

Announcements

- CAPA homework 1 due on Thursday Sept. 13 at 10 AM
- CAPA homework 2 due on Tuesday Sept 18 at 10 AM
- CAPA homework 3 due on Thursday Sept 20 at 10 AM
- Please register your iclicker through LON-CAPA
 - ◆ if you want to receive credit
 - ◆ only a few of you have not
 - ◆ first iclicker question today: **note: if I see anyone with two iclickers, I will take both of them**
- Help room hours (1248 BPS)
 - ◆ Ian La Valley(TA)
 - ◆ Mon 4-6 PM (except not Monday Sept 17)
 - ◆ Tues 12-3 PM
 - ◆ Wed 6-9 PM
 - ◆ Fri 10 AM-noon

Motion

- Position

- ◆ location in space relative to an origin (x,y,z); often-times we will just quote an x position for simplicity

- Velocity

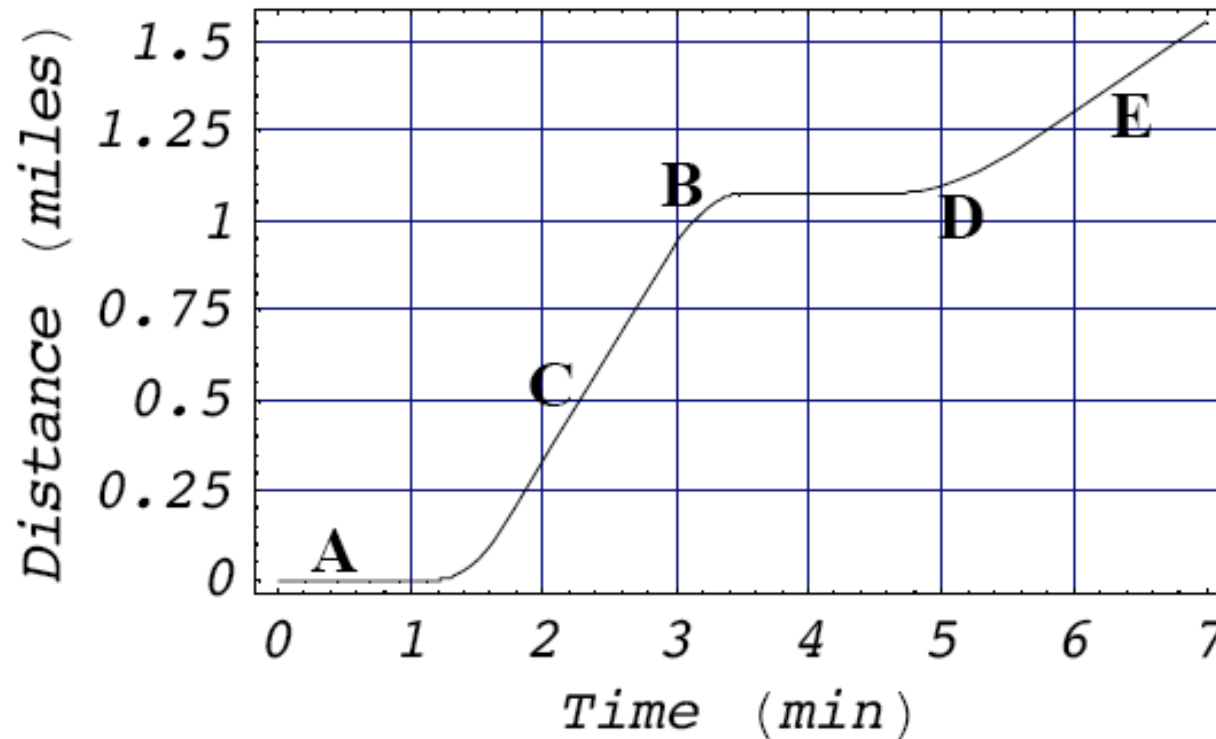
- ◆ rate of change of position

- Acceleration

- ◆ rate of change of velocity

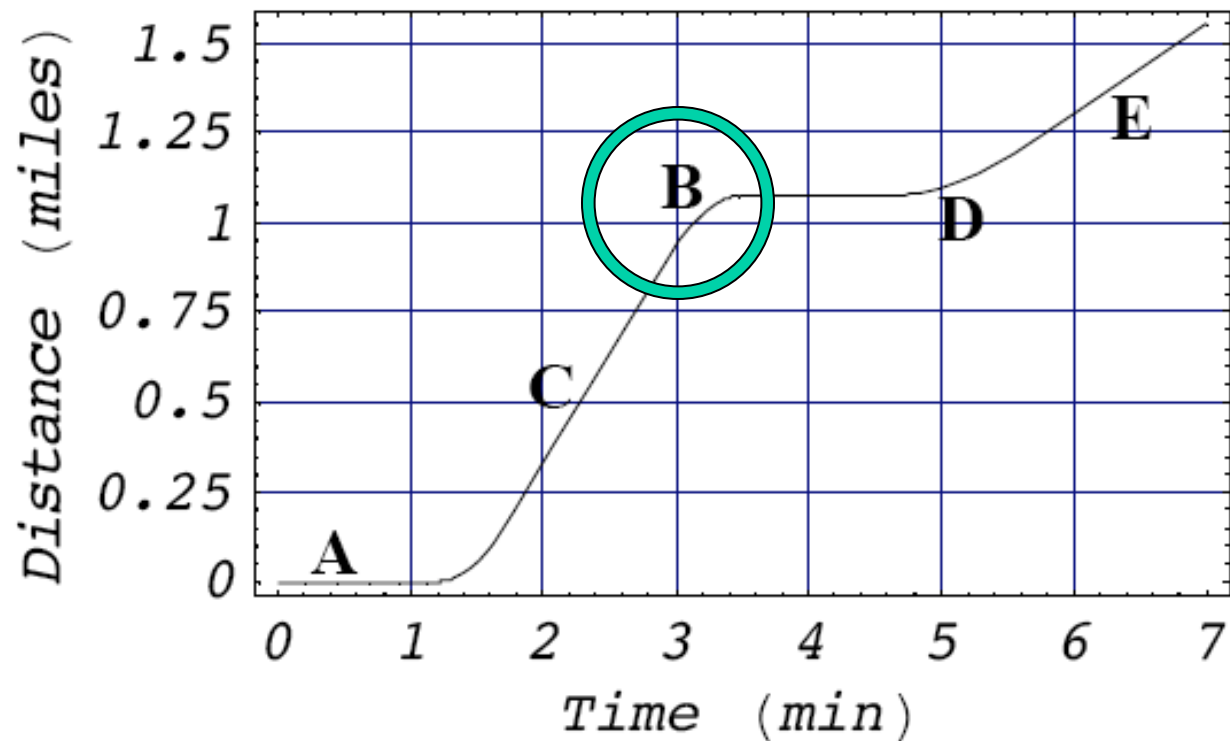
LON-CAPA question

Where is the acceleration negative?



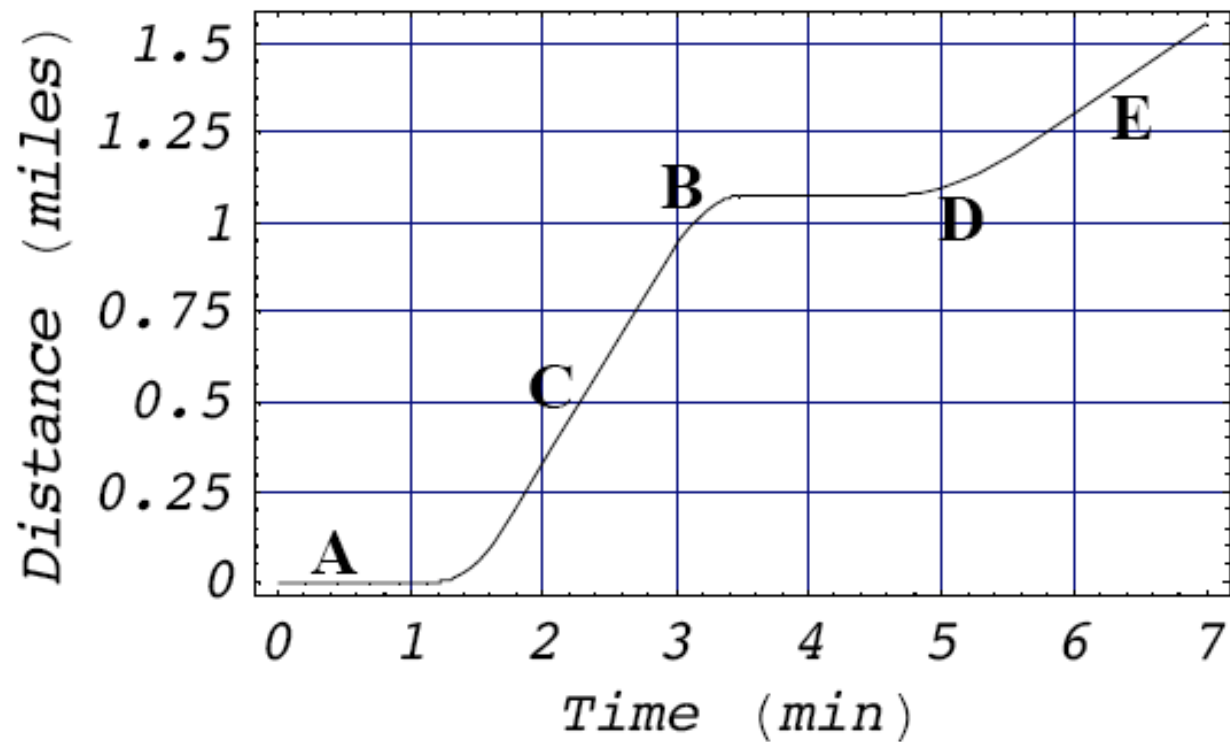
x direction $\rightarrow$ + is to the right, - is to the left

Where is the acceleration negative?



