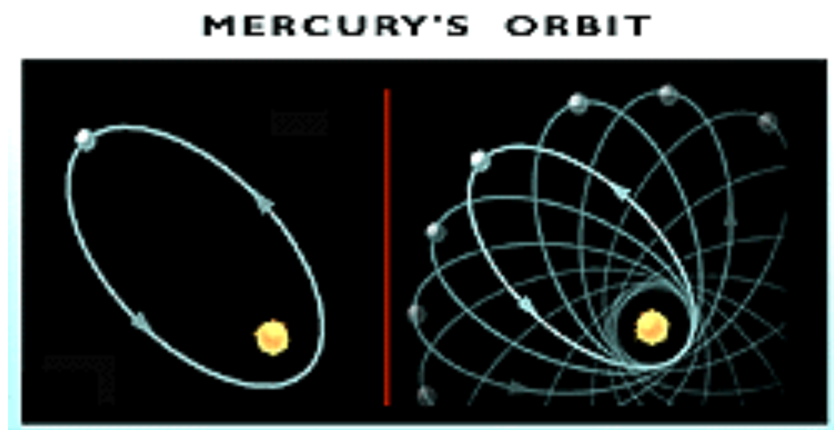
# Mapping of dark matter

- Can use the bending of the light of distant galaxies by dark matter between us and the distant galaxies to map out the distribution of dark matter



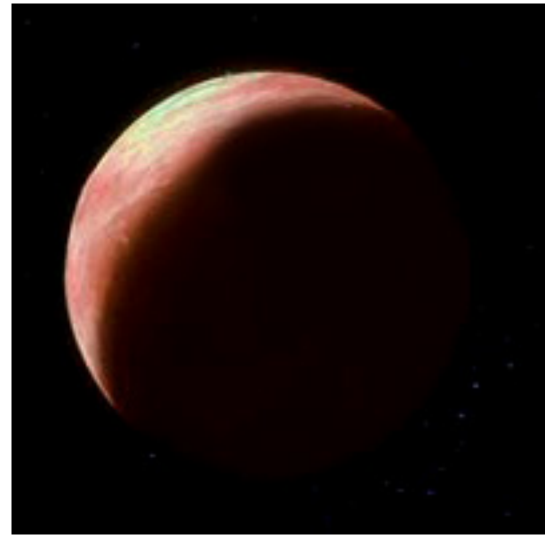
# Advance of the perihelion of Mercury

- Since almost two centuries earlier astronomers had been aware of a small flaw in Mercury's orbit around the Sun, as predicted by Newton's laws. As the closest planet to the Sun, Mercury orbits a region in the solar system where spacetime is disturbed by the Sun's mass. Mercury's elliptical path around the Sun shifts slightly with each orbit such that its closest point to the Sun (or "perihelion") shifts forward with each pass. Newton's theory had predicted an advance only half as large as the one actually observed. Einstein's predictions exactly matched the observation.



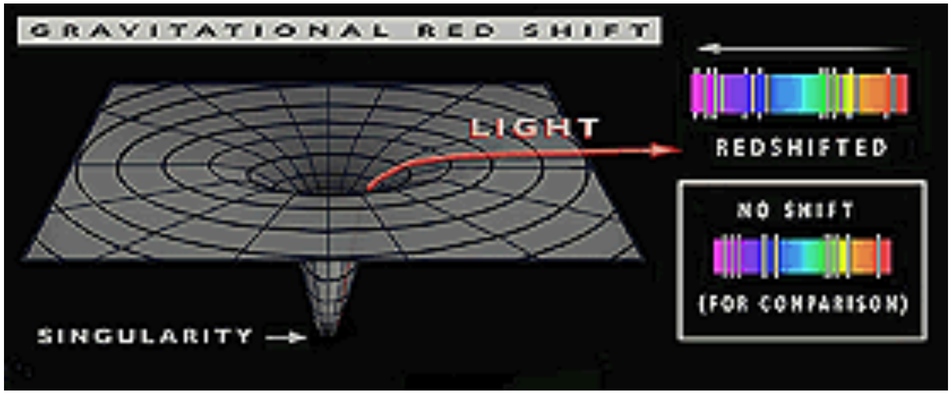
# The planet Vulcan

- Before general relativity, what other possibilities existed  that could explain away the problems with Mercury’s orbit?
- One hypothesis was that Mercury’s orbit was affected by the gravitational tugs from another planet even closer to the Sun than Mercury
- We couldn’t see it because it was so close to the Sun
- The planet was named Vulcan, after the Roman god of forging
- This is where Spock was supposed to be from in the Star Trek series
- But the temperature of this fictional planet would have been over 1000 degrees C



