NLO QCD corrections to $pp \rightarrow t\bar{t}b\bar{b}$ at the LHC

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based on
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Physics at TeV colliders
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Outline of the talk

(1) **Introduction** - NLO corrections to multi-leg processes, $t\bar{t}b\bar{b}$ production

(2) **Virtual corrections** - Feynman diagrams, tensor reduction, rational terms

(3) **Real corrections** - Dipole subtraction

(4) **Numerical results** - LHC cross section, CPU performance
(1) Introduction

Importance of **multi-leg processes** at the LHC

- huge W, Z and top-quark production rates + multiple jet emission
- multi-particle signatures with leptons and missing E
- serious backgrounds to Higgs and new-physics signals (often not fully accessible to measurements)

Importance of **NLO QCD corrections** at the LHC

- reduce scale uncertainties (high powers of $\alpha_s$!); improve description of jets
- systematics better than Tevatron; very high statistics

**Technical problems** for $2 \to 3, 4, \ldots$ processes

- numerical instability of virtual corrections (Gram determinants)
- number and complexity of diagrams grow very fast

**Challenges** for NLO programs

- reliable predictions: **numerical stability**
- **sufficient speed**: distributions require $> 1$ event/sec!
Feynman diagrams have provided hadronic NLO cross sections for several nontrivial $2 \rightarrow 3$ processes

- $pp \rightarrow t\bar{t}H, b\bar{b}H$  
  Beenakker/Dittmaier/Krämer/Plümpfer/Spira/Zerwas;  
  Dawson/Reina/Wackeroth/Orr/Jackson;  
  Peng/Wen-Gan/Hong-Shen/Ren-You/Yi  

- $pp \rightarrow HHH$  
  Plehn/Rauch;  
  Binoth/Karg/Kauer/Rückl  

- $pp \rightarrow Hjj$  
  Del Duca/Kilgore/Oleari/Schmidt/Zeppenfeld;  
  Campbell/Ellis/Zanderighi  
  Ciccolini/Denner/Dittmaier  

- $pp \rightarrow jjj$  
  Bern/Dixon/Kosower;  
  Kunszt/Signer/Trocsanyi;  
  Giele/Kilgore/Nagy  

- $pp \rightarrow Vjj$  
  Bern/Dixon/Kosower;  
  Ellis/Veseli;  
  Campbell/Ellis;  

- $pp \rightarrow VVj$  
  Dittmaier/Kallweit/Uwer;  
  Campbell/Ellis/Zanderighi;  

- $pp \rightarrow VVV$  
  Lazopoulos/Melnikov/Petriello;  
  Hankele/Zeppenfeld;  
  Binoth/Ossola/Papadopoulos/Pittau  

- $pp \rightarrow V + b\bar{b}$  
  Ferbes Cordero/Reina/Wackeroth  

- $pp \rightarrow t\bar{t}j$  
  Dittmaier/Uwer/Weinzierl  

- $pp \rightarrow t\bar{t}Z$  
  Lazopoulos/McElmurry/Melnikov/Petriello

*NLO calculations related to $pp \rightarrow t\bar{t}H$ (like $pp \rightarrow t\bar{t}b\bar{b}$)*
... and a few pioneering $2 \rightarrow 4$ results for $e^+e^-$ and $\gamma\gamma$ colliders

- $e^+e^- \rightarrow 4f$ (EW)  Denner/Dittmaier/Roth/Wieders '05
- $e^+e^- \rightarrow HH\nu\bar{\nu}$ (EW)  Boudjema/Fujimoto/Ishikawa/Kaneko/Kurihara/Shimizu/Kato/Yasui '05
- $\gamma\gamma \rightarrow t\bar{t}b\bar{b}$ (QCD)  Lei/Wen-Gan/Liang/Ren-You/Yi '07

Questions

- Can one repeat this success at hadron colliders?
- Should one switch to alternative methods like unitarity cuts?
### Les Houches ’05/’07 prioritized wishlist