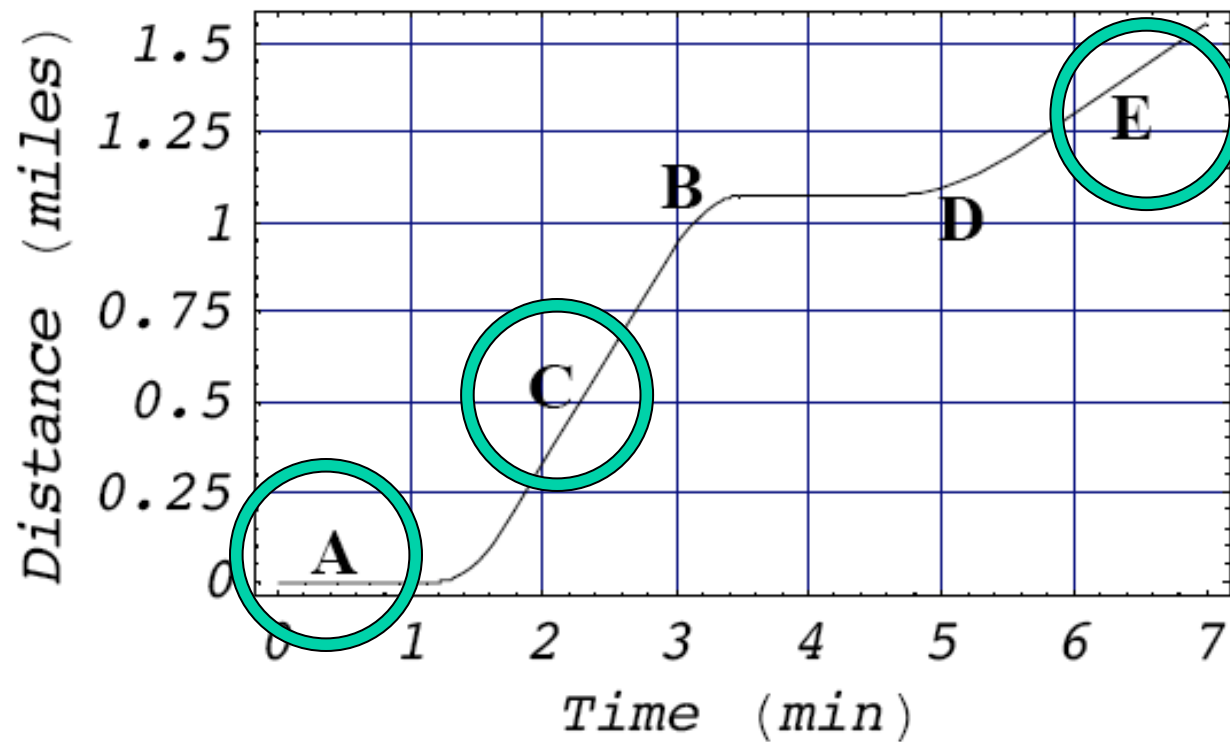
x direction $\rightarrow$ + is to the right, - is to the left

Where is the acceleration zero?



x direction $\rightarrow$ + is to the right, - is to the left

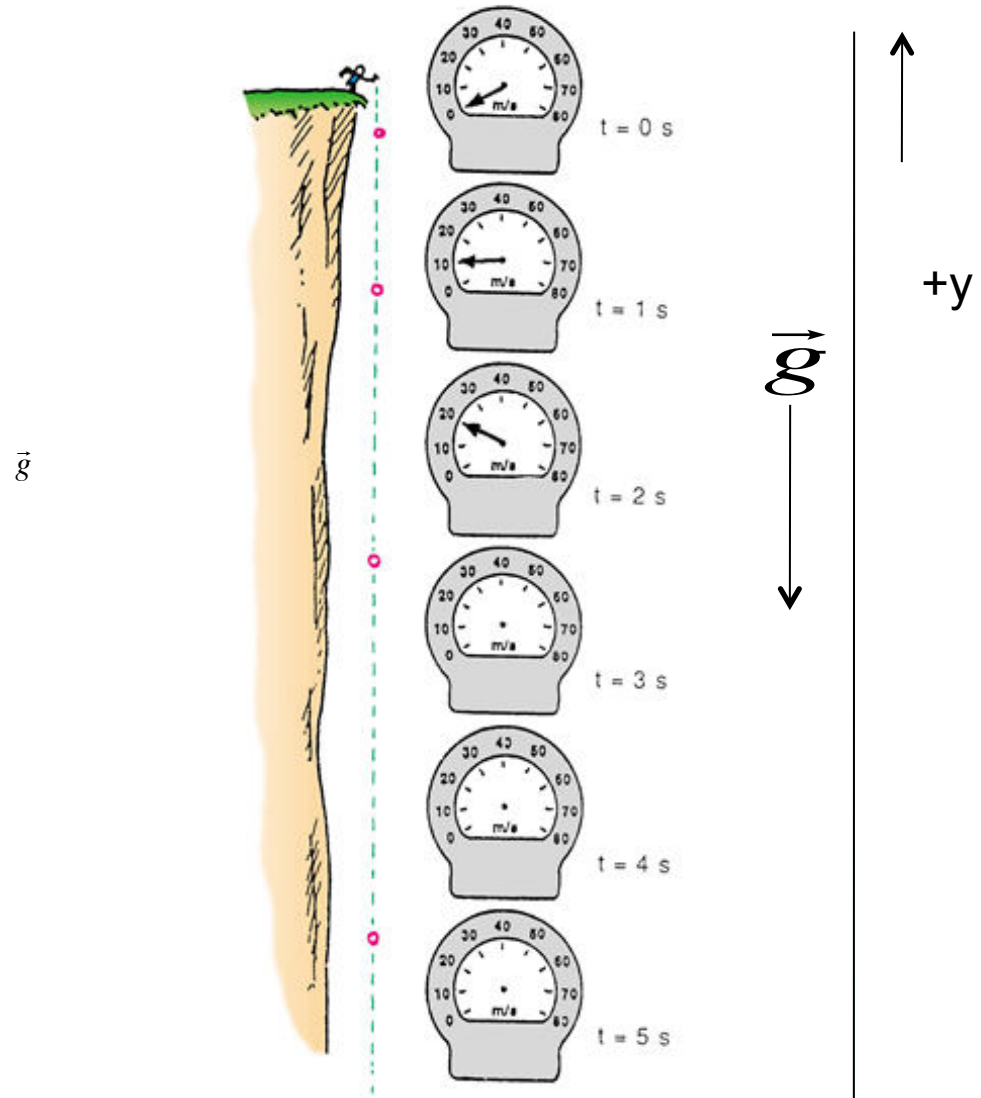
Where is the acceleration zero?



x direction $\rightarrow$ + is to the right, - is to the left

Acceleration

- An example of an accelerated object is an object falling under the influence of gravity
 - ◆ $y = y_0 + 1/2at^2$
 - ◆ here I've used y instead of x for the axis (from convention)
 - ◆ I can also write this as $y = y_0 - 1/2gt^2$
 - ◆ where $a = -g$ (the acceleration is in the $-y$ direction)
 - ◆ and $g = 9.8 \text{ m/s}^2$
- We will study later in this lecture the force of gravity and why $g = 9.8 \text{ m/s}^2$, but for the moment assume this (constant) value (and direction) at the Earth's surface



Acceleration

- Suppose you drop a ball off of a cliff and you note that it takes 6 seconds to hit the ground

- ◆ how high is the cliff?
(ignore air resistance)