# Gravitational redshift

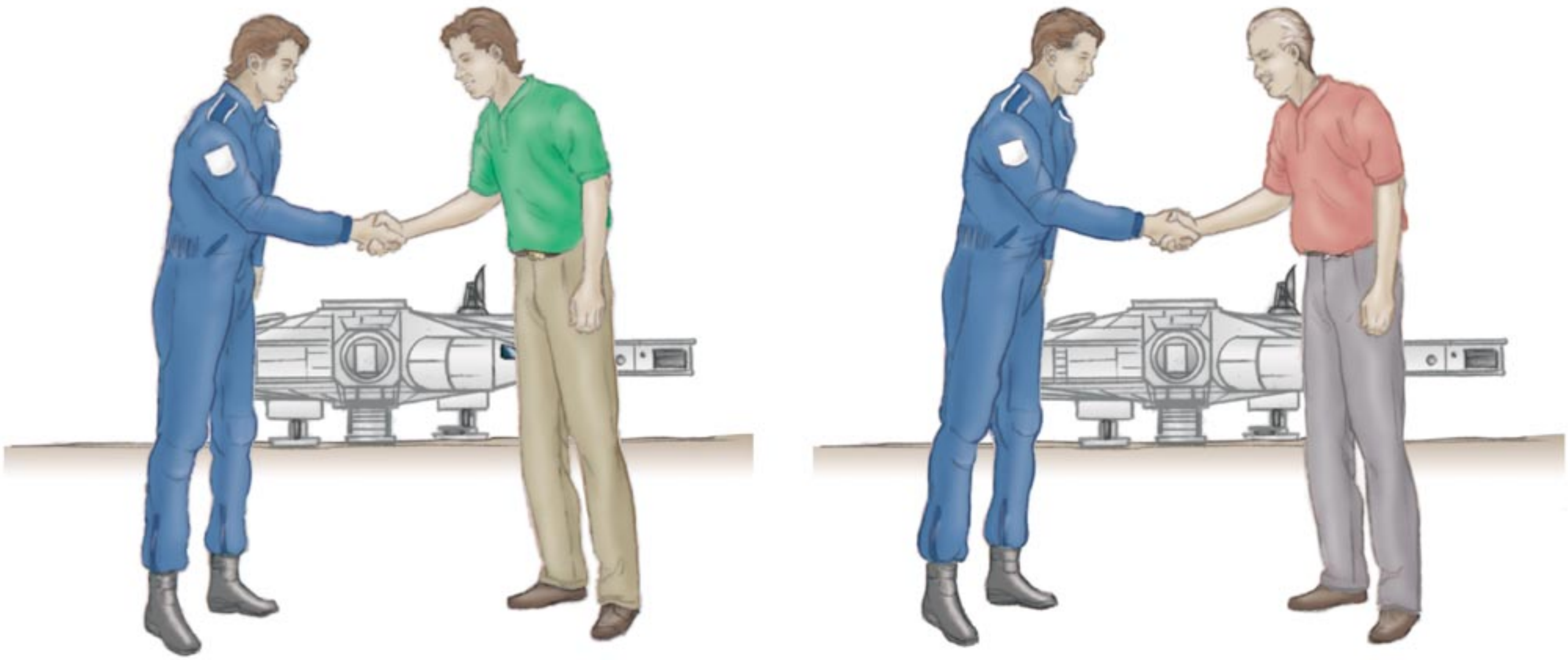
- According to General Relativity, the wavelength of light (or any other form of electromagnetic radiation) passing through a gravitational field will be shifted towards redder regions of the spectrum. To understand this **gravitational redshift**, think of a baseball hit high into the air, slowing as it climbs. Einstein's theory says that as a photon fights its way out of a gravitational field, it loses energy and its color reddens. (It can't lose speed since light can only travel at c.) Gravitational redshifts have been observed in diverse settings, including laboratory experiments.



# Time dilation

- We said that time passed more slowly for objects moving at great speeds
- But, time also passes more slowly for objects in gravitational fields
- Suppose I do an experiment where I take two atomic clocks (incredibly precise), synchronize them and keep one of the ground while the other flies in a commerical jet around the world
- Do the clocks agree when they're brought back together?
- The clock on the jet slowed down because it was travelling at a greater speed, but the clock on the ground was in a stronger gravitational field
- The gravitational effect was stronger, so the clock on the ground ended up counting off less time than the one in the jet (by about 100 ns); this was first carried out in 1971
- GPS satellites experience the effects of both special and general relativity
  - they are moving fast and are in a weaker gravitational field
  - corrections have to be made to the signals so that your navigator has the 5-10 m accuracy you're expecting

# Twin paradox



(a) (b)

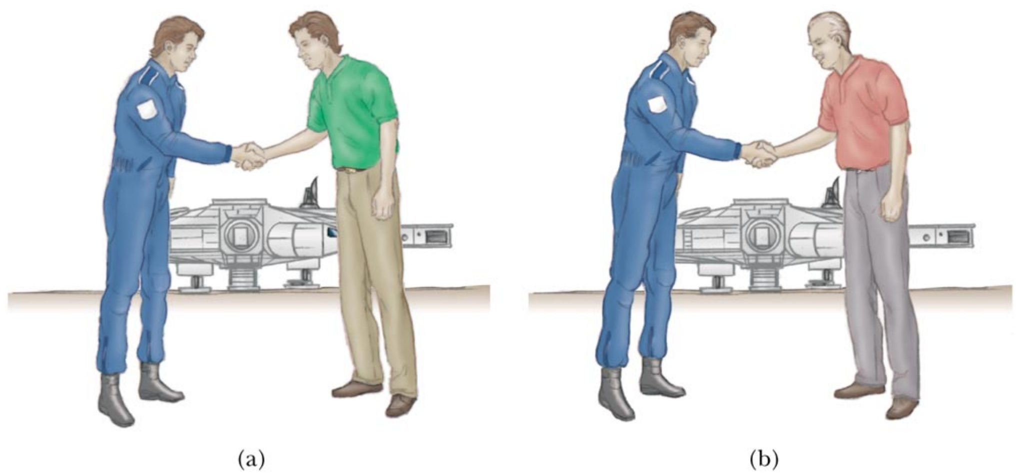


Two twins aged 21. One stays home on earth. The other heads off in a spaceship travelling close to the speed of light ($\gamma$=25; 99.9% of the speed of light)

After 50 years have passed for the twin on earth, only 2 years have passed for the twin in the spaceship. Or is it the other way around?

