FIRST STEPS TOWARDS A DUALITY RELATION AT TWO LOOPS

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Abstract
We illustrate a duality relation between one–loop integrals and single-cut phase–space integrals. This duality relation is realized by a Lorentz covariant modification of the customary $+i0$ prescription of the Feynman propagators and can be extended to generic one–loop quantities, such as Green's functions, in any relativistic, local and unitary field theories. Additionally, we comment on first steps towards a two–loop duality relation.

1. INTRODUCTION

The duality method provides a method to numerically compute multi–leg one–loop cross sections in perturbative field theories by defining a relation between one–loop integrals and single phase–space integrals [1–3]. This is done by properly regularizing propagators by a complex Lorentz–covariant prescription, which is different from the customary $+i0$ prescription of the Feynman propagators. The duality method is valid for massless as well as for real and virtual massive propagators and can straightforwardly be applied not only for the evaluation of basic one–loop integrals but also for complete one–loop quantities such as Green's functions and scattering amplitudes [3]. An extension to two–loop order is more involved and needs the treatment of occurring dependences on one of the two integration momenta in the modified $+i0$ description, which would lead to branch cuts in the complex energy plane. This extension is currently under investigation.

One motivation for deriving the duality relation is given by the fact that the computation of cross sections at next-to-leading order (NLO) requires the separate evaluation of real and virtual radiative corrections. Real (virtual) radiative corrections are given by multi–leg tree–level (one–loop) matrix elements to be integrated over the multi–particle phase space of the physical process. The loop–tree duality discussed here, as well as other methods that relate one–loop and phase–space integrals, have the attractive feature that they recast the virtual radiative corrections in a form that closely parallels the contribution of the real radiative corrections [1, 4–7]. This close correspondence can help to directly combine real and virtual contributions to NLO cross sections. In particular, using the duality relation, one can apply mixed analytical and numerical techniques to the evaluation of the one–loop virtual contributions [1]. The infrared or ultraviolet divergent part of the corresponding dual integrals can be analytically evaluated in dimensional regularization. The finite part of the dual integrals can be computed numerically, together with the finite part of the real emission contribution. Partial results along these lines are presented in Refs. [1, 2] and further work is in progress.

2. THE DUALITY RELATION AT ONE–LOOP ORDER

Consider a generic one–loop integral over Feynman propagators, where $q_i = q + \sum_{k=1}^{i} p_k$ are the momenta of the internal lines, q being the loop momentum, and p_i ($\sum_{i=1}^{N} p_i = 0$) the external (outgoing and clockwise ordered) momenta. The Feynman propagators have two poles in the complex plane of the loop energy q_0, the pole with positive (negative) energy being slightly displaced below (above) the real axis encoded by the additional $+i0$ term in the propagator. Using the Cauchy residue theorem in the complex q_0–plane, with the integration contour closed at ∞ in the lower half–plane, we obtain a sum over terms given by the integral evaluated at the poles with positive energy only. Hence a one–loop integral with N internal propagators leads to N contributions, one for each propagator for which the residue is taken. It can be shown that this residue is equivalent to cutting that line by including the corresponding on–shell propagator $\delta_+(q_i^2) = \theta(q_i^0)\delta(q_i^2)$. The remaining propagators of the expression are shifted to

$$\prod_{j\neq i} \frac{1}{q_j^2 + i0}\bigg|_{q_i^2=-i0} = \prod_{j\neq i} \frac{1}{q_j^2 - i0\,\eta(q_j - q_i)}\;, \tag{1}$$

where η is a future-like vector, i.e. a d-dimensional vector that can be either light-like $(\eta^2 = 0)$ or time-like $(\eta^2 > 0)$ with positive definite energy $(\eta_0 \geq 0)$. The calculation of the residue at the pole of the i^{th} internal line modifies the $i0$ prescription of the propagators of the other internal lines of the loop. This modified regularization is named 'dual' $i0$ prescription, and the corresponding propagators are named 'dual' propagators. The dual prescription arises, because the original Feynman propagator $1/(q_j^2 + i0)$ is evaluated at the *complex* value of the loop momentum q, which is determined by the location of the pole at $q_i^2 + i0 = 0$. The presence of η is a consequence of the fact that the residue at each of the poles is not a Lorentz–invariant quantity, since a given system of coordinates has to be specified to apply the residue theorem. Different choices of the future-like vector η are equivalent to different choices of the coordinate system. The Lorentz–invariance of the loop integral is, however, recovered after summing over all the residues. For a one–loop integral, the term $\eta(q_j - q_i)$ is always solely proportional to external momenta and hence defines a fixed pole in the q_0–plane.

Note that an extension to real and virtual massive propagators and full scattering amplitudes is straightforward and described in detail in Ref. [3].

3. FIRST STEPS TOWARDS TWO–LOOP ORDER

The fact that the term $\eta(q_j - q_i)$ is proportional to external momenta only, is not valid anymore once going to the next loop order and considering a generic two–loop n–leg diagram. Taking the residues loop by loop for the two integration momenta introduces in some cases a dependence on one of the integration momenta in the difference of $\eta(q_j - q_i)$. Hence we encounter not poles but rather branch cuts in the complex energy plane. To avoid this and more generally to avoid any dependence on integration momenta in the $\eta(q_j - q_i)$–terms demands a reformulation of the propagators into another basis, which fulfills the required properties. First steps towards a two–loop expression obtained by such a transformation have been undertaken, while the full general two–loop expression is still under investigation.

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