<table>
<thead>
<tr>
<th>Reaction</th>
<th>background for</th>
<th>existing NLO cross sections</th>
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<tr>
<td>1. $V V j$</td>
<td>$t\bar{t}H$, new physics</td>
<td>$WW j$: Dittmaier/Kallweit/Uwer ’07; $WW j$: Campbell/Ellis/Zanderighi ’07; $WW j/ZZ j$: Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)</td>
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<tr>
<td>2. $t\bar{t}b\bar{b}$</td>
<td>$t\bar{t}H$</td>
<td><strong>complete NLO:</strong> Bredenstein/Denner/Dittmaier/P. ’08-’09</td>
</tr>
<tr>
<td>3. $t\bar{t}jj$</td>
<td>$t\bar{t}H$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>4. $VVb\bar{b}$</td>
<td>$VBF \rightarrow H \rightarrow VV$, $t\bar{t}$, NP</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>5. $VVjj$</td>
<td>$VBF \rightarrow H \rightarrow VV$</td>
<td><strong>VBF:</strong> Jäger/Oleari/Zeppenfeld ’06 + Bozzi ’07</td>
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<td>6. $V jjj$</td>
<td>new physics</td>
<td><strong>leading colour approximation:</strong> Ellis/Giele/Kunszt/Melnikov/Zanderighi ’08; Ellis/Melnikov/Zanderighi ’09; Berger/Bern/Dixon/Ferbes Cordero/Forde/Gleisberg/Ita/Kosower/Maitre ’09</td>
</tr>
<tr>
<td>7. $VVV$</td>
<td>new physics</td>
<td>$ZZZ$: Lazopoulos/Melnikov/Petriello ’07; $WW Z$: Hankele/Zeppenfeld ’07; $VVV$: Binoth/Ossola/Papadopoulos/Pittau ’07</td>
</tr>
<tr>
<td>8. $b\bar{b}b\bar{b}$</td>
<td>Higgs and new physics</td>
<td><strong>qq channel (virtual):</strong> Binoth/Guffanti/Guillet/Heinrich/Karg/Kauer/Mertsch/Reiter/Reuter/Sanguinetti</td>
</tr>
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</table>

**virtual amplitudes for several 2→4 processes** van Hameren/Papadopoulos/Pittau ’09
Technical motivation for $t\bar{t}b\bar{b}$: excellent benchmark calculation

Validate/compare NLO algorithms by solving a non-trivial problem

- 2 → 4 process with $\mathcal{O}(10^3)$ diagrams involving hexagons up to rank 4
- 6 coloured legs, massless and massive quarks, gluons

Phenomenological motivation for $t\bar{t}b\bar{b}$: irreducible background to $t\bar{t}H(H \to b\bar{b})$

Associated $t\bar{t}H(H \to b\bar{b})$ production

- opportunity to observe $H \to b\bar{b}$ channel and exploit dominance of its branching ratio for $M_H < 135$ GeV
- measurement of top Yukawa coupling
- ATLAS TDR indicated discovery potential (disappeared after more reliable background estimates)
- the background has a dramatic impact
Idea of $t\bar{t}H(H \rightarrow b\bar{b})$ analysis

- consider semileptonic decay channel: $b\bar{b}b\bar{b}jjl\nu$
  final state with 4 b-quarks!
- **identify $b\bar{b}$ pair from Higgs decay**
- observe resonance in $m_{b\bar{b}}$ distribution

Main problem: b-quark combinatorics

- only 1/6 possible b-quark pairs is correct!
- perform full $t\bar{t}$ reconstruction to identify b-quarks from top (and Higgs) decay
- very difficult due to presence of $\geq 6$ jets and bottom/light-jet misidentification
- **rate of correct b-pairings only 1/3!**

Consequences

- **dilution of Higgs resonance**
- increase of background in resonance region

\[ \text{dilution of Higgs resonance in } t\bar{t}H \]

\[ \text{ATLAS CSC-note, CERN-OPEN-2008-020} \]
Background and systematic uncertainty

Backgrounds (ATLAS analysis)

- $t\bar{t}b\bar{b}$ (AcerMC, $\mu_{QCD} = m_t + m_{b\bar{b}}/2$)
- $t\bar{t}jj$ (MC@NLO, $\mu_{QCD} = m_t^2 + <p_T,t>$)

Statistics and systematics (30 fb$^{-1}$)

- $S/\sqrt{B} \simeq 2$ sufficient for measurement
- $S/B \simeq 0.1$ implies that $\Delta B/B$ systematic uncertainty of $O(10\%)$ kills measurement!

Strategy for precise determination of $B$

- similar shape of $S$ and $B$ prevents direct extraction of $B$ from data
- measure $B$ normalization in signal-free region and extrapolate to signal-rich region using precise shape predictions
- **impossible without** $t\bar{t}b\bar{b}$ and $t\bar{t}jj$ at NLO!
2 → 4 and 2 → 5 Feynman diagrams