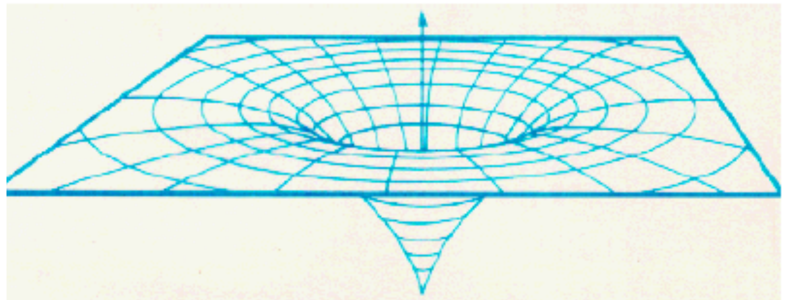
# Answer: not so easy to understand

- It's the twin in the spaceship who doesn't age as much
- The two frames of reference (the Earth and the spaceship) are not equivalent
- One accelerates (the spaceship) and the other doesn't



# Einstein field equations

- Einstein viewed gravity as being due to the curvature of space
- A large mass curves space more than a small mass
- The curvature is described by the Einstein field equations, which we briefly mentioned earlier when we were discussing tensors



- The EFE describe how mass and energy are related to the curvature of spacetime. In abstract index notation, the EFE reads as shown to the right, where $G_{ab}$ is the Einstein tensor, $\Lambda$ is the cosmological constant, $c$ is the speed of light in a vacuum and $G$ is the gravitational constant, which comes from Newton's law of gravity.

$$G_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

This is a simple-looking equation, but is very difficult to solve in practice

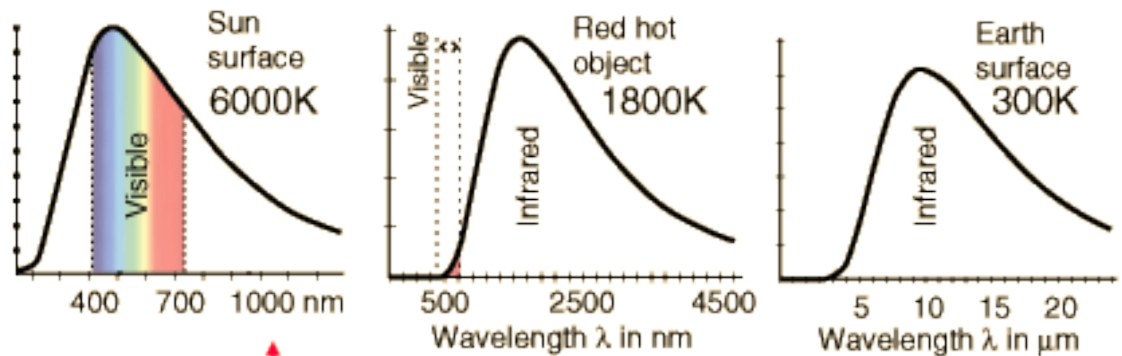
Elegant Universe

# Einstein field equations

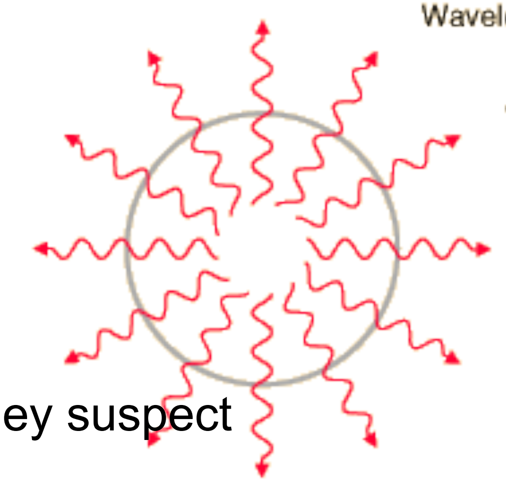
$$G_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

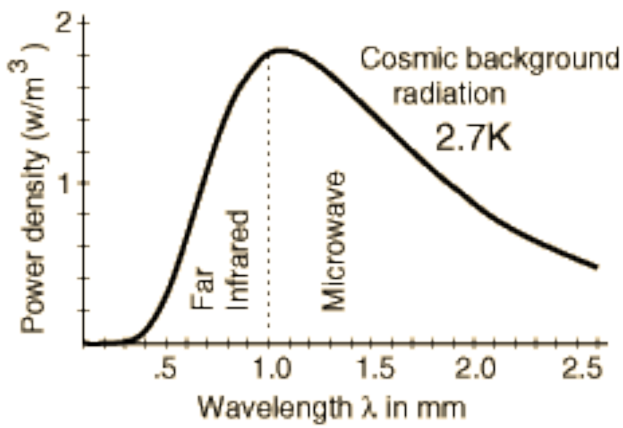
“My greatest blunder.” sincerely, Albert Einstein

