# Announcements

- CAPA homework 2 due on Tuesday Sept 18 at 10 AM
- CAPA homework 3 due on Thursday Sept 20 at 10 AM
- Please register your iclicker through LON-CAPA
  - if you want to receive credit
  - only a few of you have not
  - first iclicker question today**: note: if I see anyone with two iclickers, I will take both of them**
- Help room hours (1248 BPS)
  - Ian La Valley(TA)
  - Mon 4-6 PM (except not Monday Sept 17)
  - Tues 12-3 PM
  - Wed 6-9 PM
  - Fri 10 AM-noon

# Universal law of gravitation

- Incredibly simple and beautiful relation

$$\text{Force} \propto \frac{\text{mass}_1 X \text{mass}_2}{\text{distance}^2}$$

$$\text{Force} \propto \frac{\text{m}_1 m_2}{\text{d}^2}$$

- The gravitational force between two masses is proportional to the product of the two masses and is inversely proportional to the square of the distance between them

- Before we dealt with mass in the equation below, technically known as the *inertial* mass

$$F = ma$$

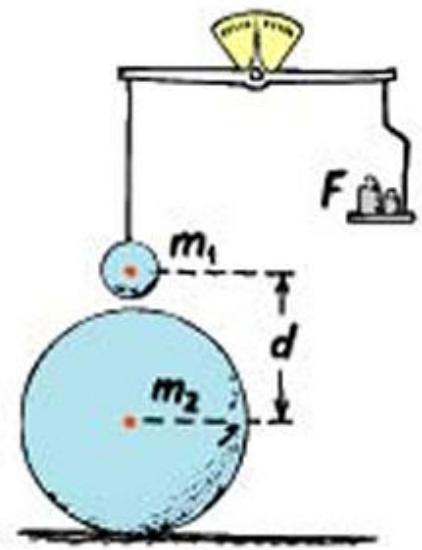
- Here we are dealing with the *gravitational* mass
- It is experimentally established that the two are equal and in fact the equivalence of the two is an integral part of the general theory of relativity

# Universal law of gravitation

- We need a real equation, which means we need a constant of proportionality

$$F = G\frac{m_1 m_2}{d^2}$$

- Newton didn't know the value of G by itself, only the product of G times the mass of the Earth
- The measurement of G was big news in 1798
- Knowing G meant that the mass of the Earth could be calculated
  - $6X10^{24}$kg




G was determined by measuring the attraction of two masses; a difficult measurement since the force is very small

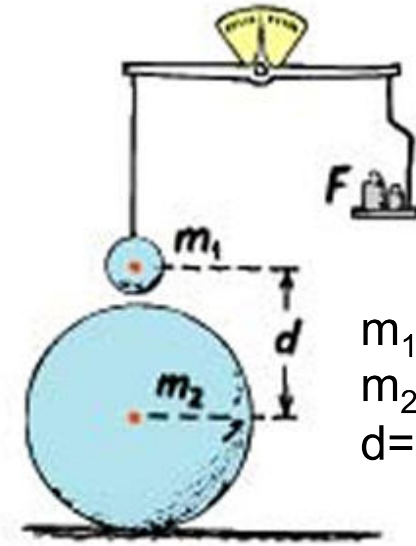
$G=6.67X10^{-11}\ Nm^2/kg^2$

# How small of a force?

$$F = G\frac{m_1 m_2}{d^2}$$

$$F = 6.67X10^{-11} Nm^2/kg2\frac{(1kg)(5000kg)}{(1m)^2}$$

$$F = 3.34X10^{-7} N$$




$m_1$=1kg
$m_2$=5000kg
d=1m

G was determined by measuring the attraction of two masses; a difficult measurement since the force is very small

$G=6.67X10^{-11} Nm^2/kg^2$

The small value of G is an indication of how weak the gravitational force is.

# Weight

- Let's consider another force, your weight, i.e. the force the Earth exerts on you
- Suppose you weigh 60 kg
- $m_1=m_{Earth}=6X10^{24}$ kg
- $m_2=m_{you}=60$ kg
- $d=R_{earth}=6.37X10^6$ m

$$F = G\frac{m_1 m_2}{d^2}$$

$$F = 6.67X10^{-11} Nm^2/kg^2 \frac{(6X10^{24}kg)(60kg)}{(6.37X10^6 m)^2}$$

$$F = 591N$$

# Clicker question

- What is the gravitational force that you exert on the Earth?
  - A) 0 N
  - B) 591 N
  - C) 6.67X10$^{-11}$ N
  - D) 6X10$^{24}$ N
  - E) cannot be determined with the information given

$$F_{Earth} = G\frac{m_1 m_2}{d^2}$$

$$F_{Earth} = 6.67X10^{-11} Nm^2/kg^2 \frac{(6X10^{24} kg)(60kg)}{(6.37X10^6 m)^2}$$

$$F_{Earth} = 591N$$

# Acceleration

- What is your acceleration due to the gravitational force of the Earth?

$$a = \frac{F}{m_{you}} = \frac{G\frac{m_{Earth}m_{you}}{d^2}}{m_{you}}$$

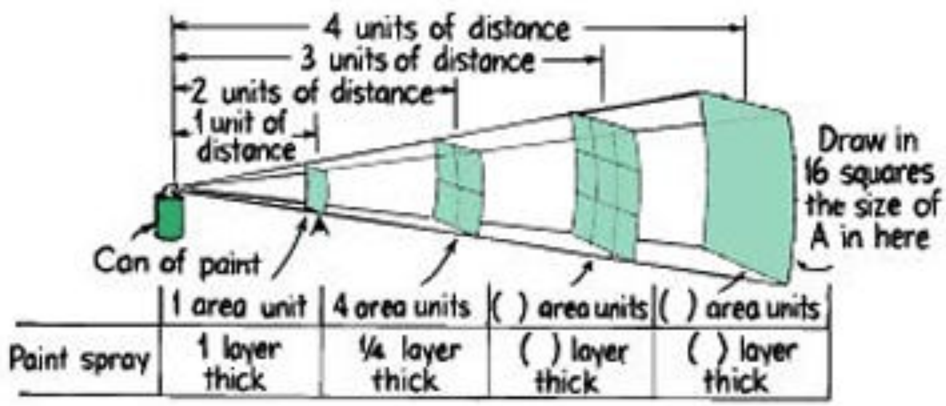
$$a = G\frac{m_{Earth}}{d^2} = 9.8N/kg = 9.8m/s^2 = g$$

- That is, your acceleration is the same as your friend who has twice the mass that you do (and weighs twice as much)

# Inverse square law

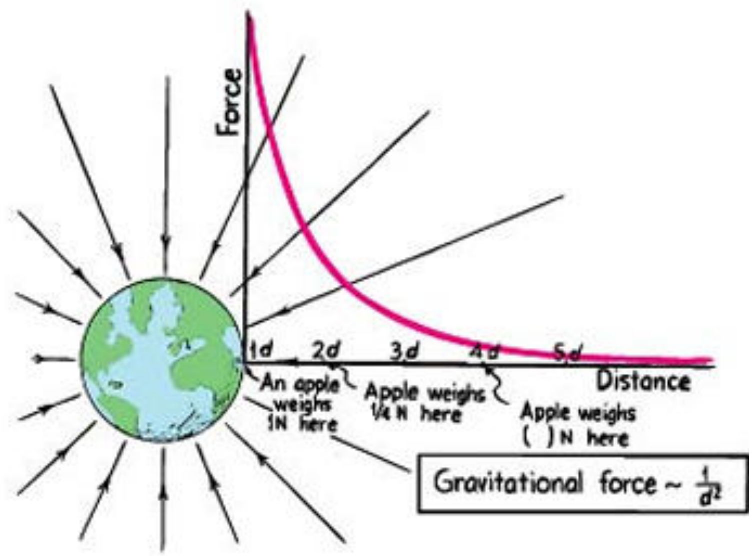
$$F = G\frac{m_1 m_2}{d^2}$$

The gravitational force grows weaker as the square of the distance. Inverse square law.