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

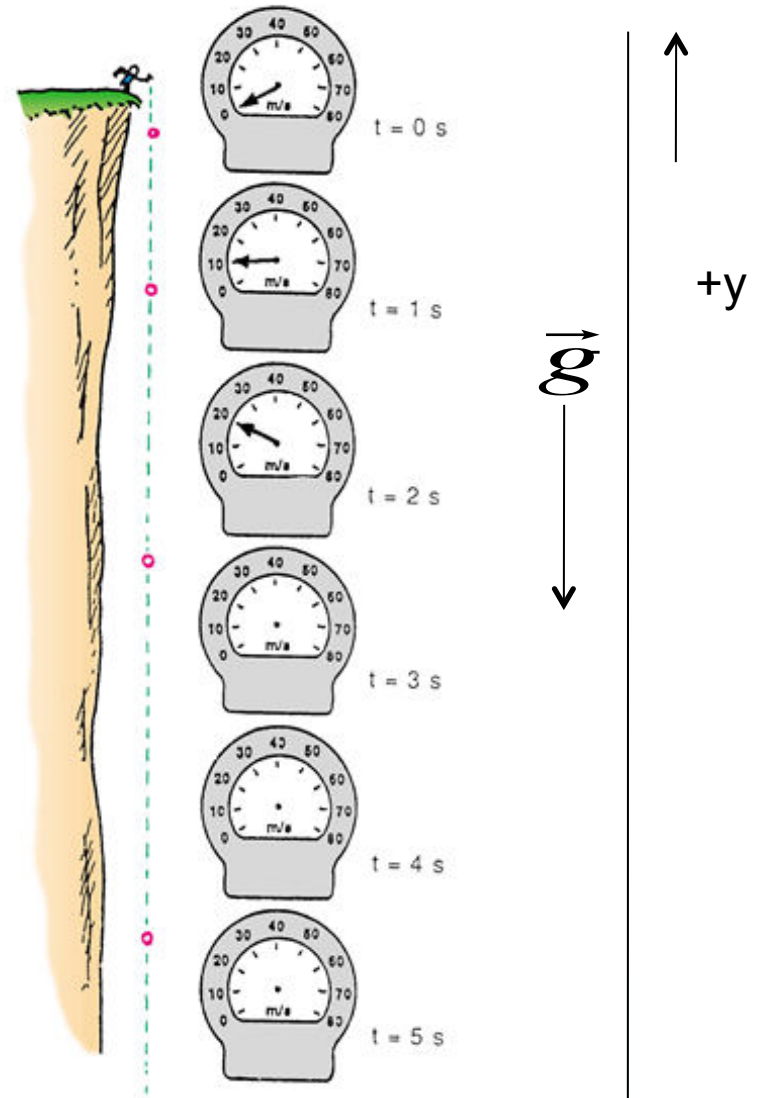
$$y = y_0 - \frac{1}{2} g t^2$$

- ◆ at $t=6$ s, $y=0$

$$y_0 = \frac{1}{2} g t^2 = \frac{1}{2} (9.8 \text{ m/s}^2) (6 \text{ s})^2$$

$$y_0 = 176.4 \text{ m}$$

- ◆ how would your answer change if air resistance was a factor?



Acceleration

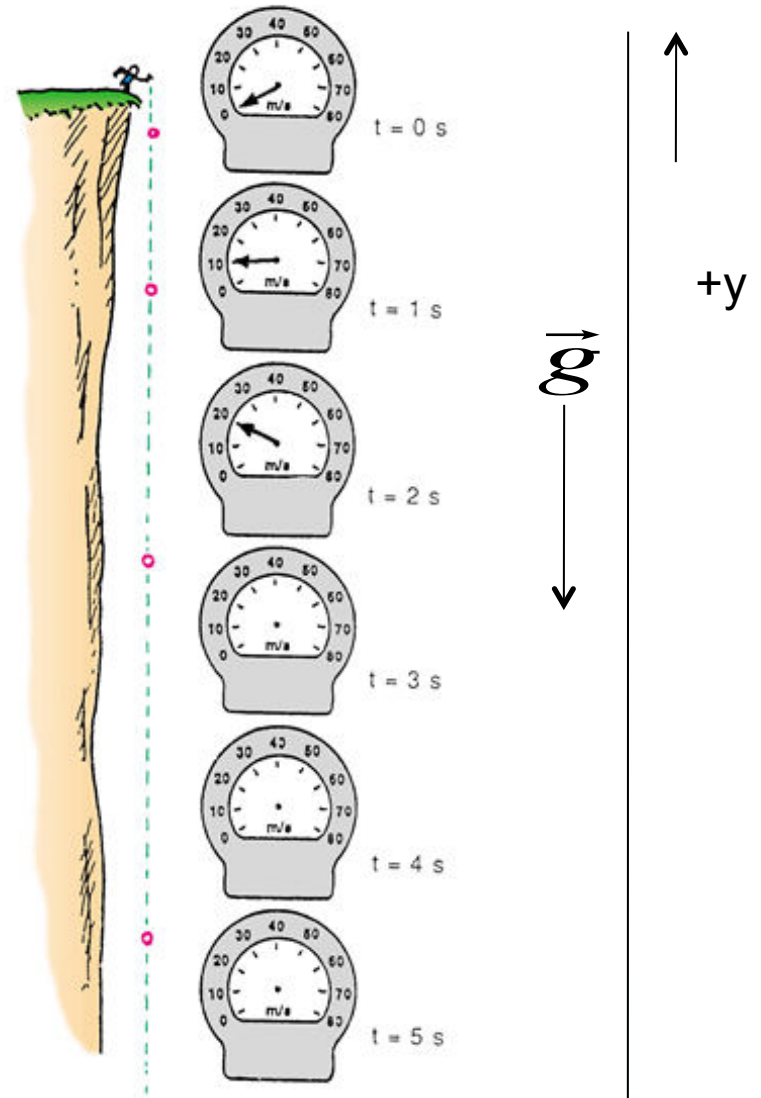
- What's happening to the velocity?
 - ◆ $v = v_0 - gt$
 - ◆ ...or $v = -gt$
 - ◆ under constant acceleration, the velocity increases uniformly with time
- What is the velocity when the ball hits the ground?

$$v = v_0 + at$$

$$v = 0 - gt$$

$$v = -(9.8 \text{ m/s}^2)(6 \text{ s}) = -58.8 \text{ m/s}$$

- What would happen to the speed if air resistance were a factor?

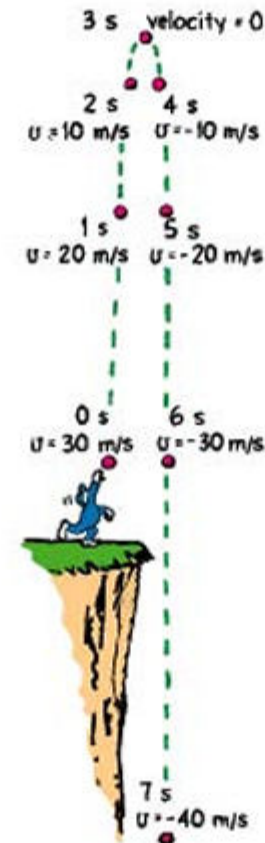


Acceleration

- Suppose that there is an initial velocity
- What does the path look like?
 - ◆ $y = y_0 + v_0 t + \frac{1}{2}at^2$
 - ◆ ...or $y = y_0 + v_0 t - \frac{1}{2}gt^2$
- According to this picture, it reaches its maximum height at a time of ~3 seconds
- How fast is it traveling then?
- How high has it gone?

$$y = y_0 + (30\text{ m/s})(3\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(3\text{ s})^2$$

$$y = y_0 + 45.9\text{ m}$$



Acceleration

- What's happening to the velocity?

- ◆ $v = v_o + at$

- ◆ ...or $v = v_o - gt$

- What is the velocity at $t \sim 3$ s?

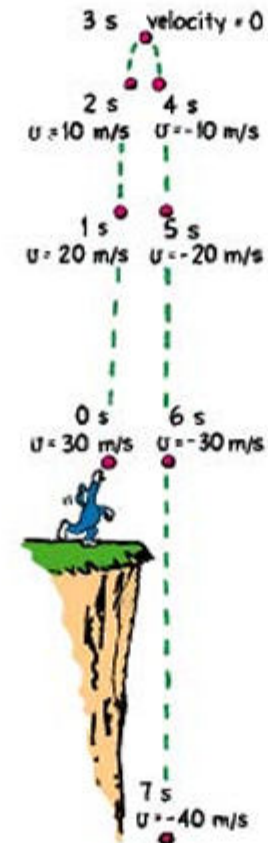
$$v = 30 \text{ m/s} - (9.8 \text{ m/s}^2)(3 \text{ s}) = 0.6 \text{ m/s}$$

$$v \sim 0$$

- When is the velocity exactly 0?

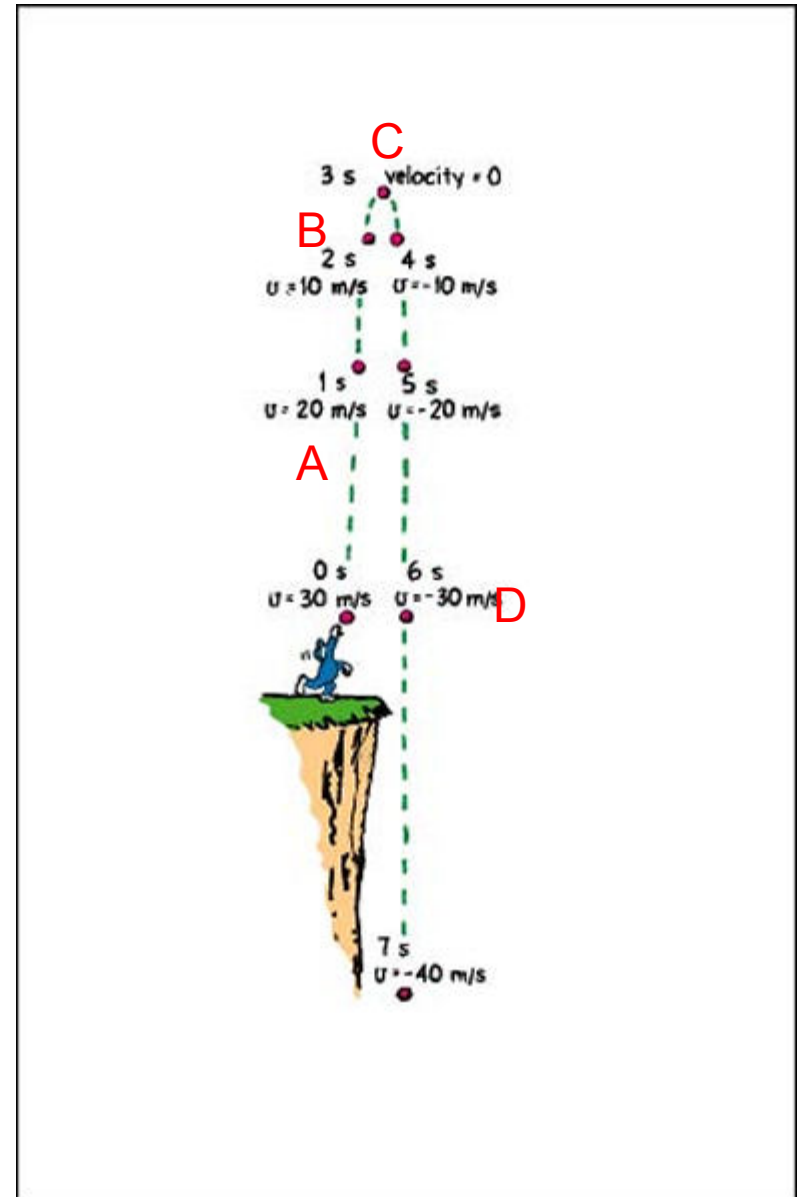
$$0 = v_0 - gt$$

$$t = \frac{v_o}{g} = \frac{30 \text{ m/s}}{9.8 \text{ m/s}^2} = 3.06 \text{ s}$$