…because without the cosmological constant the universe could not be static and would have to be expanding



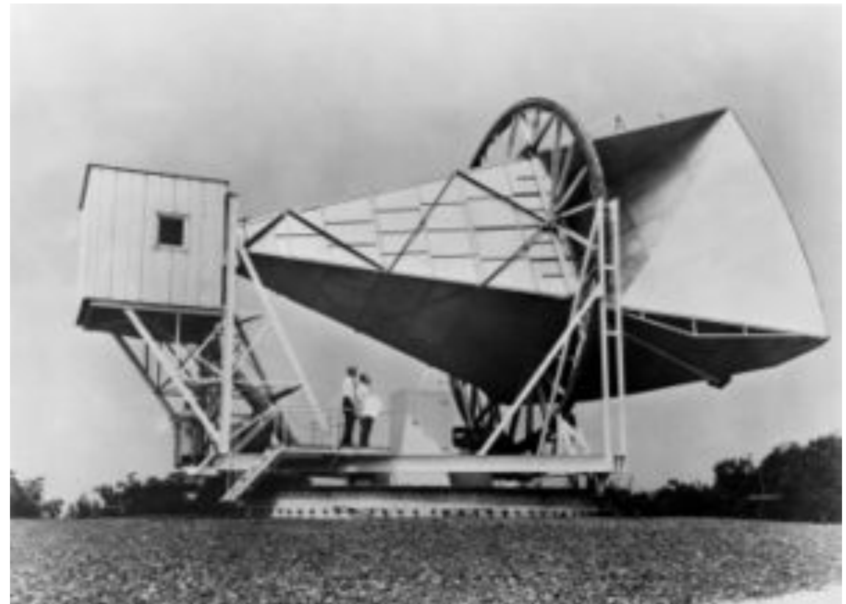

But in the 1960’s Penzias and Wilson discovered the remnants of the Big Bang that started the universe expanding

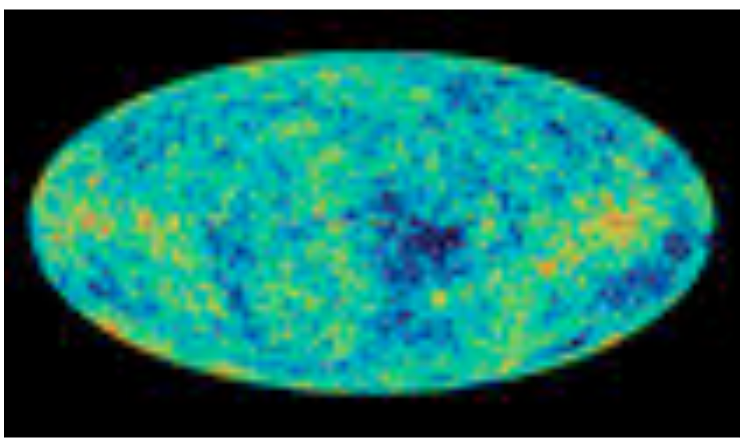




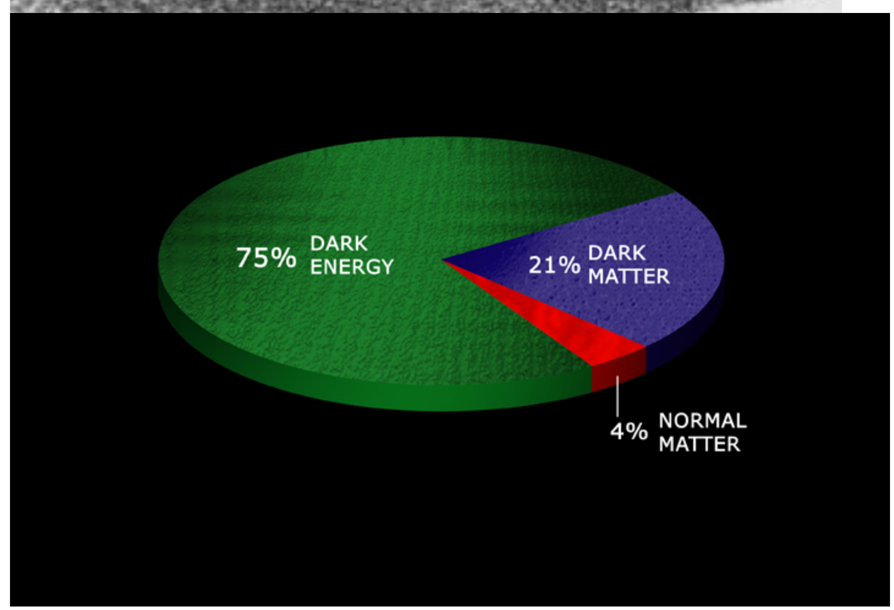

What were they really looking for?
What non-exotic explanation did they suspect for their signal?