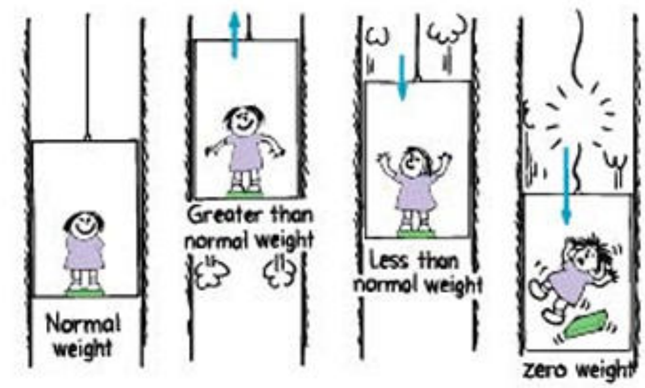
# Inverse square law

- The further away from the center of the Earth that you travel, the smaller your weight will be
- If you weigh 600 N at sea level,  you'll weigh 598 N on top of Mt Everest
- The gravitational force falls off to zero as the distance from the center of the Earth goes to infinity

# Weightlessness

- Suppose that you're in an elevator that's stationary (or moving with a constant velocity)
  - you stand on a scale and it gives your normal weight
- Suppose the elevator accelerates upward
  - the scale registers a larger weight; you feel heavier
- The elevator accelerates downward
  - the scale registers a lower weight; you feel lighter
- The elevator cable snaps
  - the scale registers zero weight; you feel weightless

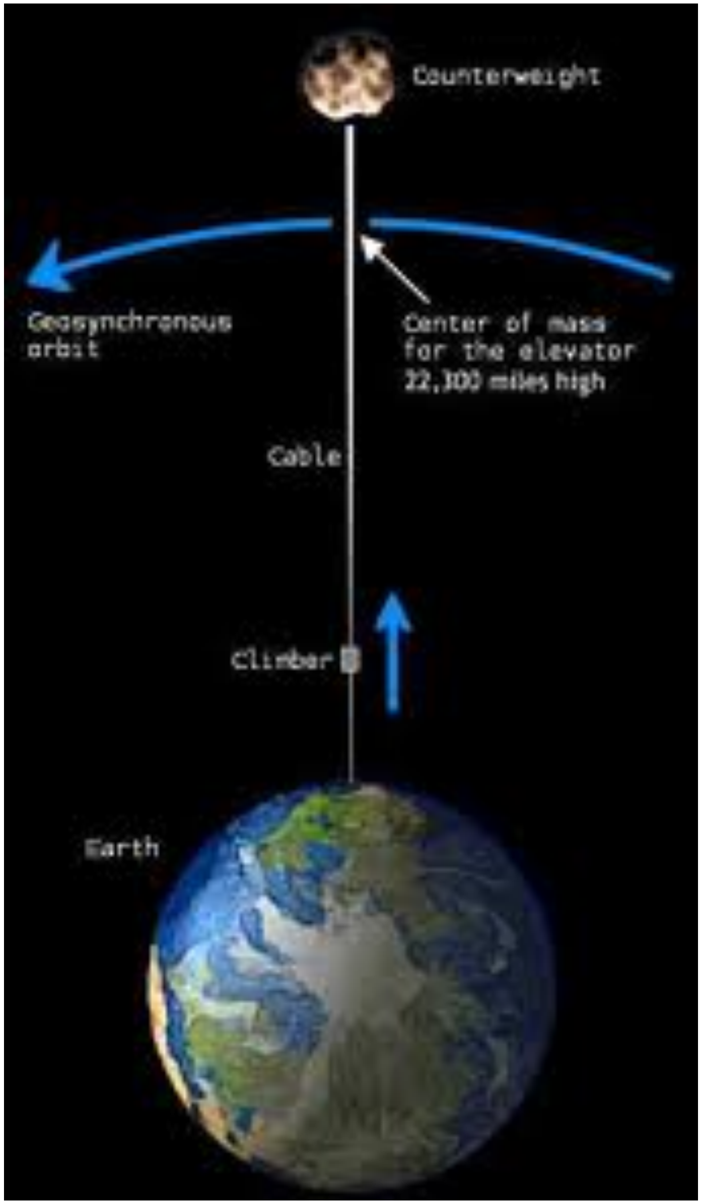


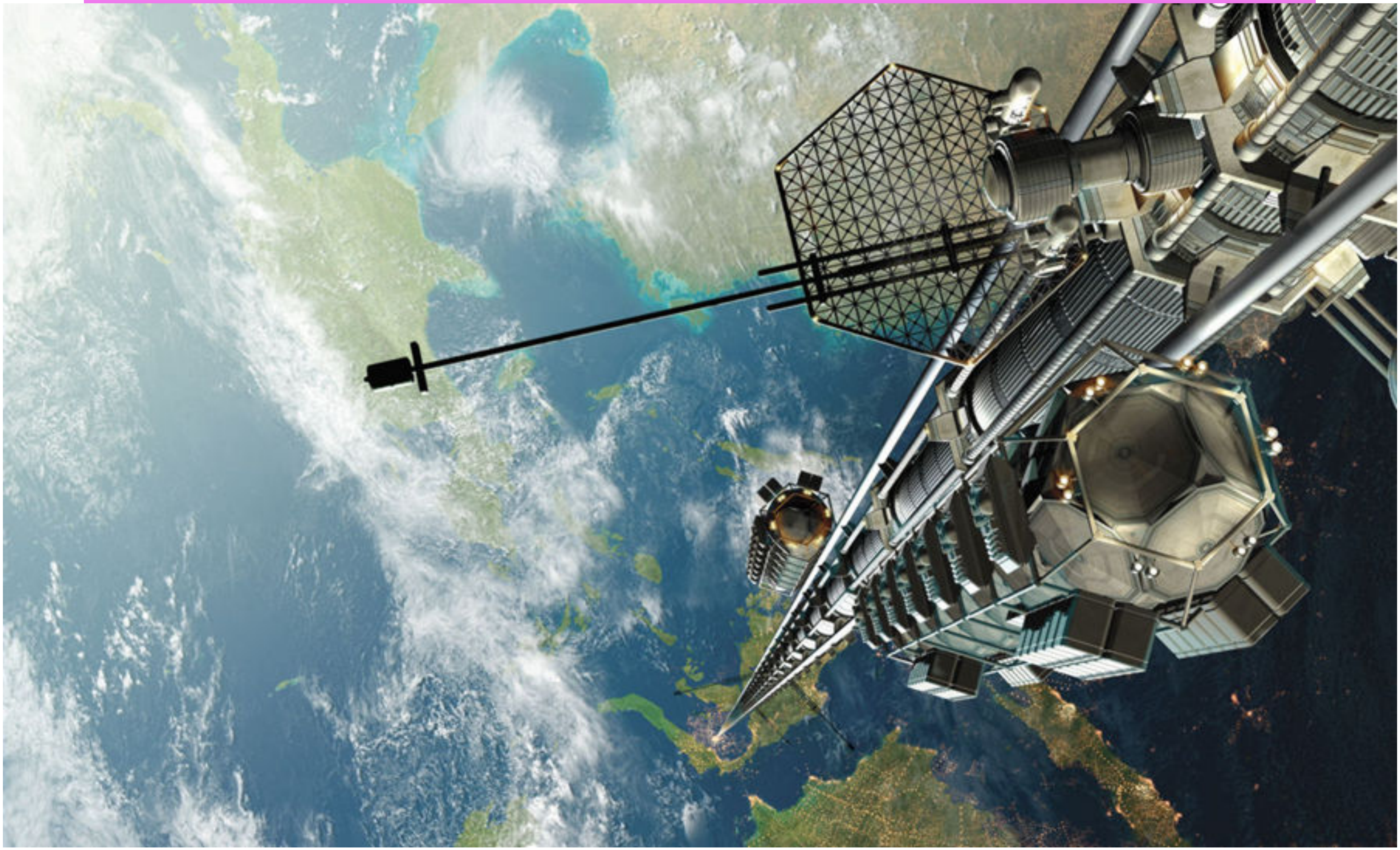

# Weightlessness

- If you have enough money, you book a flight on a ‘vomit comet’ that lets you experience weightlessness for a brief period of time
- Or you can spend some time on the International Space Station
  - if you can get there