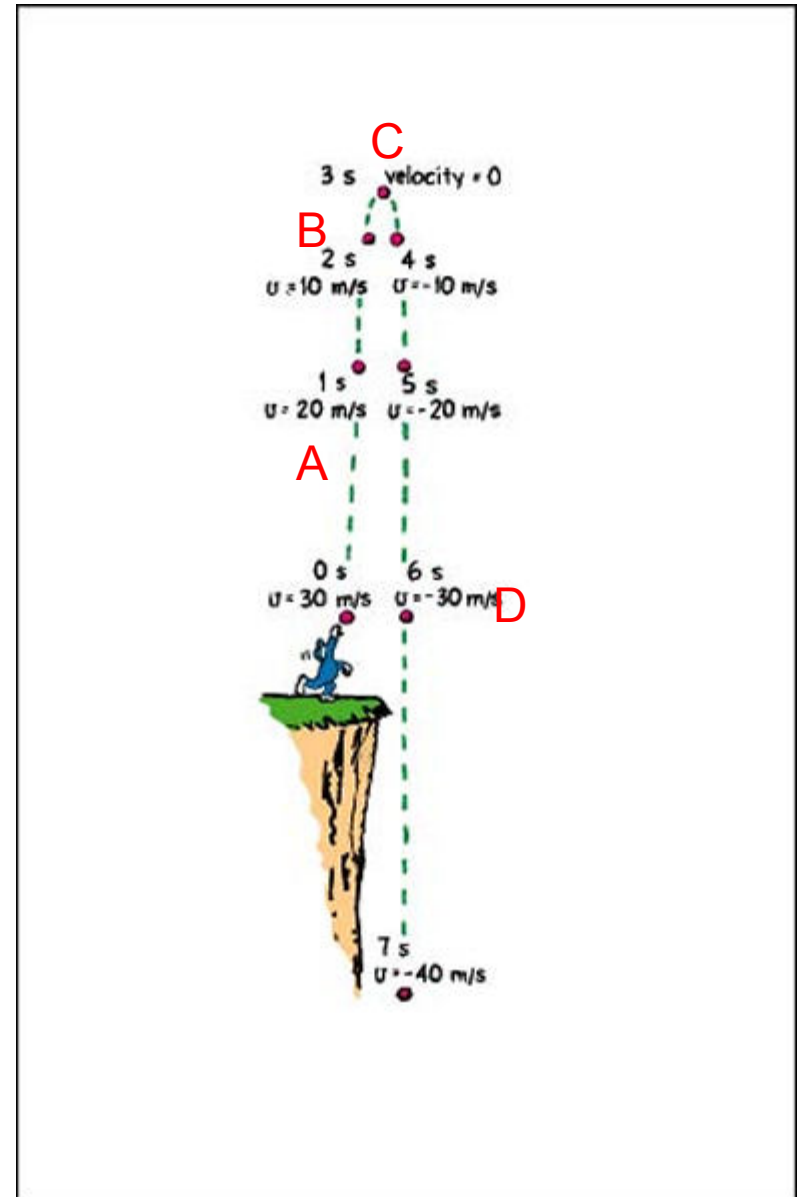
iclicker: acceleration

- Where is the acceleration the largest?
 - ◆ A
 - ◆ B
 - ◆ C
 - ◆ D
 - ◆ E (everywhere the same)



Clicker: acceleration

- Where is the acceleration the largest?
 - ◆ A
 - ◆ B
 - ◆ C
 - ◆ D
 - ◆ E (everywhere the same)



Hang-time

- What is the “hang-time” of a great basketball player?
- As you leap off the floor, your initial velocity v_0 decreases due to the acceleration from gravity (-9.8 m/s^2)
- At the top of the leap, your velocity is 0
- The time to fall is the same as the time to rise
 - ◆ hang time = time up + time down

$$y - y_0 = h = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(1.25\text{m})}{9.8\text{m/s}^2}} = 0.50\text{s}$$

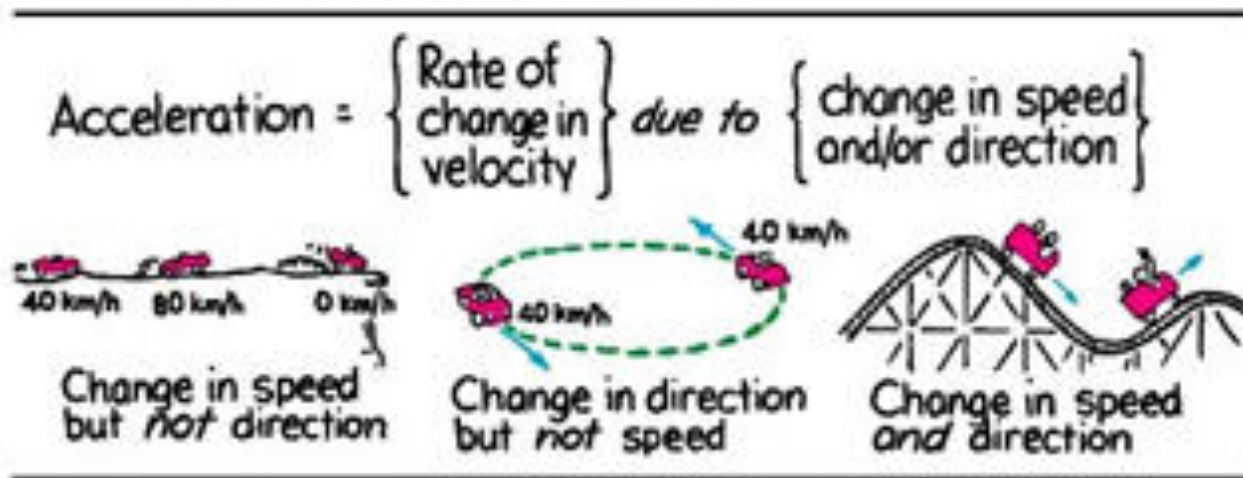
- Total time = $0.50\text{s} + 0.50\text{s} = 1\text{s}$



A very good jump is 1.25 m

Acceleration

Reminder: acceleration is the rate of change of velocity. It doesn't mean increase in speed, or even change in speed, just change in the velocity, either direction or magnitude.



Understanding motion

- Aristotle (384 BC – 322 BC)
 - ◆ *natural* motions (vertical)-objects seek their natural location
 - ◆ *violent* motions (horizontal) require active agents
- Galileo (1564-1642)
 - ◆ used experimentation to deduce the laws of motion
 - ◆ introduced concept of inertia
 - ◆ spent a great deal of time trying to understand acceleration
- Newton (1643-1727)
 - ◆ developed quantitative connections between forces and motion

Isaac Newton

- In 1687, the world changed
- Isaac Newton published his Mathematical Principles of Natural Philosophy
 - ◆ Newton was able to give quantitative descriptions of motions both in our world and in the rest of the universe
- In the 20th Century, we've come to realize that Newtonian physics is an approximation
 - ◆ relativity and quantum mechanics give a better description in regimes where the speeds are close to that of light, and where the distances are at the atomic level or smaller



Nature and nature's laws lay hid in night;
God said, "Let Newton be" and all was light

Alexander Pope, 18th Century British poet

Isaac Newton and Q

- Coincidental resemblance, I think not



What is a force?