# Penzias and Wilson

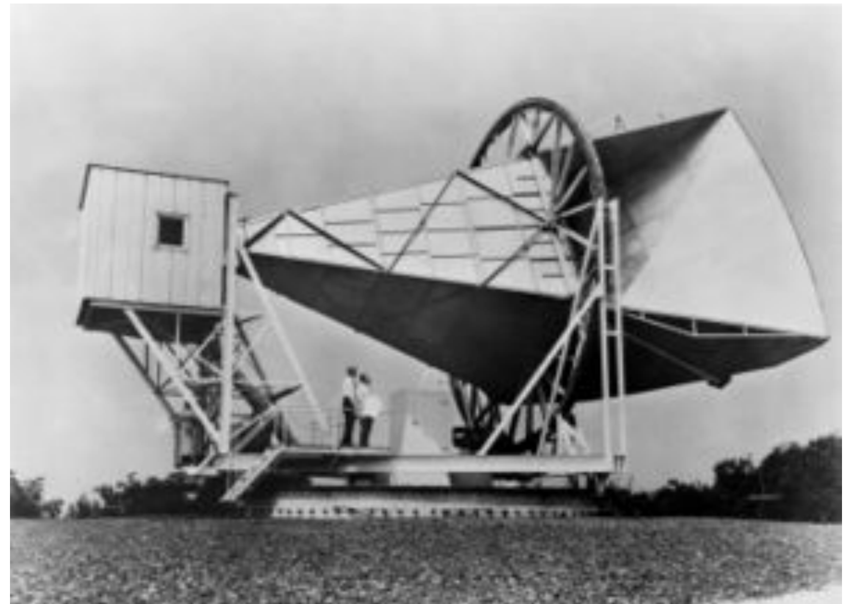


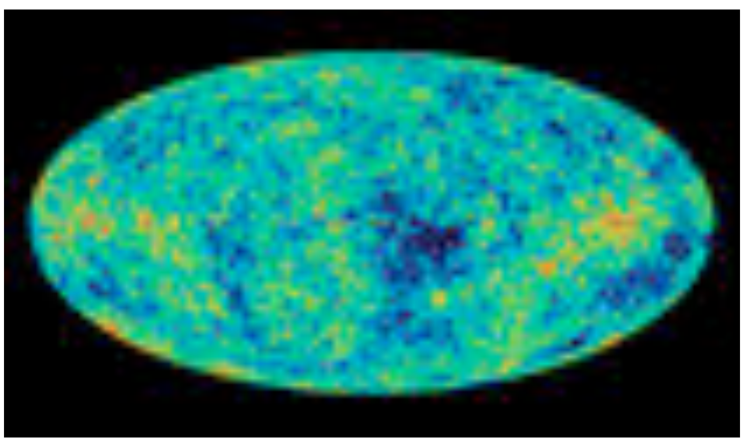


latest WMAP results



# Penzias and Wilson





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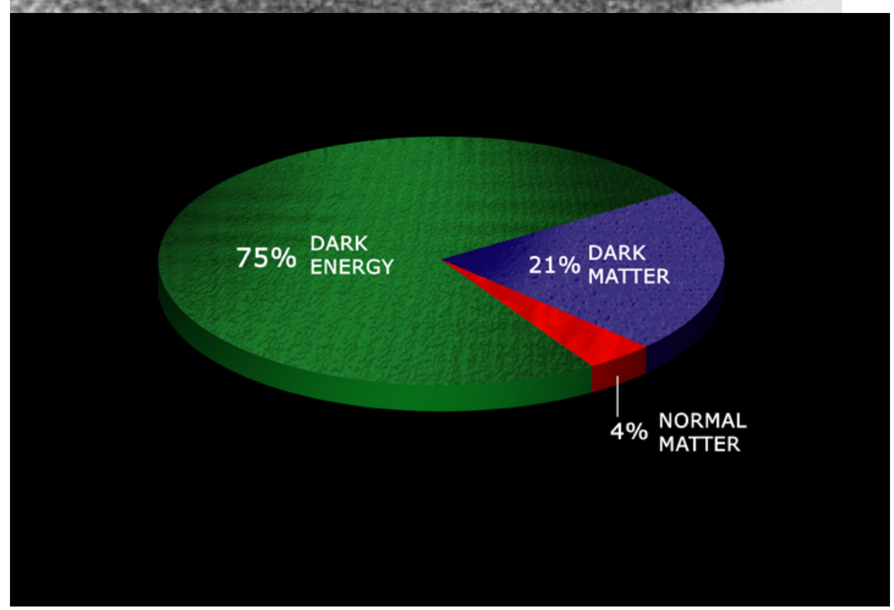
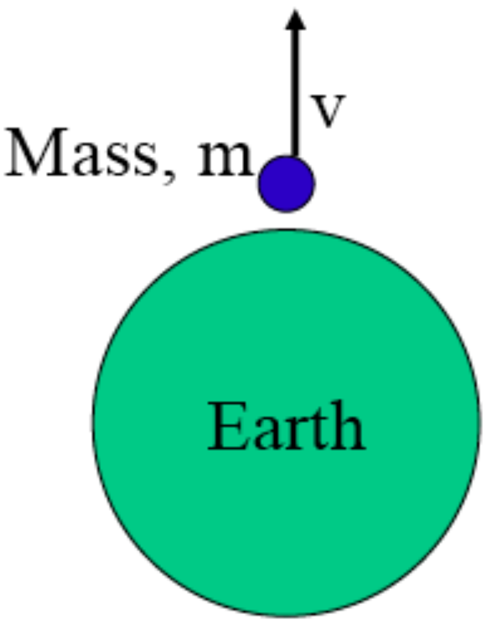


# Escape velocity

…using Newton's laws, we can calculate the escape velocity of a planet




The velocity to completely escape the gravity of a planet is:

$$KE(leaving) = PE(far\ away)$$

$$\frac{1}{2}mv^2 = \frac{GmM_{planet}}{R_{planet}}$$

$$v = \sqrt{\frac{2GM_{planet}}{R_{planet}}}$$

The escape velocity for the Earth is about 11 km/s.