# Space elevator

- One of the problems refers to a space elevator
- This would be a convenient way of transporting material into a geosynchronous orbit without the use of any rockets
- One end of the elevator is attached to the surface of the Earth, the other to a counterweight
- It's stable
- But we don't currently have the technology to build a strong enough tower
- Carbon nanotubes are close; they would work for an elevator on the Moon or on Mars





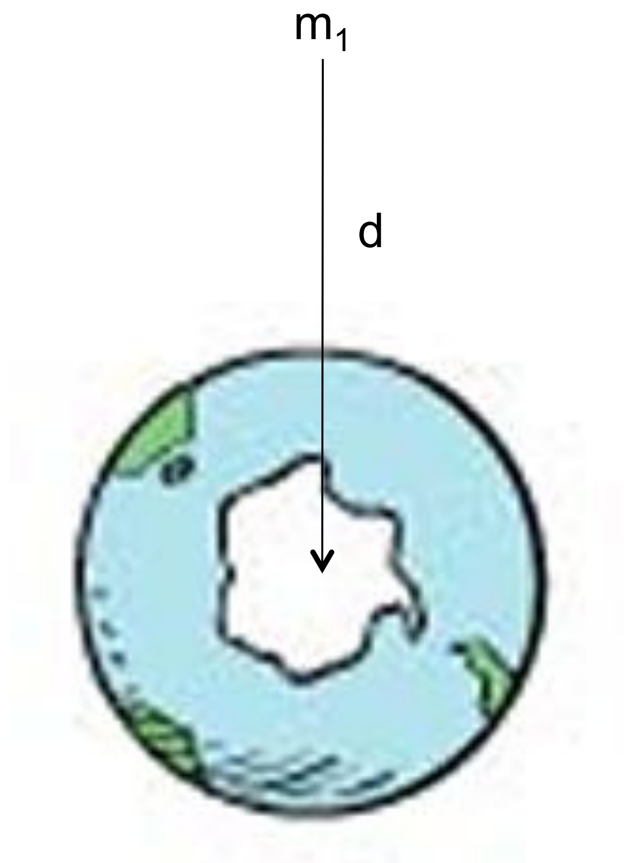
http://www.pbs.org/wgbh/nova/space/space-elevator.html

# Central force

- Suppose I have a mass $m_1$ and I want to calculate the gravitational force on this mass from the Earth
- What is the direction of the force on the mass?
  - towards the center of the Earth
- What is the direction of the force on the Earth?
  - along the line joining $m_1$ and the center of the Earth, pointing towards $m_1$
- What is the magnitude of the force?

$$F = G\frac{m_1 m_2}{d^2}$$

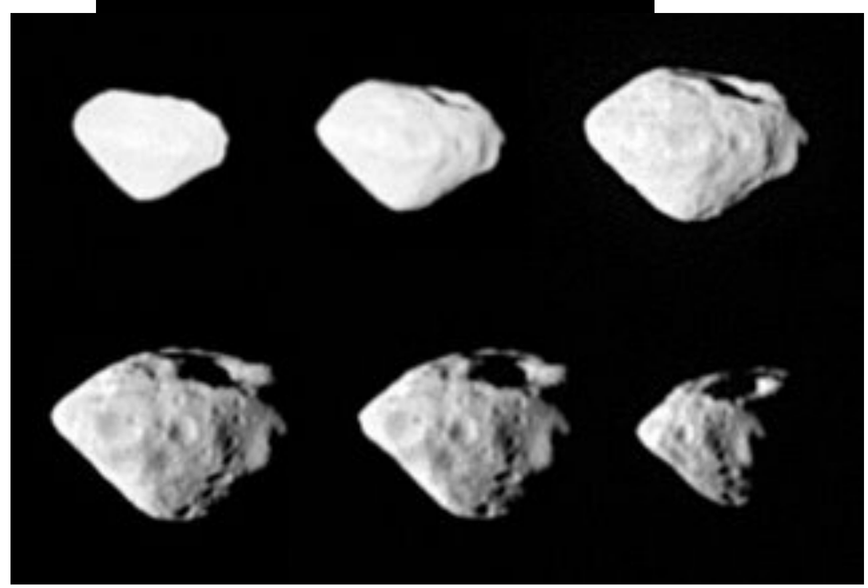
- The Earth has a radius of $6.37 \times 10^6$ m, but from a gravitational point of view, it acts like all of the mass is concentrated in a point at the center of the Earth




How do we know that? Isaac Newton had to develop calculus to prove it.

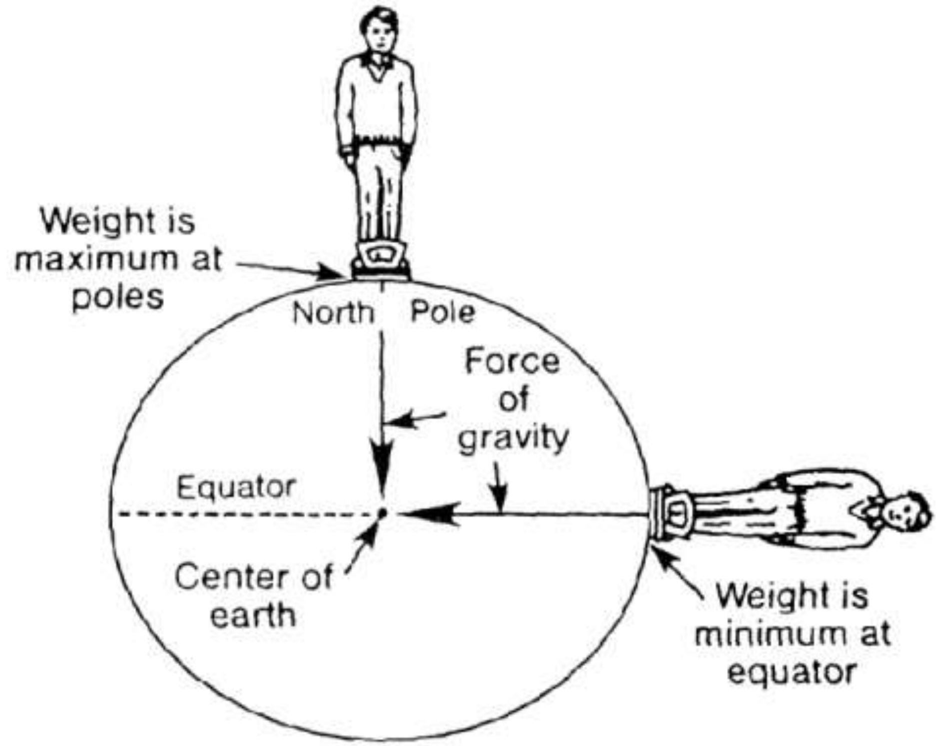
# Roundness

- Why is the Earth round?
- Because every part of the Earth attracts every other part and so the Earth is pulled together as tightly as possible
  - a sphere
- The Moon is round for a similar reason
- The asteroids, except for the largest ones, are not round, because they don’t have sufficient mass