Quark-antiquark and gluon induced processes

arXiv:0807.1248

arXiv:0905.0110

σ_{LO} [fb]  pp → t\bar{t}b\bar{b} + X

\begin{align*}
\text{# diagrams and impact on } \sigma_{tot} & \quad q\bar{q} \quad gg \quad qg \\
\hline
\text{LO} & 7 & 36 \\
\text{virtual} & 188 & 1003 \\
\text{real} & 64 & 341 & 64 \\
\frac{(\sigma/\sigma_{tot})_{LO}}{\sigma_{tot}} & 5\% & 95\% \\
\frac{(\sigma/\sigma_{tot})_{NLO}}{\sigma_{tot}} & 3\% & 92\% & 5\%
\end{align*}

m_t = 172.6 \text{ GeV} \\
\mu_0/2 < \mu < 2\mu_0 

m_{b\bar{b}, \text{cut}} [\text{GeV}]
Step 1: quark anti-quark channel

Motivation

- 6-fermion process related to $e^+e^- \rightarrow 4f$
- proof of principle of feasibility of calculation

Main results [arXiv:0807.1248]

- small NLO correction (K=1.03)
- strong reduction of scale dependence
- very high CPU performance of 13 ms/PS-point
Tree and one-loop contributions to $q\bar{q}/gg \rightarrow t\bar{t}b\bar{b}$

Tree and one-loop sample diagrams in the $q\bar{q}$ and $gg$ channels

Two independent calculations

- diagrams generated with *FeynArts 1.0 / 3.2* [Kühlbeck/Böhm/Denner '90; Hahn '01]
- one calculation uses *FormCalc 5.2* [Hahn '06] for preliminary algebraic manipulations (Dirac algebra, covariant decomposition)
- bulk of reduction with two in-house *MATHEMATICA* programs
- numerics with two independent *Fortran77* codes (two libraries for tensor integrals)

Top quarks massive and bottom quarks massless
Structure of the calculation

Standard matrix elements and colour structures for individual diagrams

\[ D = \left[ \sum_{k} a_k C_k(\{c\}) \right] \sum_{i} \mathcal{F}_i S_i(\{p\}, \{\lambda\}) \]

Form factors \( \mathcal{F}_i \) in terms of tensor integrals

\[ \mathcal{F}_i = \sum_{j_1 \ldots j_R} \mathcal{K}_{i,j_1 \ldots j_R} T_{j_1 \ldots j_R}(\{p\}) \]

computed numerically diagram by diagram (no analytic reduction to scalar integrals)

Main goals

- reduction to small set of standard matrix elements \( S_i \)
- fast and stable numerical evaluation of tensor integrals \( T_{j_1 \ldots j_R} \)
Colour structure

- colour basis in the $q\bar{q}$ and $gg$ channels

\[
\delta_{i_1i_2}\delta_{i_3i_4}\delta_{i_5i_6}, \quad \delta_{i_1i_2}T^a_{i_3i_4}T^a_{i_5i_6}, \quad \ldots
\] 6 elements

\[
\delta^{a_1a_2}\delta_{i_3i_4}\delta_{i_5i_6}, \quad T^a_{i_3i_4}T^a_{i_5i_6}, \quad \ldots
\] 14 elements

- colour factorization for individual diagrams ⇒ CPU time doesn’t scale with the number of colour structures

Rational terms originating from UV $1/(D - 4)$ poles of tensor loop integrals

\[
K_{j_1\ldots j_R}(D) \underbrace{T_{j_1\ldots j_R}}_{\frac{R_{j_1\ldots j_R}}{(D-4)}} = K_{j_1\ldots j_R}(4)T_{j_1\ldots j_R} + K'_{j_1\ldots j_R}(4)R_{j_1\ldots j_R} + \mathcal{O}(D - 4)
\]

+ UV–finite part

- residues $R_{j_1\ldots j_R}$ of UV poles of tensor integrals explicitly available

- after $(D - 4)$-expansion continue calculation in $D = 4$

- **rational terms from IR poles** appear in intermediate expressions but **cancel at amplitude level and can thus be ignored** (proven for any scattering amplitude in App. A of JHEP 0808, 108 (2008) [arXiv:0807.1248])
Cancellation of rational terms originating from IR poles

Explicit calculations in terms of tensor and/or scalar integrals contain "rational" terms arising from double and single \textbf{IR poles} of tensor loop integrals

\[
\mathcal{K}_{j_1\ldots j_R}(D) \quad T_{j_1\ldots j_R} \quad \Rightarrow \quad \mathcal{K}'_{j_1\ldots j_R}(4) R_{1,j_1\ldots j_R} + \frac{1}{2} \mathcal{K}''_{j_1\ldots j_R}(4) R_{2,j_1\ldots j_R} + \ldots
\]

\[
\quad \frac{R_{2:j_1\ldots j_R}}{(D-4)^2} + \frac{R_{1:j_1\ldots j_R}}{(D-4)} \quad \text{+ IR-finite part}
\]