# Escape velocity for a very dense object

- What is the escape velocity for an object with the mass of the Sun and a radius of 1 km?
- $M_{sun}$=1.99X10$^{30}$ kg
- G=6.67X10$^{-11}$ Nm$^2$/kg$^2$

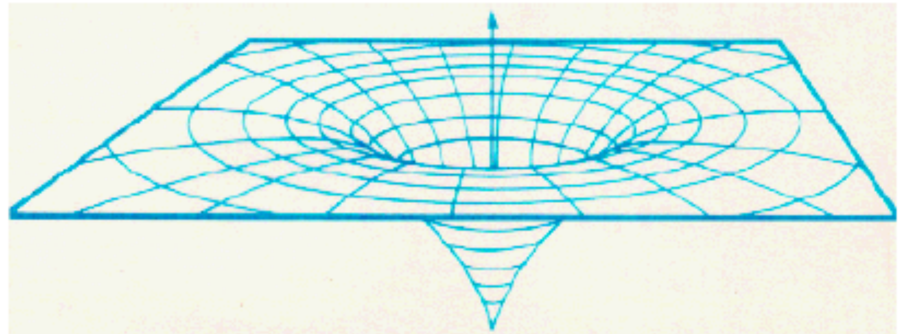
$$v = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2(6.67X10^{-11}Nm^2/kg)(1.99X10^{30}kg)}{1000m}}$$

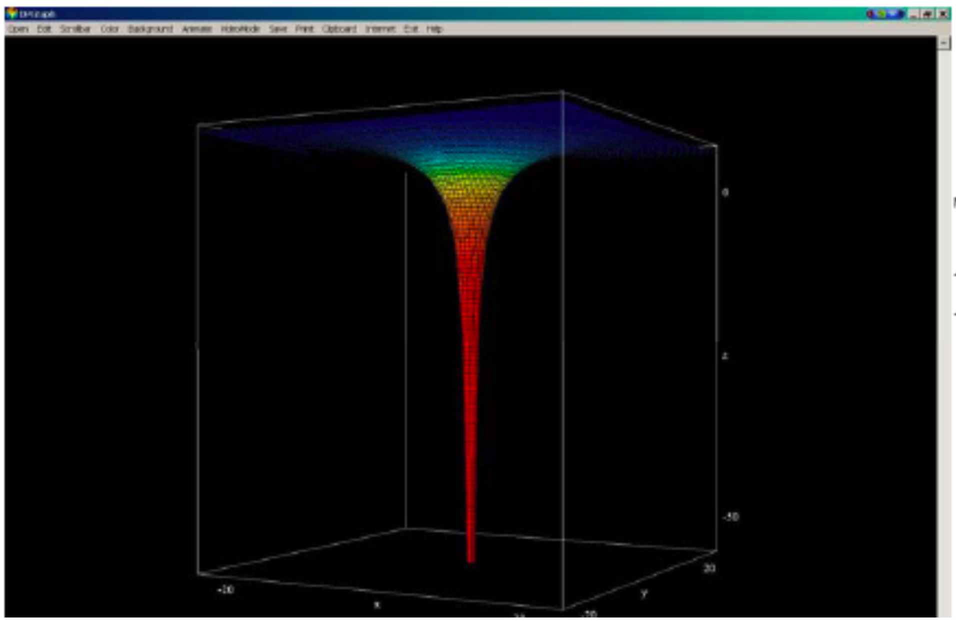
$$v = 5.2X10^8 m/s$$

If you could shrink the Sun down to a radius of 1 km, nothing could escape from it, not even light.

# i.e. a black hole

The concentrated mass stretches space.