# Oblateness of the Earth

- We discussed that gravity is responsible for the planets being round
- But they're not exactly round due to the rotational motion of the Earth
- The Earth tends to be somewhat thicker at the equator than at the poles
- Not by much
  - 12756 km at equator
  - 12714 km at the poles
- So a round sphere is still a very good representation of the shape of the Earth




How different is the force of gravity at the North Pole and at the Equator? Take a mass of 50 kg.

$$F = G\frac{m_1 m_2}{d^2}$$

# Oblateness of the Earth

- The Earth tends to be somewhat thicker at the equator than at the poles
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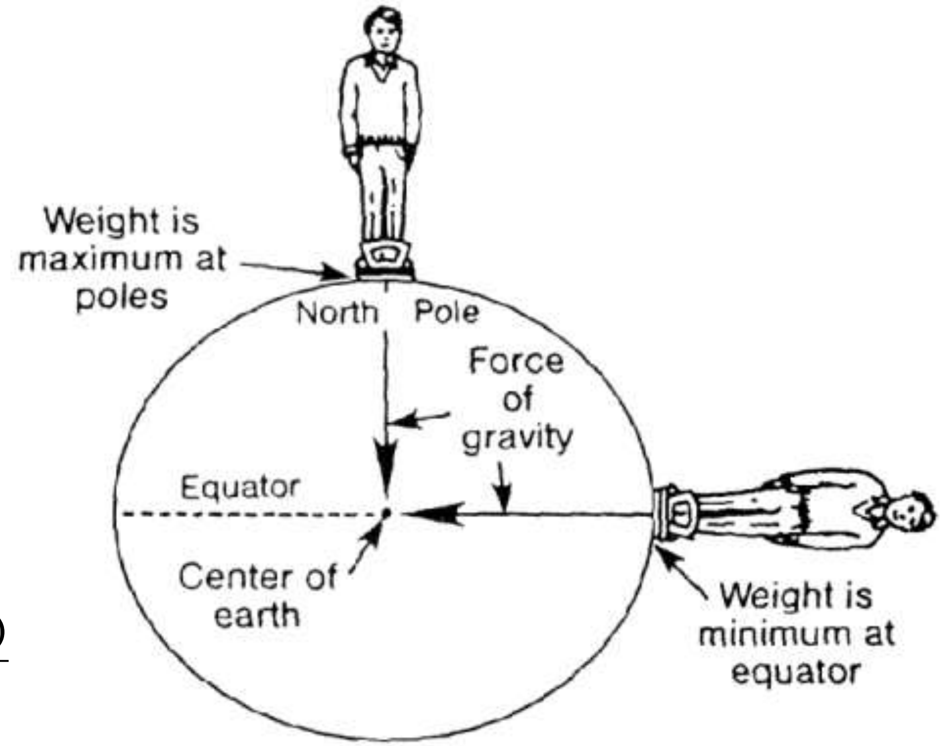



$$F_{equator} = W_{equator} = G\frac{mm_{Earth}}{d^2}$$

$$F_{equator} = (6.67X10^{-11}Nm^2/kg^2)\frac{(50kg)(6X10^{24}kg)}{(6.378X10^6m)^2}$$

$$F_{equator} = 491.9N$$

$$F_{pole} = (6.67X10^{-11}Nm^2/kg^2)\frac{(50kg)(6X10^{24}kg)}{(6.357X10^6m)^2}$$

$$F_{pole} = 495.2N$$

How different is the force of gravity at the North Pole and at the Equator? Take a mass of 50 kg.

How much did the mass change?

$$F = G\frac{m_1 m_2}{d^2}$$

# Jupiter

- Jupiter is even more oblate since
  - it's much larger than the Earth
  - it rotates much faster (one day = 10 hours)
  - it's composed mostly of fluid
- How much would this 50 kg person weigh on the equator of the surface of Jupiter?
- Jupiter is 300 times as massive as the Earth
- Why isn't the weight 300 times as much?

$$F = W = G\frac{mm_{Jupiter}}{d^2}$$

$$F = (6.67X10^{-11}Nm^2/kg^2)\frac{(50kg)(1.90X10^{27}kg)}{(7.13X10^7m)^2}$$

$$F = 1246N$$

# Neutron star

- A neutron star has a mass of 4 X $10^{30}$ kg (about twice the sun's mass) and a radius of 10 km (about 1/70000th that of the sun)
- What would be the weight of a 50 kg person on the surface of this neutron star?

$$F = G\frac{mm_{neutronstar}}{d^2}$$

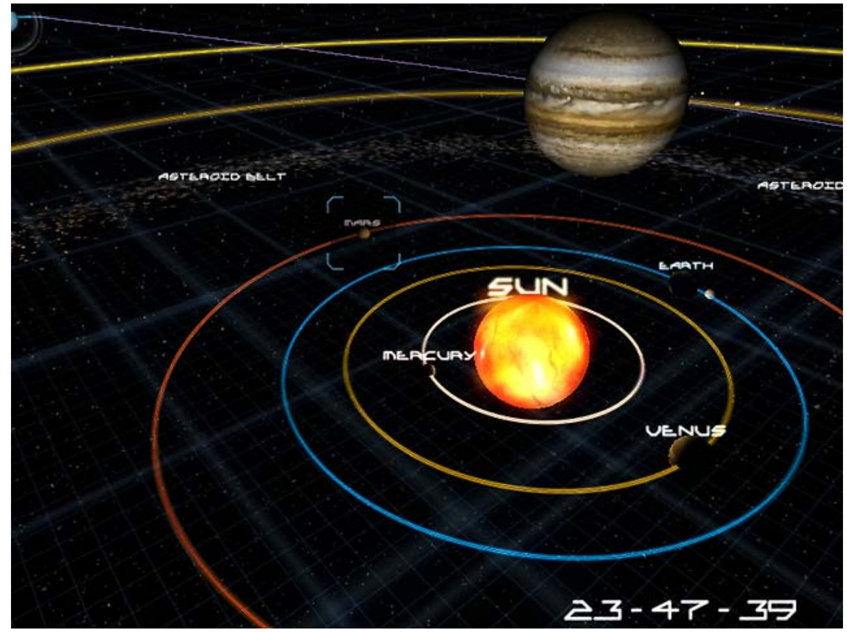
$$F = (6.67X10^{-11} Nm^2/kg^2)\frac{(50kg)(4X10^{30}kg)}{(1X10^4 m)^2}$$

$$F = 1.33X10^{14} N$$

- Or about 300 billion times as much as on the surface of the Earth
- A neutron star has the same density as the nucleus of an atom