- Practical definition:
it's a push or pull
- It can cause a body to accelerate
- It's a vector so it has both a magnitude and direction
- Force is the rate of change of momentum

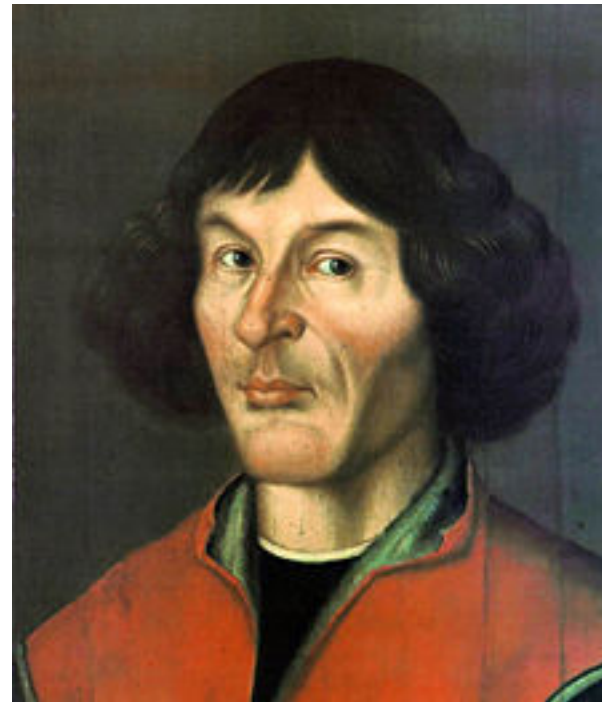
● Newton's 3 laws

1. If the net force on an object is zero, the object will not accelerate
▲ Galileo's law of inertia
2. The amount of acceleration depends on the net force and on the mass of the object
3. For every force there is an equal and opposite force

First law

- In 1543, on his deathbed, Nicholas Copernicus published his life's work, *De Revolutionibus Orbium Coelestium*, in which he proposed the Earth moves around the sun
- Why did he wait so long?
 - ◆ he was afraid of the reaction of the church
 - ◆ he didn't understand the first law
- He couldn't conceive of a force big enough to keep the Earth moving
 - ◆ not realizing that once in motion, it will stay in motion

If the net force on an object is zero, the object will not accelerate



Second law

- The amount of acceleration depends on the net force and on the mass of the object

$$\text{Acceleration} = \frac{\text{net force}}{\text{mass}} \quad \vec{a} = \frac{\vec{F}}{m}$$

- then the unit for acceleration must be the unit for force (N) divided by the unit for mass (kg), or N/kg
- but earlier we said that the unit for acceleration is m/s²
- it can be either depending on whether we are doing a motion problem or a force problem



low acceleration

Force of hand
accelerates
the brick



Twice as much force
produces twice as
much acceleration



Twice the force on
twice the mass gives
the same acceleration

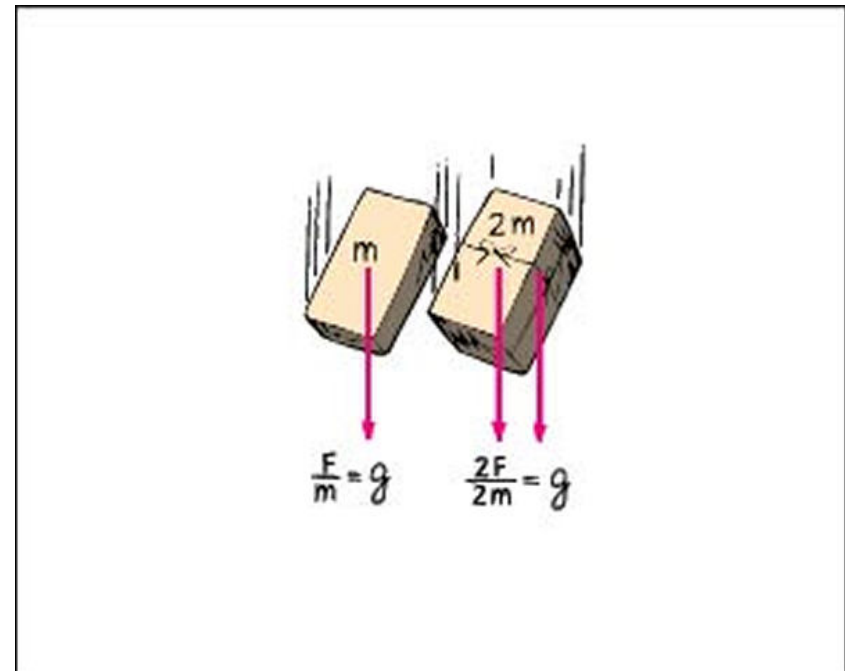


What if the acceleration is due to gravity?

- $g = 9.8 \text{ m/s}^2 = 9.8 \text{ N/kg}$

$$\vec{F} = m\vec{g}$$

- The greater the mass of an object, the greater is the gravitational force
- The gravitational force on 2 bricks is twice as large as the force on 1 brick, but the acceleration is the same



Free fall with air resistance

- Air resistance on skydiver builds up as speed increases
- Downward force is always $|F| = mg$

- The net force is

$$\vec{F}_{net} = m\vec{g} + \vec{R}$$

$$F_{net} = R - mg$$

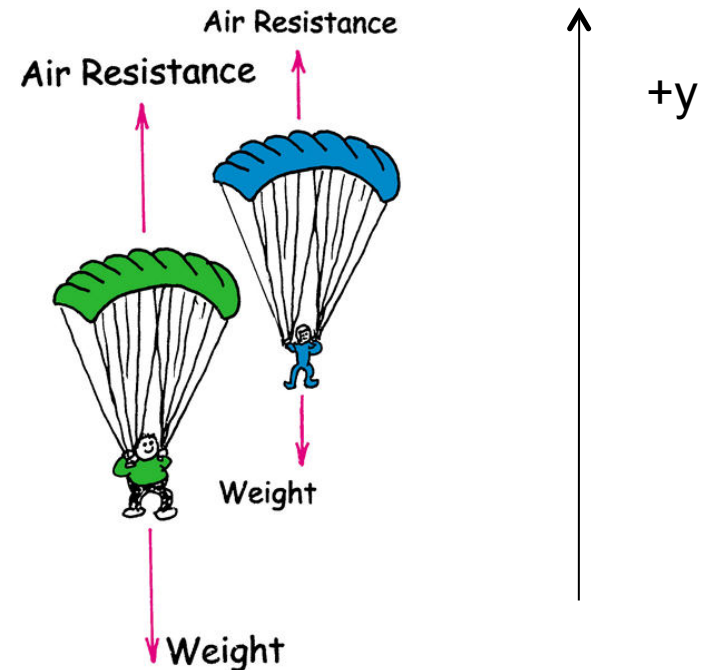
♦ note the signs

R depends on v

- So air resistance reduces the net force, and reduced force means reduced acceleration

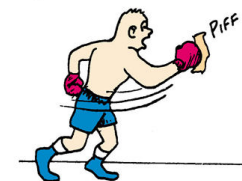
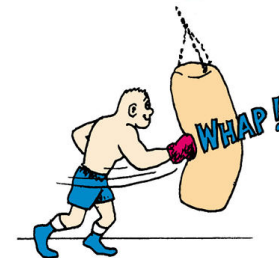
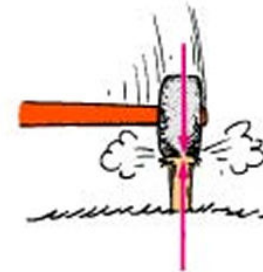
$$a = \frac{F_{net}}{m} = \frac{R}{m} - g$$

- What happens when the force due to air resistance equals (in magnitude) the force due to gravity?



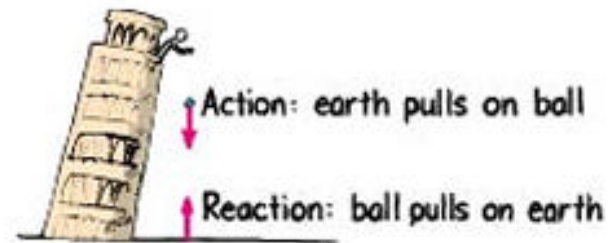
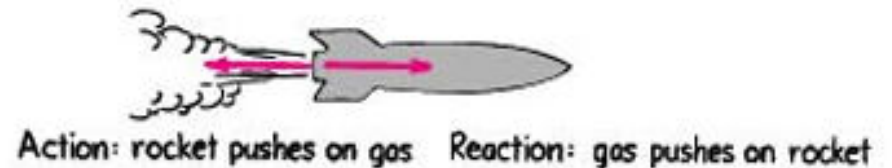
Newton's 3rd law

- For every force there is an equal and opposite force
 - or, to every action, there is an equal and opposite reaction
- So, if I push on the wall, the wall is pushing back on me
- The stake is pushing back on the hammer
- The punching bag is pushing back on the boxer
 - ◆ note that the boxer can only exert a small force on the tissue, since the tissue isn't exerting much force back on the boxer
- We call one force the action force and the other, the reaction force
 - ◆ doesn't matter really which is which



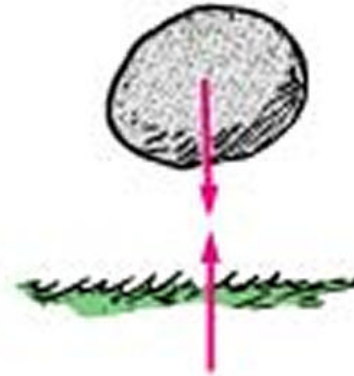
Action and reaction

- Action: object A exerts a force on object B
- Reaction: object B exerts an equal and opposite force on object A
- The force on the bullet is the same as the force acting on the rifle
 - ◆ why does the bullet have a larger acceleration?



Newton's 3rd law

- The rock pulls on the Earth as much as the Earth pulls on the rock
- If there's no reaction force, then there's no action force
- So, no (or little) traction when walking on ice



Another example

- Your friend (mass 80 kg) and you (mass 60 kg) are both at rest wearing frictionless roller skates. You push against your friend's back with a force of 60 N. How hard is your friend's back pushing against you?
- What is your acceleration? What is your friend's acceleration?