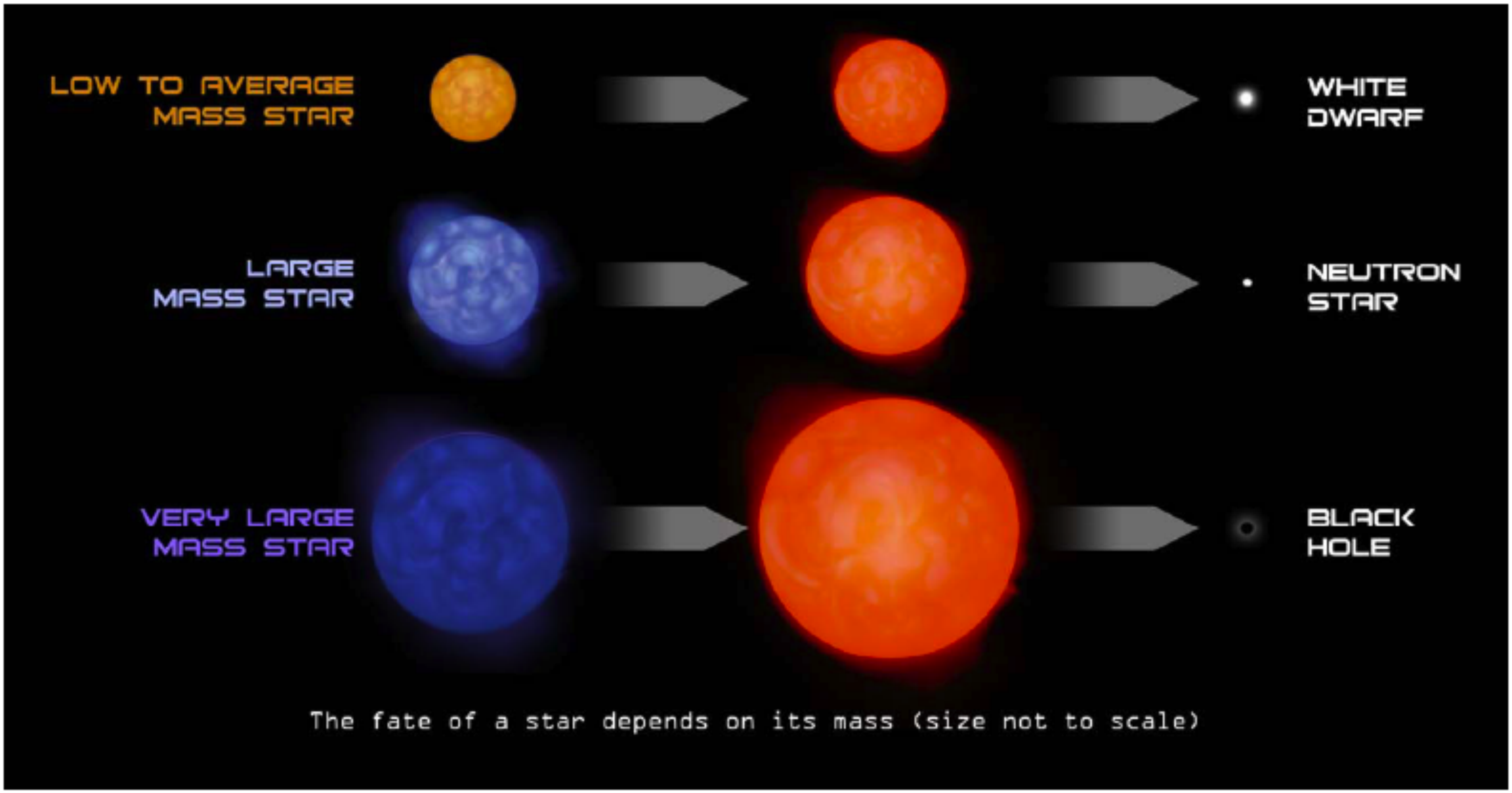




The “hole” in space is so deep that light can not escape.

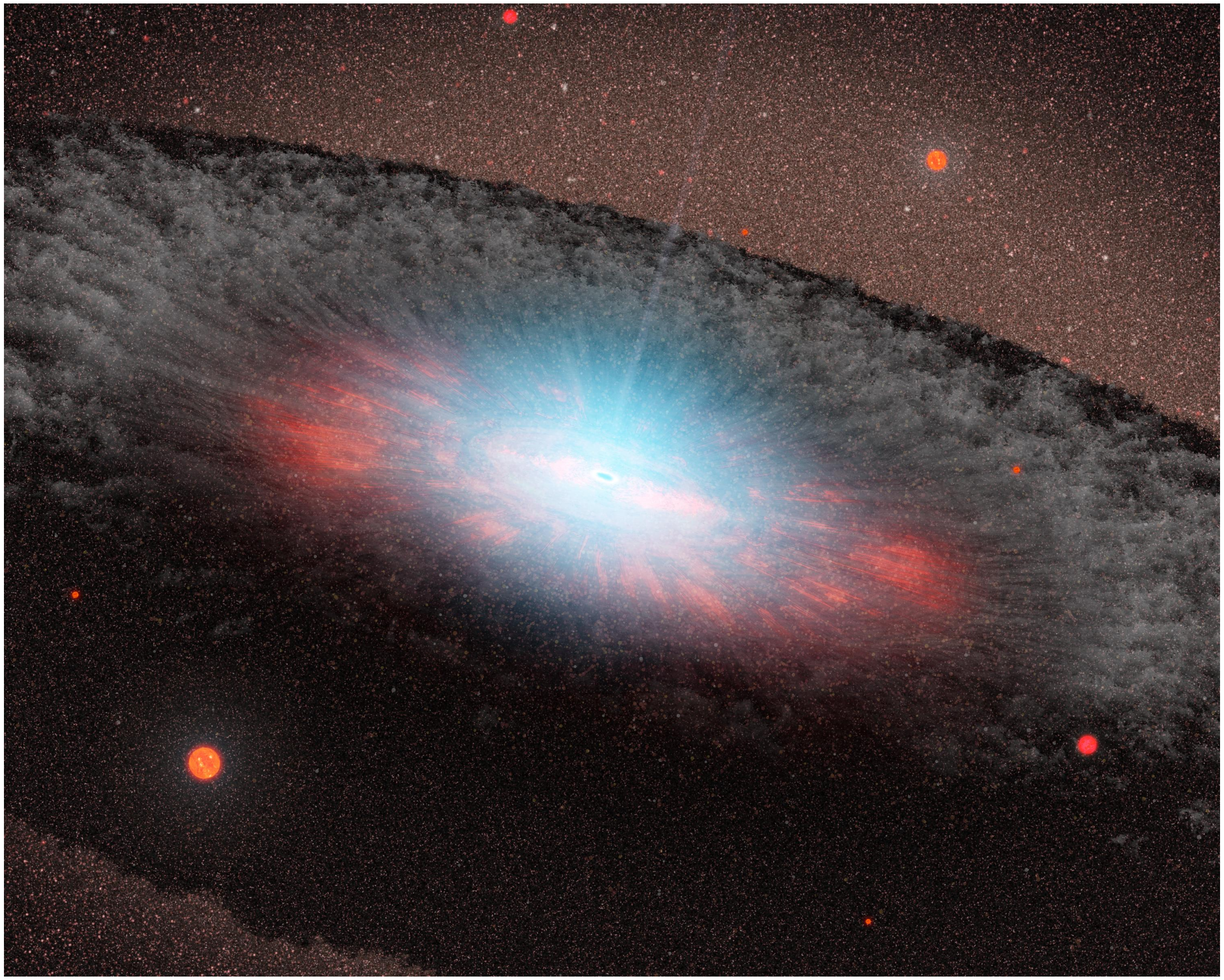
# Where do black holes come from?