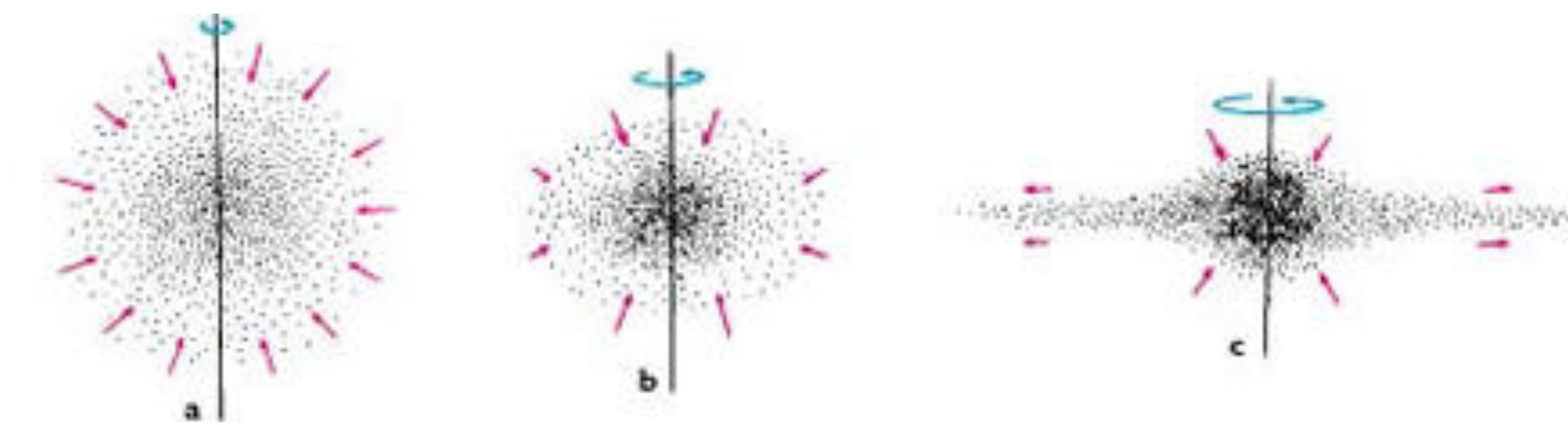
# Gravity

- What's responsible for keeping the solar system together?
  - gravity
- What would happen if the sun were replaced by a black hole of the same mass?
  - nothing, except that it would get dark
  - the gravitational force of the sun already acts as if its originating from a point in the center of the sun
- What's responsible for keeping the Milky Way together?
  - gravity, except there's not enough visible matter
  - most of the universe appears to be composed of *dark matter*




# Origin of the solar system

- A slightly rotating ball of interstellar gas contracts due to gravitational attraction and speeds up to conserve angular momentum
- The increased momentum causes them to sweep in wider paths around the rotational access, producing an overall disk shape
- The planets condense out of eddies in the cooling disk

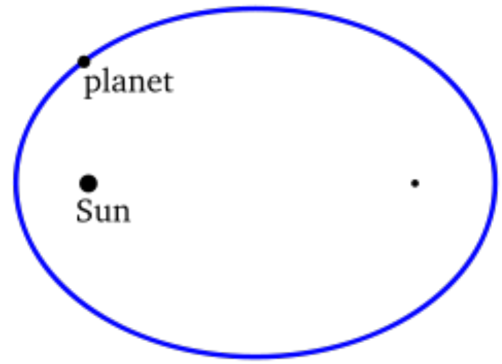


# Johannes Kepler

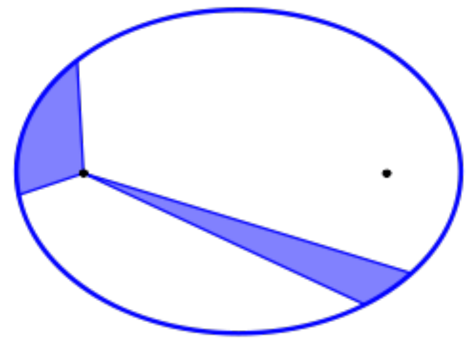
- 1571-1630
- He took the detailed observations of Tycho Brahe on the motions of the planets and was able to formulate 3 laws that describe the motions of the planets

# Kepler's 3 laws

- Every planet has an elliptical orbit with the Sun at one focus of the ellipse




- A line joining the planet and the Sun sweeps out equal areas in equal times
  - so the planet must move fastest when it's closest to the Sun



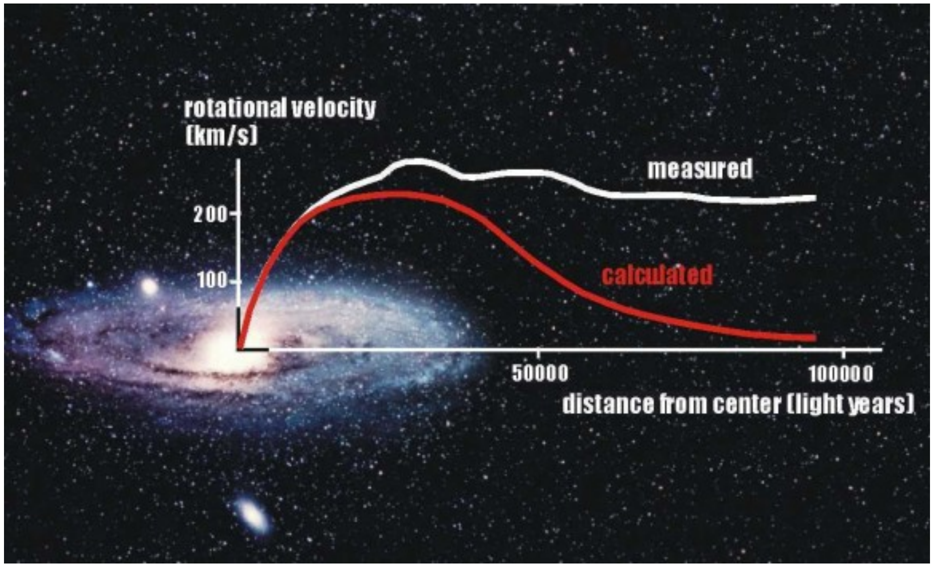
- The square of the period of the orbit of a planet is proportional to the cube of the radius (semi-major axis)

$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM_{sun}}$$

Empirical observations from Kepler; can be derived using Newton's law of gravitation

# Rotation of the galaxy and dark matter

- Remember earlier I said that you would expect stars further from the center of the galaxy to rotate more slowly than ones at the center
- The fact that this does not happen is evidence of the presence of dark matter around our galaxy (10X as much dark matter as regular matter)




$$\left(\frac{P}{2\pi}\right)^2 = \frac{a^3}{GM_{sun}}$$

Empirical observations from Kepler; can be derived using Newton's law of gravitation

# Planets

Table of the largest objects in the Solar System – Wikipedia, the free encyclopedia