- can require lot of algebraic work (second-order expansion)
- cancel completely in truncated (!) one-loop amplitudes (observed in various calculations)
- explicit proof based on the following two observations

\begin{enumerate}
\item \textbf{Tensor-reduction identities are free from rational terms of IR type}
\begin{itemize}
\item Only $g^{\mu\nu}$-type components of tensor integrals ($T_{00\ldots}$) receive $D$-dependent factors in reduction identities
\item while soft and collinear regions ($q^\mu \rightarrow xp^\mu$) yield only $p_i^\mu p_j^\mu \ldots$ tensor structures
\end{itemize}
\end{enumerate}
(B) IR-divergent part of (individual) loop diagrams can be expressed in terms of integrals with $D$-independent prefactors

(B.1) consider all diagrams involving IR-divergent integrals

(B.2) look for trace-like contractions, $g_{\nu\lambda} \Gamma^{\nu\lambda}$, which can produce $D$-dep. coefficients:

$$g_{\nu\lambda} g^{\nu\lambda} = D, \quad g_{\nu\lambda} \gamma^\nu \not{\!\!} \gamma^\lambda = (2 - D) \not{\!\!} \phi, \ldots$$ (1)

(B.3) Manipulating the IR-divergent part of the integrand in D dim (using appropriate loop parametrizations and taking into account double-counting subtleties)

$$\int d^D q \, \epsilon^{\nu\lambda}_* (q + 2p)_\lambda g_{\mu\nu} - (2q + p)_\mu g_{\nu\lambda} + (q - p)_\nu g_{\lambda\mu} \frac{\Gamma^{\nu\lambda}}{q^2(q + p)^2} (q)$$

we find that the IR-singular part is free from trace-like contractions (due to the collinear limit $\epsilon^{\mu*}(2q + p)_\mu \to 0$ in the above example)
Practical consequence

- "rational" terms from IR poles can be systematically neglected
- subtlety related to scaleless 2-point functions

\[ B_0(0, 0, 0) = \left( \frac{2}{D-4} \right)_{\text{IR}} - \left( \frac{2}{D-4} \right)_{\text{UV}} = 0 \]

\[ (D-4)B_0(0, 0, 0) \Rightarrow (D-4)\left( \frac{-2}{D-4} \right)_{\text{UV}} = -2 \]

Generality and applicability

- cancellation applies to any truncated QCD amplitude and is independent from reduction algorithm (OPP, unitarity, PV, ...)
- proven for Feynman t’Hooft or similar gauges (power-1 propagators)
- \( \delta Z \) factors contain rational terms of IR origin
Reduction of Standard Matrix Elements (SMEs): strategy/simple examples

(A) \( \bar{q} q \) channel: after standard \( D \)-dim manipulations, \( \mathcal{O}(10^3) \) Dirac struct. of type

\[
\begin{bmatrix}
\bar{v}(p_1) \ldots \gamma^\mu \gamma^\nu \not{p}_3 \ldots u(p_2) \\
q\bar{q} \text{ chain}
\end{bmatrix}
\begin{bmatrix}
\bar{v}(p_3) \ldots \gamma^\mu \gamma^\nu \gamma^\rho \not{p}_6 \ldots u(p_4) \\
t\bar{t} \text{ chain}
\end{bmatrix}
\begin{bmatrix}
\bar{v}(p_5) \ldots \gamma^\rho \not{p}_2 \not{p}_3 \ldots u(p_6) \\
b\bar{b} \text{ chain}
\end{bmatrix}
\]

Needs additional identities in order to eliminate \( \not{p} \)-terms and \( \gamma \)-contractions

- without introducing new denominators that might spoil numerical stability

After cancellation of \( 1/(D - 4) \) poles we work in 4 dimensions and introduce

\[
\omega_{\pm} = \frac{1}{2}(1 \pm \gamma^5)
\]

\[
\bar{v}(p_i) \Gamma u(p_j) \Rightarrow \sum_{\lambda=\pm} \bar{v}(p_i) \Gamma \omega_\lambda u(p_j)
\]

This permits to use the \textbf{Chisholm identity}

\[
i \varepsilon^{\alpha \beta \gamma \delta} \gamma_\delta \gamma^5 = \gamma^\alpha \gamma^\beta \gamma^\gamma - g^\alpha \beta \gamma^\gamma + g^\alpha \gamma \gamma^\beta + g^\beta \gamma \gamma^\alpha
\]