Another example

- Your friend (mass 80 kg) and you (mass 60 kg) are both at rest wearing frictionless roller skates. You push against your friend's back with a force of 60 N. How hard is your friend's back pushing against you?
- The forces are an action-reaction pair, so your friend's back is pushing on you with a force of 60 N as well
- What is your acceleration? What is your friend's acceleration?

$$a_{you} = \frac{F}{m_{you}} = \frac{60N}{60kg} = 1N/kg = 1m/s^2$$

$$a_{friend} = \frac{F}{m_{friend}} = \frac{60N}{80kg} = 0.75N/kg = 0.75m/s^2$$

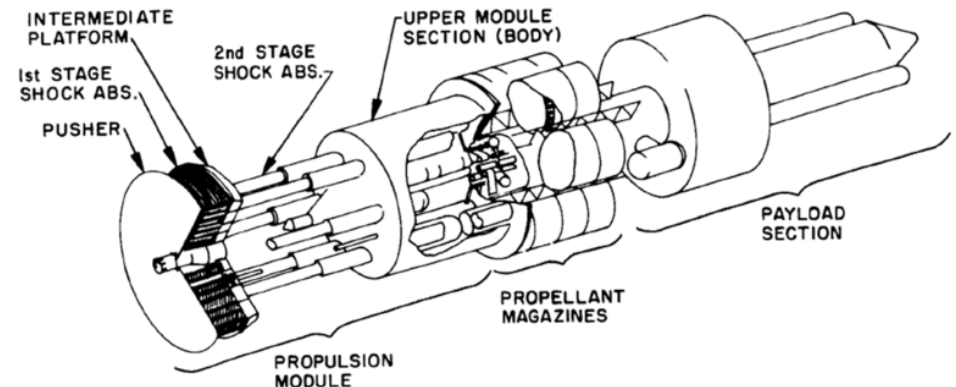
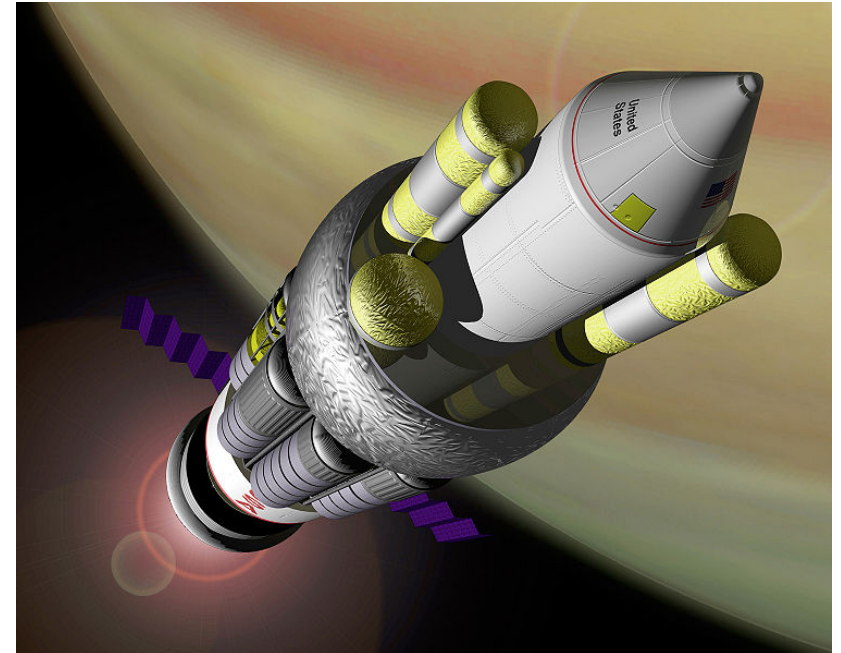
Robert Goddard and the New York Times

- Jan. 13, 1920 Editorial
- "after the rocket quits our air and really starts on its longer journey it will neither be accelerated nor maintained by the explosion of the charges it then might have left. To claim that it would be is to deny a fundamental law of dynamics, and only *Dr. Einstein and his chosen dozen, so few and fit, are licensed to do that.*" It expressed disbelief that Professor Goddard actually "*does not know of the relation of action to reaction, and the need to have something better than a vacuum against which to react*" and even talked of "*such things as intentional mistakes or oversights.*" Goddard, the Times declared, apparently suggesting bad faith, "*only seems to lack the knowledge ladled out daily in high schools.*"[
- The New York Times published a correction on July 17, 1969



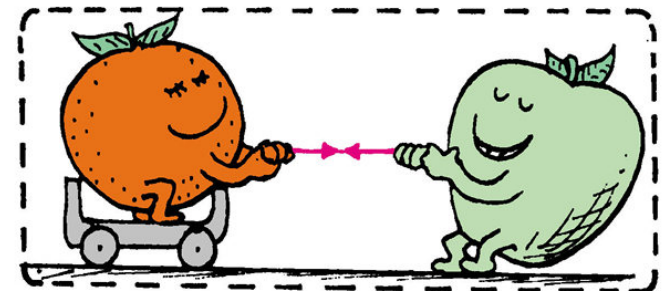
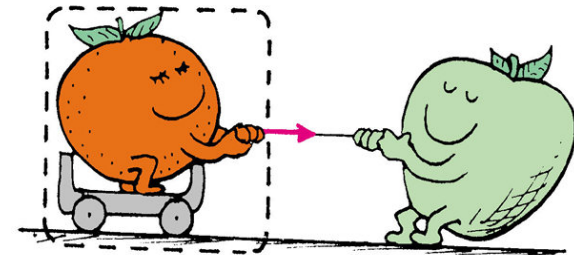
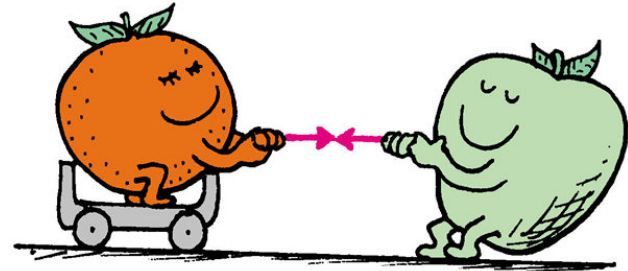
Project Orion

- During the late 50's and early 60's, there was a project to design a spaceship powered by detonating nuclear bombs underneath the ship
- The shock wave would push against a “pusher” plate and would cause the ship to accelerate
- The Nuclear Test Ban treaty killed the idea



Defining your system

- So if the action-reaction pair of forces are equal and opposite, does that mean that there is no net force and thus no acceleration
- Depends on the system you're talking about



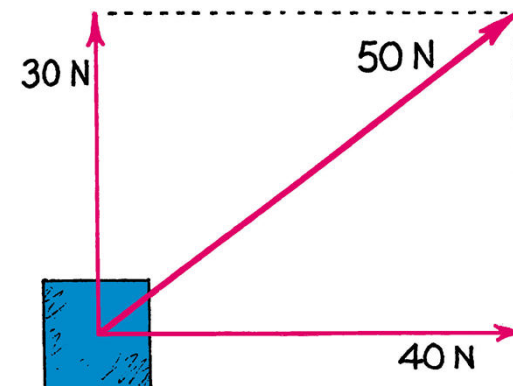
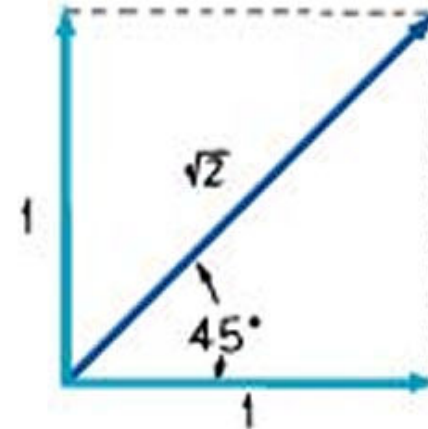
Action-reaction

- Supposedly Baron Munchhausen yanked himself out of the sea by pulling on his own hair (or bootstraps depending on the version of the story)
 - ◆ this is where the phrase “pulling yourself up by your own bootstraps” comes from
- Try it yourself to see if there's any net force



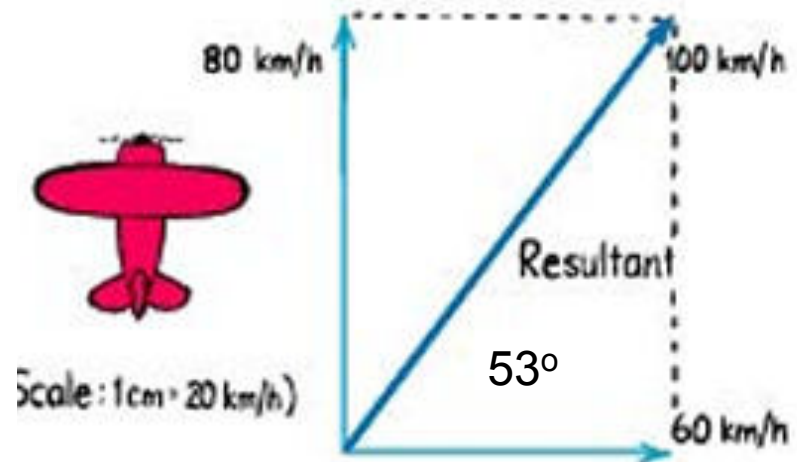
More vectors

- We have already said that quantities such as velocity, force and acceleration require both magnitude and direction
- They are vector quantities
- We've talked about how to add vectors