http://en.wikipedia.org/wiki/Attributes_of_the_largest_solar_system_bodies

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## Planets

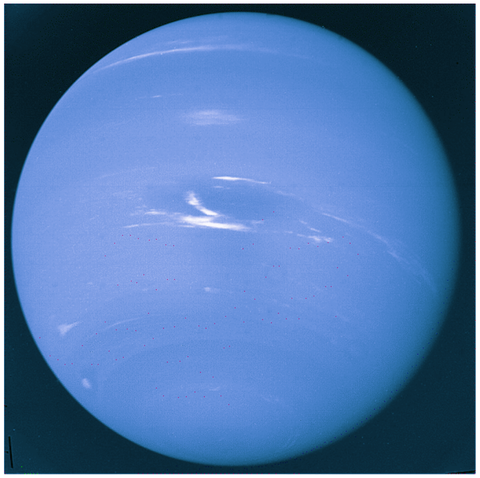
| | | Mercury | Venus | Earth | Mars | Jupiter | Saturn | Uranus | Neptune |
|---|---|---|---|---|---|---|---|---|---|
| Astronomical symbol | | ☿ | ♀ | ⊕ | ♂ | ♃ | ♄ | ♅ | ♆ |
| Mean distance from Sun | km | 57,909,175 | 108,208,930 | 149,597,870 | 227,936,640 | 778,412,010 | 1,426,725,400 | 2,870,972,200 | 4,498,252,900 |
| | AU | 0.38709893 | 0.72333199 | 1 | 1.52366231 | 5.20336301 | 9.53707032 | 19.19126393 | 30.06896348 |
| Mean radius | km | 2,439.64 | 6,051.59 | 6,378.15 | 3,397.00 | 71,492.68 | 60,267.14 | 25,557.25 | 24,766.36 |
| | :E[1] | 0.3825 | 0.9488 | 1 | 0.53226 | 11.209 | 9.449 | 4.007 | 3.883 |
| Surface area | km² | 75,000,000 | 460,000,000 | 510,000,000 | 140,000,000 | 64,000,000,000 | 44,000,000,000 | 8,100,000,000 | 7,700,000,000 |
| | :E[1] | 0.1471 | 0.9010 | 1 | 0.2745 | 125.5 | 86.27 | 15.88 | 15.10 |
| Volume | $km^3$ | $6.083\times10^{10}$ | $9.28\times10^{11}$ | $1.083\times10^{12}$ | $1.6318\times10^{11}$ | $1.431\times10^{15}$ | $8.27\times10^{14}$ | $6.834\times10^{13}$ | $6.254\times10^{13}$ |
| | :E[1] | 0.056 | 0.87 | 1 | 0.151 | 1,321.3 | 763.59 | 63.086 | 57.74 |
| Mass | kg | $3.302\times10^{23}$ | $4.8690\times10^{24}$ | $5.9742\times10^{24}$ | $6.4191\times10^{23}$ | $1.8987\times10^{27}$ | $5.6851\times10^{26}$ | $8.6849\times10^{25}$ | $1.0244\times10^{26}$ |
| | :E[1] | 0.055 | 0.815 | 1 | 0.107 | 318 | 95 | 14 | 17 |
| Density | $g/cm^3$ | 5.43 | 5.24 | 5.515 | 3.940 | 1.33 | 0.70 | 1.30 | 1.76 |
| Equatorial gravity | $m/s^2$ | 3.70 | 8.87 | 9.81 | 3.71 | 23.12 | 8.96 | 8.69 | 11.00 |
| Escape velocity | km/s | 4.25 | 10.36 | 11.18 | 5.02 | 59.54 | 35.49 | 21.29 | 23.71 |
| Rotation period | days[2] | 58.646225 | -243.0187[3] | 0.99726968 | 1.02595675 | 0.41354 | 0.44401 | -0.71833[3] | 0.67125 |
| Orbital period | years[2] | 0.2408467 | 0.61519726 | 1.0000174 | 1.8808476 | 11.862615 | 29.447498 | 84.016846 | 164.79132 |
| Mean orbital speed | km/s | 47.8725 | 35.0214 | 29.7859 | 24.1309 | 13.0697 | 9.6724 | 6.8352 | 5.4778 |
| Eccentricity | | 0.20563069 | 0.00677323 | 0.01671022 | 0.09341233 | 0.04839266 | 0.05415060 | 0.04716771 | 0.00858587 |
| Inclination | deg. | 7.00487 | 3.39471 | 0.00005 | 1.85061 | 1.30530 | 2.48446 | 0.76986 | 1.76917 |
| Axial tilt[4] | deg. | 0.0 | 177.3 | 23.45 | 25.19 | 3.12 | 26.73 | 97.86 | 29.58 |
| Mean surface temp. | K | 440 | 730 | 288-293 | 186-268 | 152 | 134 [5] | 76 [5] | 72 [5] |
| Mean air temp.[6] | K | | | 288 | | 165 | 135 | 76 | 73 |
| Atmospheric composition | | He $Na^+$ $P^+$ | $CO_2$ $N_2$ | $N_2$ $O_2$ | $CO_2$ $N_2$ Ar | $H_2$ He | $H_2$ He | $H_2$ He $CH_4$ | $H_2$ He $CH_4$ |
| Number of known moons | | 0 | 0 | 1 | 2 | 63 | 60 | 27 | 13 |
| Rings? | | No | No | No | No | Yes | Yes | Yes | Yes |
| Planetary discriminant[7] | | $9.1\times10^4$ | $1.35\times10^6$ | $1.7\times10^6$ | $1.8\times10^5$ | $6.25\times10^5$ | $1.9\times10^5$ | $2.9\times10^4$ | $2.4\times10^4$ |

## Dwarf planets

| | Ceres | Pluto | Makemake | Eris |
|---|---|---|---|---|

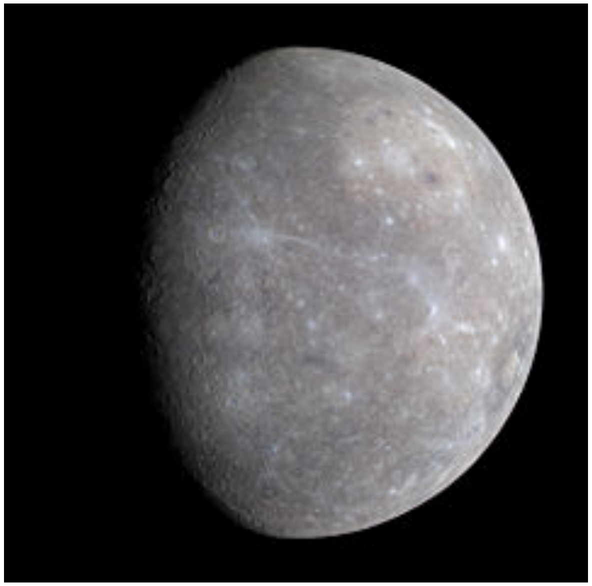
# Uranus

- First planet discovered in modern times
  - not visible to naked eye
- The largest gravitational force in the solar system is due to the Sun (most of the mass)
- But the other planets in the Solar System tug on each other and cause the planets to wobble in their orbits
- If you calculate the effects of all of the other planets on Neptune's wobble, it's not enough
- Either the universal law of gravitation doesn't work at these large distances or there's an $8^{th}$ planet
- There is an $8^{th}$ planet (Neptune) and it was where they calculated it should be