- They result from the supernova explosions of very massive stars



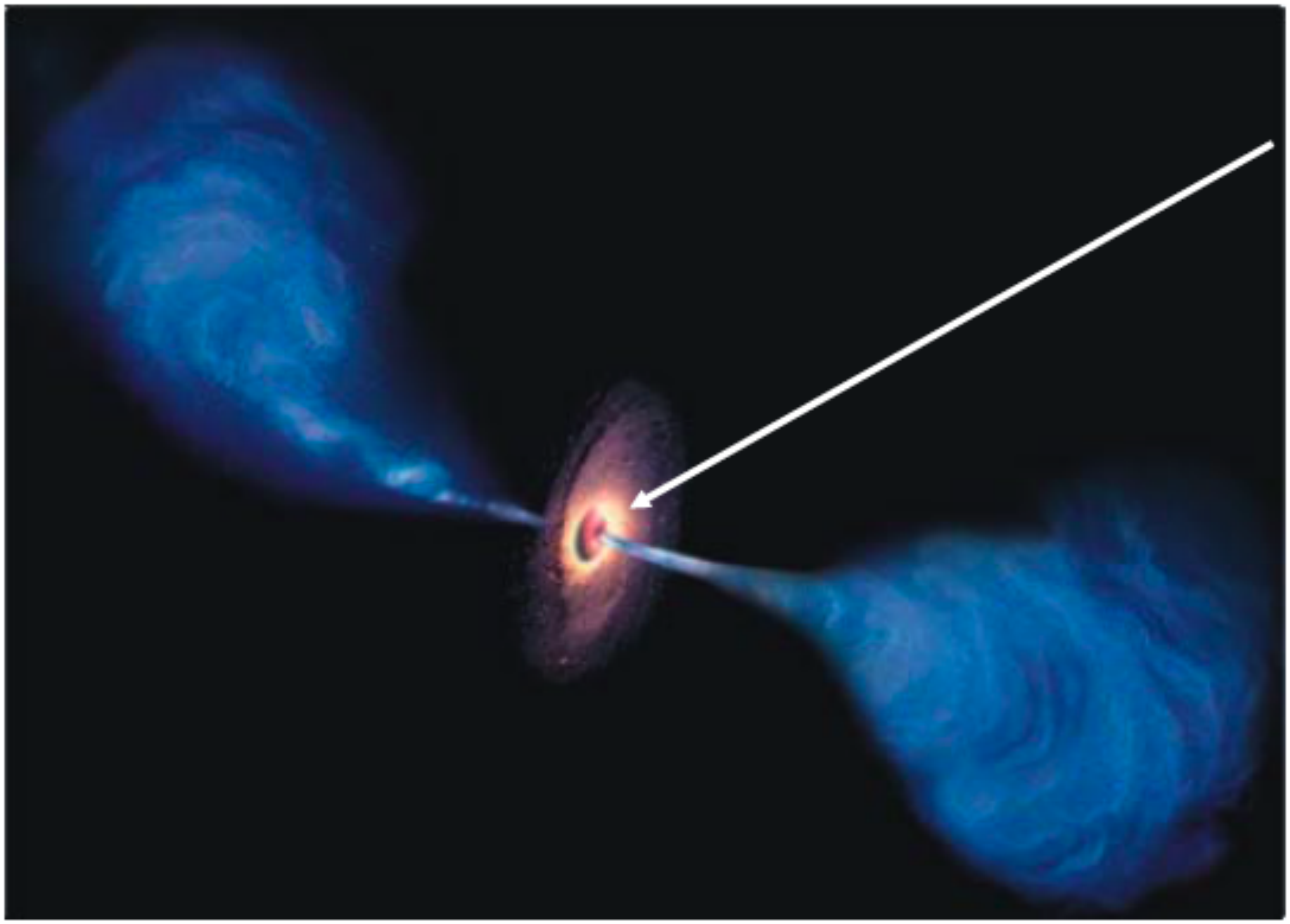
# Super-massive black holes

- Black holes with a mass of 1E6 to 1E10 solar masses
- Essentially all galaxies, including our own, contain super-massive black holes



# Active galaxies

- Have super-massive black holes at the center, and are in the process of having lunch (accreting nearby stars, gas, dust)



Supermassive black hole $10^9$ $M_{sun}$

# The real thing