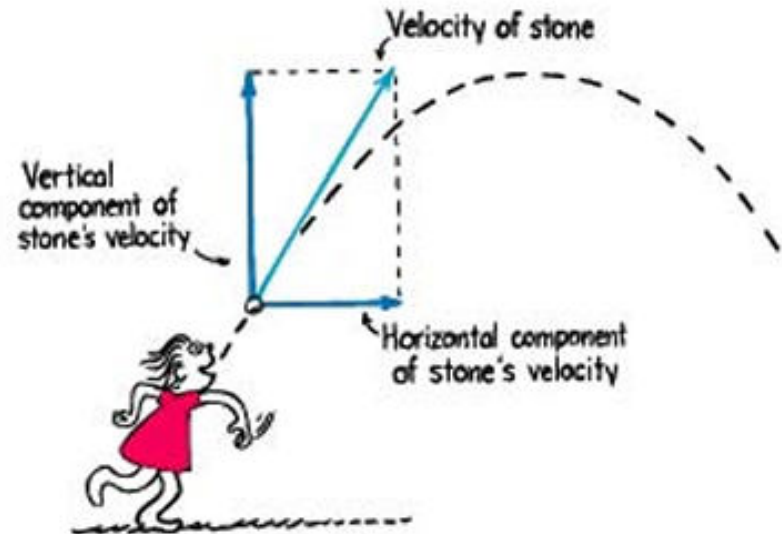
Vector addition

- Suppose a plane is heading north at 80 km/h, but there's a wind blowing east at 60 km/h
- What is the resultant (vector) velocity?
- I have two vectors I have to add
- The resultant speed is 100 km/h with a direction 53 degrees north of east



Resolving vectors

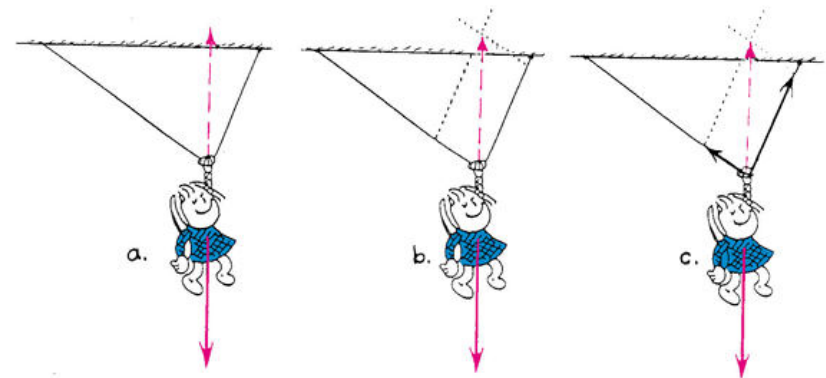
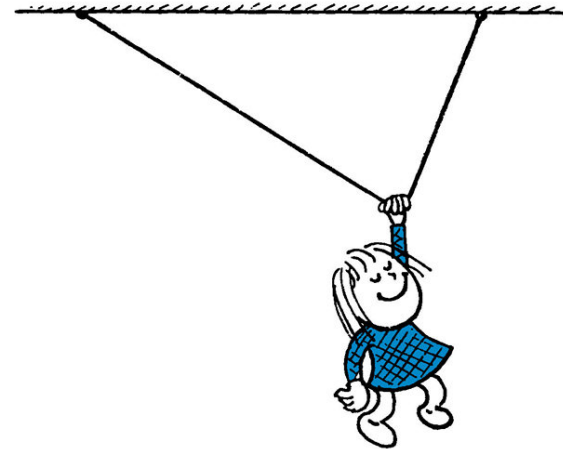
- Just as we can add any two vectors to get a resultant vector, we can also any vector into two components perpendicular to each other
 - ◆ as for example, the vertical and horizontal components of a thrown ball



Note that there is a change in the velocity of the ball. It is accelerating in the downward direction.

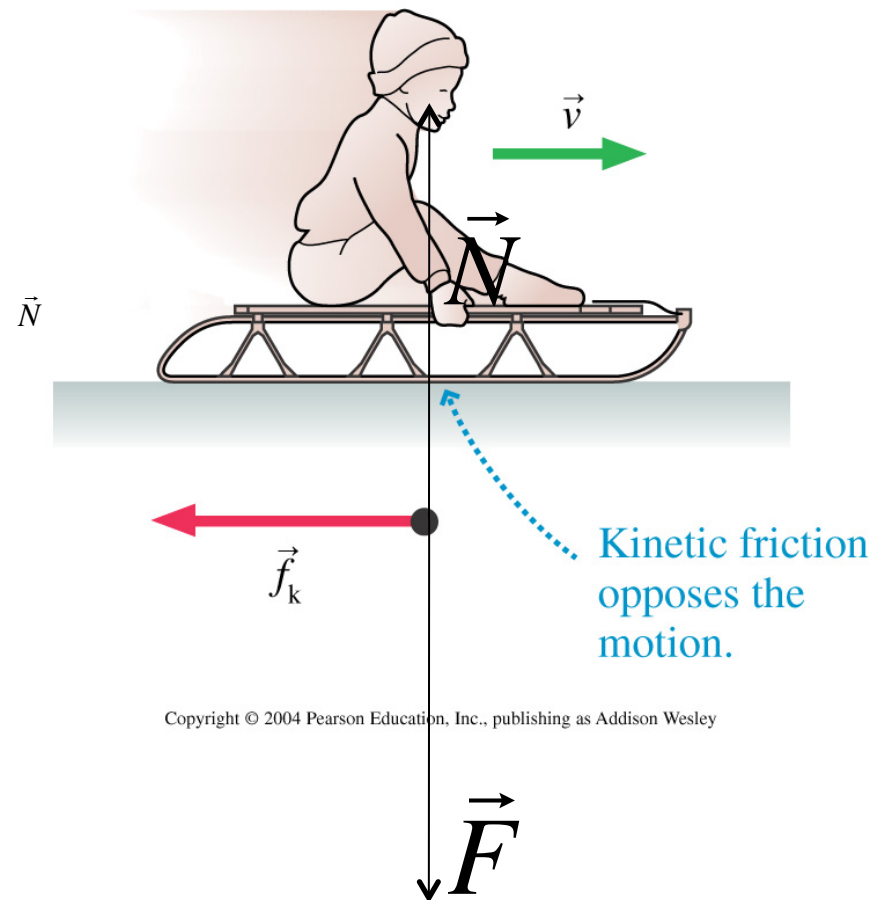
Forces

- A girl hangs from a rope
- What are the forces involved?
- Which end of the rope is more likely to break?
- The force of gravity is acting downward
- ...so there must be an equal and opposite reaction force
- That force can be divided into vector components along the two ropes
- The larger force is on the rope segment to the right...so that is where the rope is more likely to break



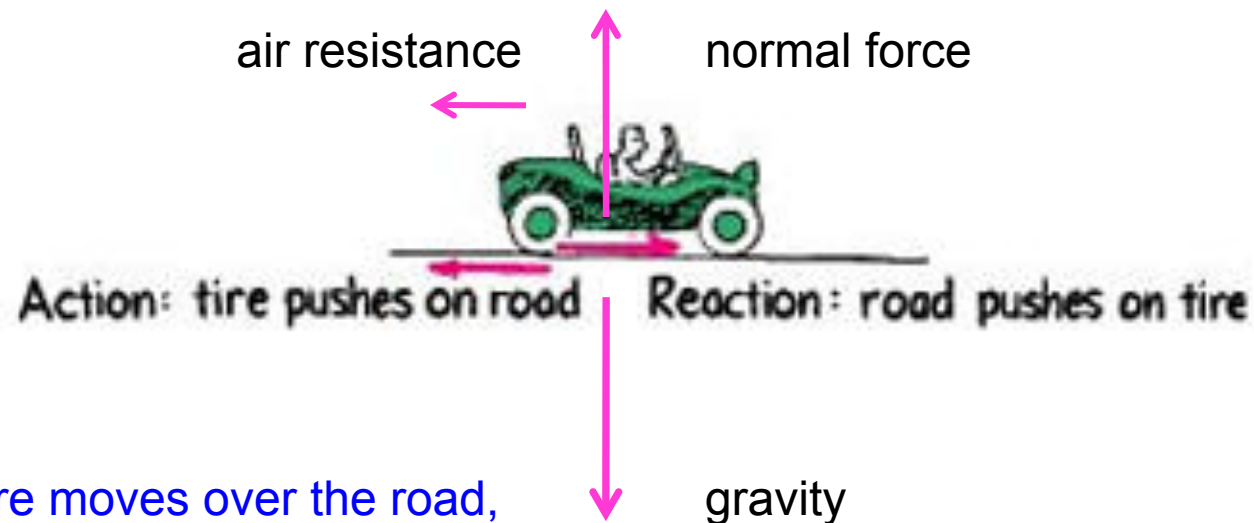
Friction

- Consider the forces acting on a sled initially moving with a velocity v
- Gravity is pulling downward with a force F
- There's an equal and opposite normal force N applied from the sled to the ground
- There's a frictional force proportional to the normal force
 - ◆ $f_k = \mu_k N$
 - ◆ where μ_k is called the coefficient of kinetic friction
- So there's no net force in the vertical direction
- Suppose there is a net force in the horizontal direction
- So what happens to the sled