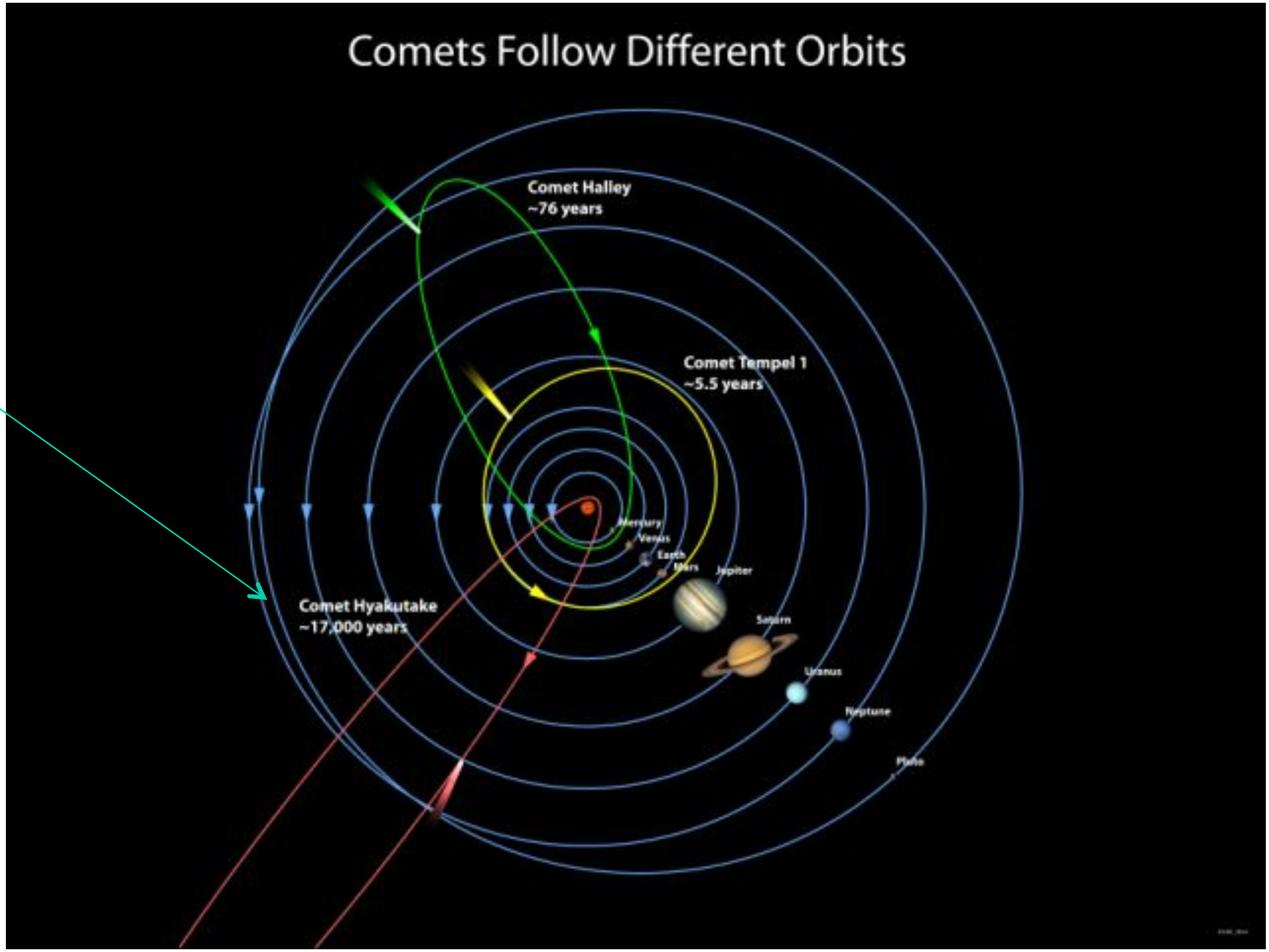
# Mercury

- Closest planet to the sun
- Tugs on Mercury perturb orbit
- Not covered by Newtonian physics
- Another planet closer to the sun?
  - Vulcan, Spock's planet
- …or effects of general relativity



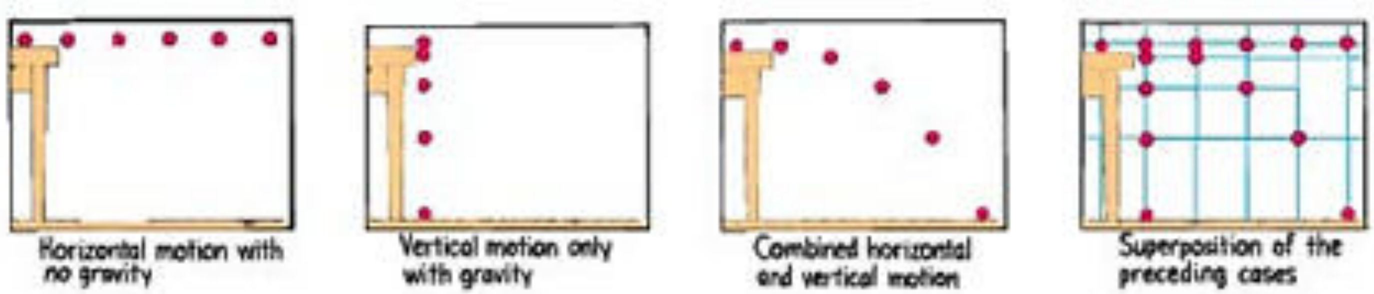
# Orbits

From 1979
to 199
Pluto was
closer to
the Sun
than Neptune



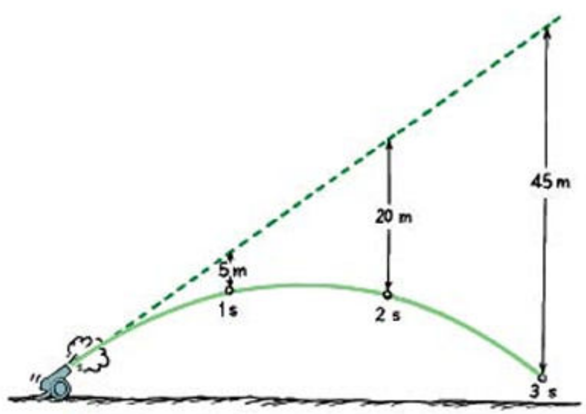
# Vertical and horizontal motions

- When both vertical and horizontal motions are present, they can be treated completely independently
- For example, below is shown a ball rolling off of a table with a constant horizontal velocity
- The constant horizontal velocity continues (ignoring any air resistance) while there is a vertical acceleration due to gravity

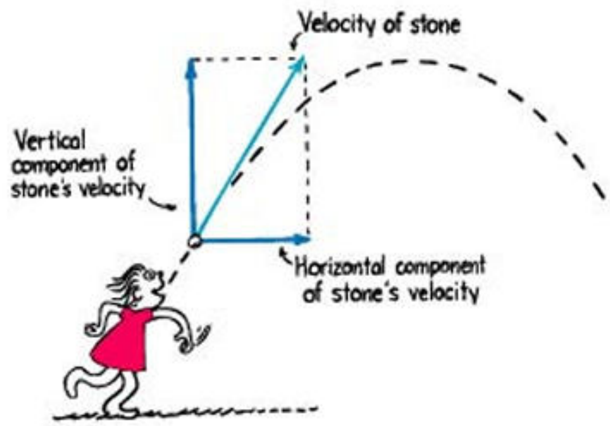


# Projectile motion

- With no gravity, the cannon ball would follow a straight line
- Because of the acceleration due to gravity, it follows a parabolic path




- If I throw a stone, it will also follow a parabolic path




simulation

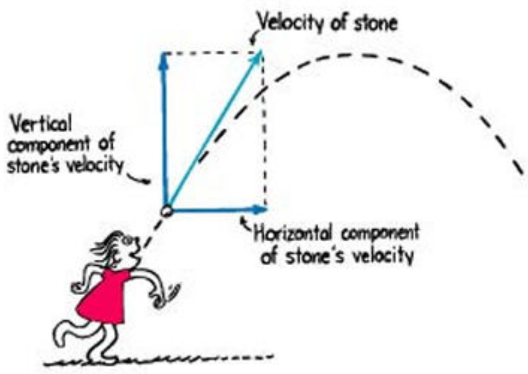
# Projectile Motion

- So we have motion in both the x and y directions
- And the two motions are independent so we can write down two separate equations for the x and y motions

$$x = x_0 + v_0^x t + \frac{1}{2} a_x t^2$$

$$y = y_0 + v_0^y t + \frac{1}{2} a_y t^2$$

- I can simplify somewhat since there is no acceleration in the x direction and I can write the acceleration in the y direction as -g

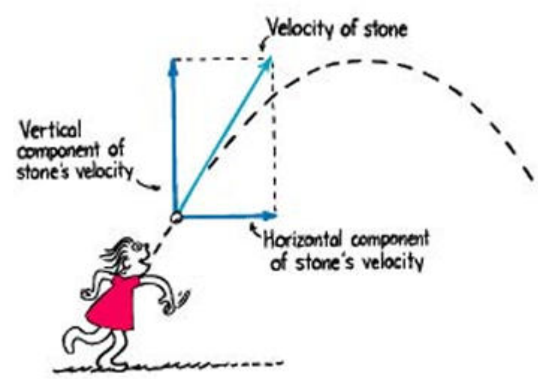



$$x = x_0 + v_0^x t$$

$$y = y_0 + v_0^y t - \frac{1}{2} g t^2$$

# Projectile Motion

- Let's start simple
- I throw the ball horizontally with a speed of 20 m/s
- How long before it hits the ground?
- How far has it travelled?