Motion of a car

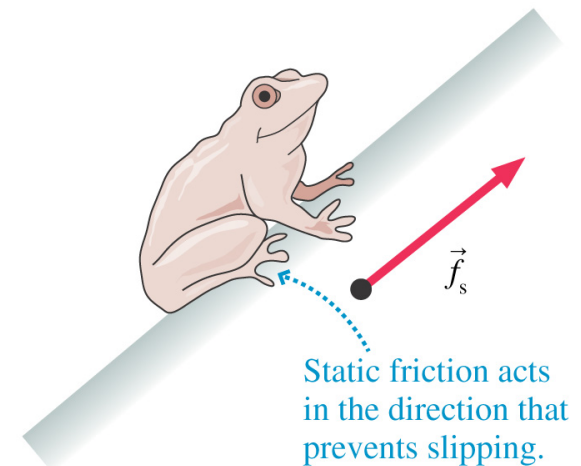
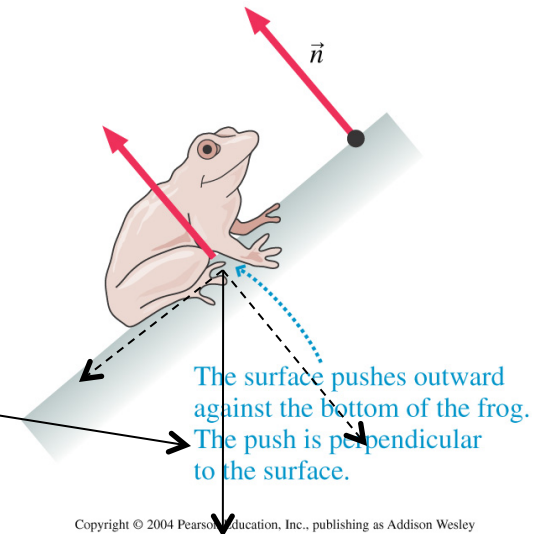
- Consider all of the forces acting in this situation



- As the tire moves over the road, the tire flattens. The force exerted by the road on the tire acts to slow the tire's rotation.
- Rolling resistance
- Less pronounced for steel wheels

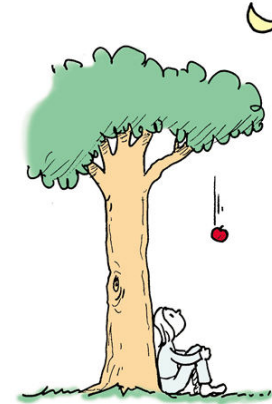
Static friction

- What about when there's no motion
- Is there still friction?
- Yes
- There's a gravitational force acting downward
- It can be resolved into two components
 - ◆ one perpendicular to the surface, balanced by the normal force
 - ◆ one along the surface
- There's a frictional force also along the surface, proportional to the normal force
 - ◆ $f_s = \mu_s N$
- If the horizontal forces are balanced, the frog stays put
- What happens if I start tilting the frog?



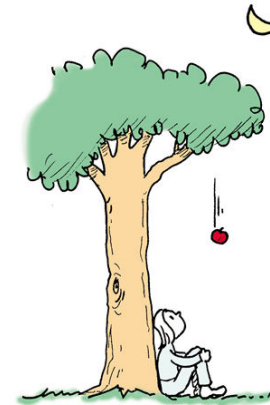
Gravitation

- Remember the story about Isaac Newton and the apple
- Maybe it didn't happen in reality, but the reasoning is roughly correct
- Newton had the insight to realize that the force between the Earth and the apple is the same as the force between the Earth and the Moon



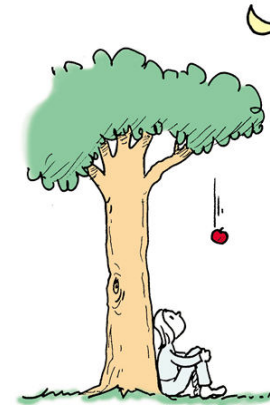
Gravitation

- The apple falls to the ground with an acceleration of 9.8 m/s^2
- The Moon falls away from the straight line path it would follow if there were no forces acting on it
 - ◆ remember Newton's first law
- Isaac Newton compared the measurements for the Moon's acceleration to his calculations
 - ◆ and found they didn't agree
 - ◆ so he put his theory away for 20 years, and worked on optics
 - ◆ and on how to turn lead into gold



Gravitation

- Inspired by the appearance of Halley's comet in 1682, he returned to the problem and found errors in the experimental data
- Now the data agreed with his predictions
- He then published the universal law of gravitation

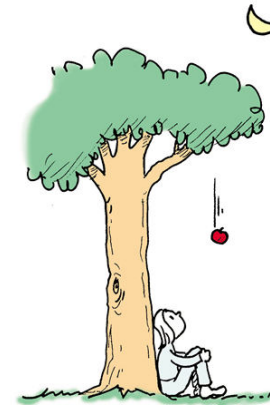


What did he find?

- He found the acceleration of the moon to be $\sim 2.7 \times 10^{-3} \text{ m/s}^2$
- Radius of Earth $= 6.37 \times 10^6 \text{ m}$
- Average Earth-Moon distance is $3.84 \times 10^8 \text{ m}$
- So the Moon is about 60 times as far away from the center of the Earth as is someone standing on the surface of the Earth

$$\frac{9.83 \text{ m/s}^2}{2.7 \times 10^{-3} \text{ m/s}^2} \approx 3600 = 60^2$$

- The acceleration due to gravity decreases as the square of the distance from the center of the Earth



Universal law of gravitation

- Incredibly simple and beautiful relation

$$\text{Force} \propto \frac{\text{mass}_1 \times \text{mass}_2}{\text{distance}^2}$$

$$\text{Force} \propto \frac{m_1 m_2}{d^2}$$

- The gravitational force between two masses is proportional to the product of the two masses and is inversely proportional to the square of the distance between them

- Before we dealt with mass in the equation below, technically known as the *inertial* mass

$$F = ma$$

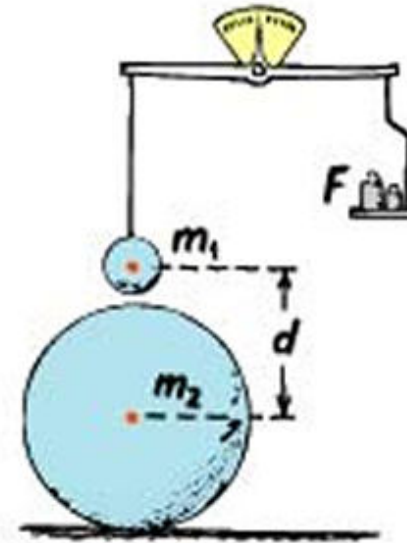
- Here we are dealing with the *gravitational* mass
- It is experimentally established that the two are equal and in fact the equivalence of the two is an integral part of the general theory of relativity

Universal law of gravitation

- We need a real equation, which means we need a constant of proportionality

$$F = G \frac{m_1 m_2}{d^2}$$

- Newton didn't know the value of G by itself, only the product of G times the mass of the Earth
- The measurement of G was big news in 1798
- Knowing G meant that the mass of the Earth could be calculated
 - ◆ $6 \times 10^{24} \text{ kg}$



G was determined by measuring the attraction of two masses; a difficult measurement since the force is very small

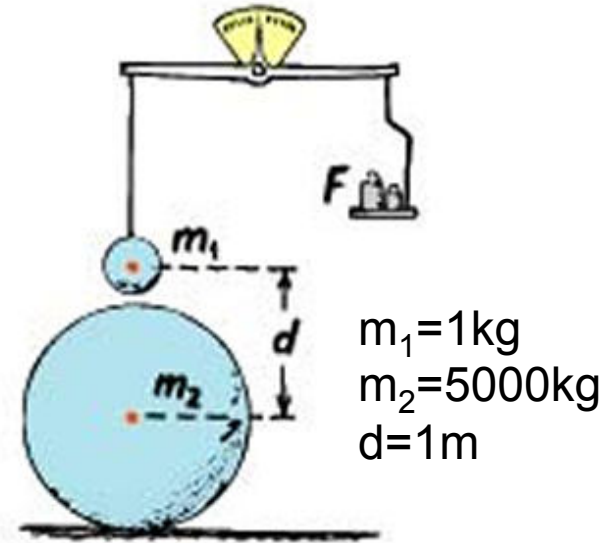
$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

How small of a force?

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2 \frac{(1\text{kg})(5000\text{kg})}{(1\text{m})^2}$$

$$F = 3.34 \times 10^{-7} \text{ N}$$



G was determined by measuring the attraction of two masses; a difficult measurement since the force is very small

The small value of G is an indication of how weak the gravitational force is.

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2$$

Weight

- Let's consider another force, your weight, i.e. the force the Earth exerts on you
- Suppose you weigh 60 kg
- $m_1 = m_{\text{Earth}} = 6 \times 10^{24} \text{ kg}$
- $m_2 = m_{\text{you}} = 60 \text{ kg}$
- $d = R_{\text{earth}} = 6.37 \times 10^6 \text{ m}$

$$F = G \frac{m_1 m_2}{d^2}$$

$$F = 6.67 \times 10^{-11} \text{ Nm}^2 / \text{kg}^2 \frac{(6 \times 10^{24} \text{ kg})(60 \text{ kg})}{(6.37 \times 10^6 \text{ m})^2}$$

$$F = 591 \text{ N}$$