$$x = x_0 + v_0^x t$$

$$y = y_0 + v_0^y t - \frac{1}{2} g t^2$$

# Projectile Motion

- Assume that I release it 2 m from the ground
- $y_o$=2m, $v_o^y$=0 m/s


$$y = y_o - \frac{1}{2}gt^2$$

$$0 = 2m - \frac{1}{2}(9.83m/s^2)t^2$$

$$t^2 = \frac{4m}{9.83m/s^2} = 0.407s^2$$

$$t = 0.64s$$

$$x = x_o + 20m/s(0.64s) = x_o + 12.8m$$

$$x = x_0 + v_0^x t$$

$$y = y_0 + v_0^y t - \frac{1}{2}gt^2$$

# Projectile Motion

- Suppose I throw it at 20 m/s at an angle of 45°
- Let's again start with the vertical motion
  - how long before it hits the ground?

$$0 = 2m + (20m/s)\sin 45^o t - \frac{1}{2}(9.83m/s^2)t^2$$

$$0 = 2m + (20m/s)(0.707)t - (4.915m/s^2)t^2$$

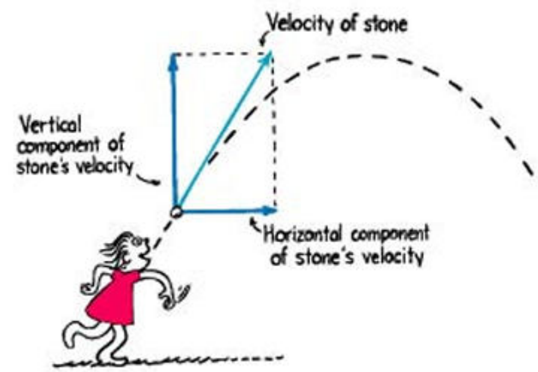
$$4.915t^2 - 14.14t - 2 = 0$$

$$t = 3.01s$$

- Now the horizontal motion

$$x = x_0 + (20m/s)\cos 45^o t$$

$$x = x_0 + (20m/s)(0.707)(3.01s) = x_0 + 42.6m$$

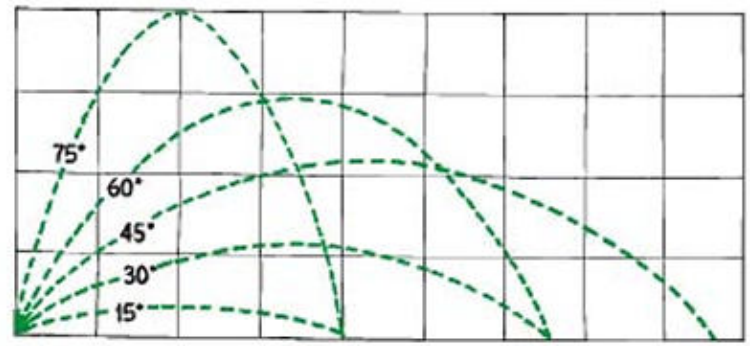


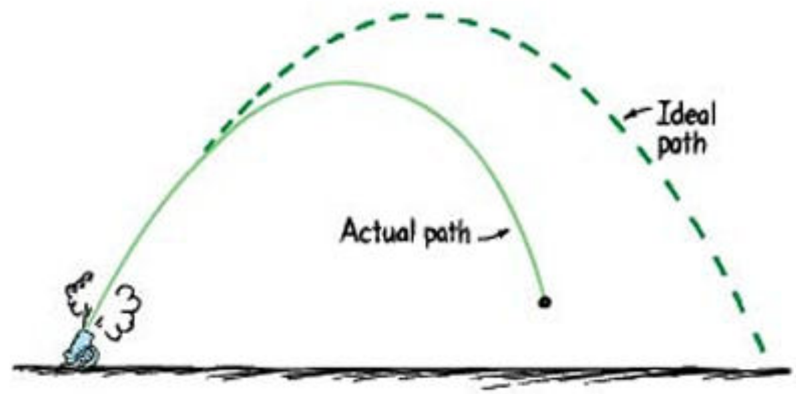

$$x = x_0 + v_0^x t$$

$$y = y_0 + v_0^y t - \frac{1}{2}gt^2$$

Projectile motion
Shoot the monkey

- What angle should you throw a ball in order for it to go the maximum distance, given that the initial release velocity is the same?
  - somewhere between 0 and 90 degrees
  - to be more precise 45 degrees
- What is the impact of air resistance?

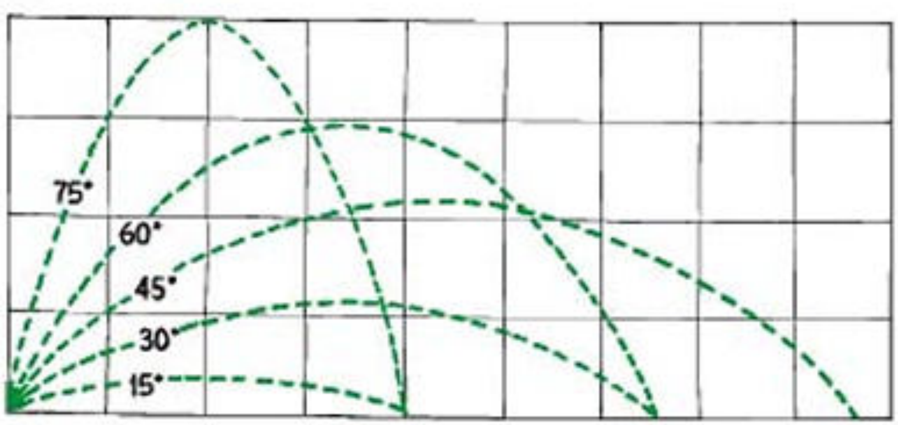


# Clicker question

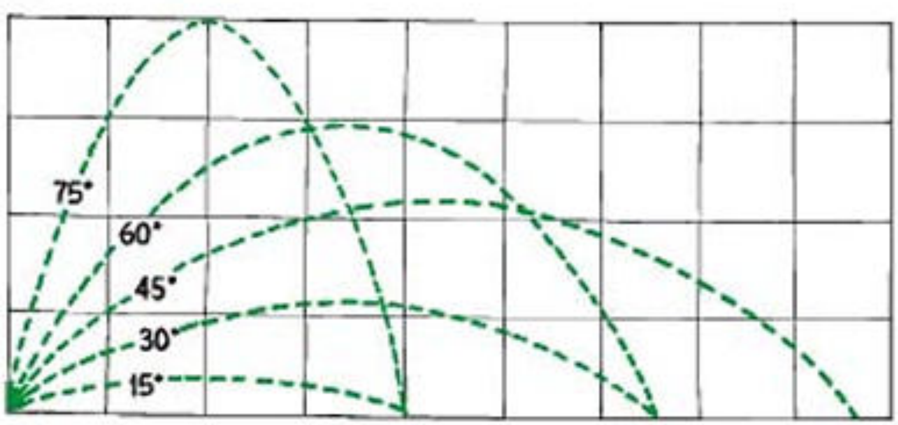
- For the possible paths for the projectile below, which has the largest vertical acceleration?

- A) 75°
- B) 60°
- C) 45°
- D) 30°
- E) they're all the same



# Clicker question

- For the possible paths for the projectile below, which has the largest vertical acceleration?




- A) 75°
- B) 60°
- C) 45°
- D) 30°
- E) **they're all the same**