Contracting with \( \otimes \gamma \gamma \gamma^5 \) and symmetrizing \( \Rightarrow \) eliminates the \( \varepsilon \)-tensor and yields

\[
i \varepsilon^{\alpha \beta \gamma \delta} \left[ \gamma_\delta \gamma^5 \otimes \gamma \gamma \gamma^5 \right] = 0 \Rightarrow \gamma^\mu \gamma^\alpha \gamma^\beta \omega_{\pm} \otimes \gamma_\mu \omega_{\mp} = \gamma^\mu \omega_{\pm} \otimes \gamma^\alpha \gamma^\beta \gamma_\mu \omega_{\mp}
\]

and similar identities that permit to exchange \( \gamma^\alpha \gamma^\beta \) between \( \gamma \)-contracted chains
Other identities involving doubly $\gamma$-contraced chains
\[
\gamma^\mu \gamma^\alpha \gamma^\nu \omega_{\pm} \otimes \gamma_\mu \gamma^\beta \gamma_\nu \omega_{\mp} = 4 \gamma^\beta \omega_{\pm} \otimes \gamma^\alpha \omega_{\mp}
\]
e etc.

can be used (for instance)

• to reduce number of $\gamma$-contractions

• or (in the inverse direction) to shift $\not{p}$-terms and then exploit Dirac equation

\[
4 \left[ \bar{v}(p_1) \cdots \not{p}^4 \omega_{\pm} u(p_2) \right] \left[ \bar{v}(p_3) \cdots \not{p}^2 \omega_{\mp} u(p_4) \right] = \left[ \bar{v}(p_1) \cdots \gamma^\mu \not{p}^2 \gamma^\nu \omega_{\pm} u(p_2) \right] \left[ \bar{v}(p_3) \cdots \gamma_\mu \not{p}^4 \gamma_\nu \omega_{\mp} u(p_4) \right] = 4(p_2p_4) \left[ \bar{v}(p_1) \cdots \gamma^\mu \omega_{\pm} u(p_2) \right] \left[ \bar{v}(p_3) \cdots \gamma_\mu \omega_{\mp} u(p_4) \right] + \text{mass-terms}
\]

In practice

• Chisolm identity + Dirac equation + momentum conservation (+ a lot of patience)

• yields several useful relations

• construct a sophisticated algorithm to reduce # of $\gamma$-contractions and $\not{p}$-terms
At the end of the day 25 types of standard matrix elements

- 10 of "massless" type: one Dirac matrix per chain

\[
\begin{align*}
\bar{v}(p_1)\gamma^\mu \omega_\alpha u(p_2) & \quad \bar{v}(p_3)\gamma^\mu \omega_\beta u(p_4) & \quad \bar{v}(p_5)\gamma^\mu \omega_\rho u(p_6) \\
\bar{v}(p_1)\gamma^\mu \gamma^\nu \omega_\alpha u(p_2) & \quad \bar{v}(p_3)\gamma^\mu \omega_\beta u(p_4) & \quad \bar{v}(p_5)\gamma^\mu \omega_\rho u(p_6)
\end{align*}
\]

- 15 of "massive" type: 2/0 Dirac matrices inside the t\bar{t} chain

\[
\begin{align*}
\bar{v}(p_1)\gamma^\mu \omega_\alpha u(p_2) & \quad \bar{v}(p_3)\gamma^\mu \gamma^\nu \omega_\beta u(p_4) & \quad \bar{v}(p_5)\gamma^\mu \omega_\rho u(p_6) \\
\bar{v}(p_1)\gamma^\mu \gamma^\nu \omega_\alpha u(p_2) & \quad \bar{v}(p_3)\gamma^\mu \gamma^\nu \omega_\beta u(p_4) & \quad \bar{v}(p_5)\gamma^\mu \omega_\rho u(p_6) \\
\bar{v}(p_1)\gamma^\mu \gamma^\nu \gamma^\rho \omega_\alpha u(p_2) & \quad \bar{v}(p_3)\omega_\beta u(p_4) & \quad \bar{v}(p_5)\gamma^\mu \omega_\rho u(p_6) \\
\bar{v}(p_1)\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \omega_\alpha u(p_2) & \quad \bar{v}(p_3)\omega_\beta u(p_4) & \quad \bar{v}(p_5)\gamma^\mu \omega_\rho u(p_6)
\end{align*}
\]

25 \times 8 chiral structures \((\omega_\alpha \otimes \omega_\beta \otimes \omega_\rho) \Rightarrow 200\text{ SMEs}\) for the q\bar{q} channel
(B) Reduction strategy for gg channel

Standard $D$-dim manipulations for quark chains and gluon-helicity vectors
$(\epsilon_1 p_1 = \epsilon_2 p_2 = \epsilon_1 p_2 = \epsilon_2 p_1 = 0)$ yield structures of type

\[
\left\{ \epsilon_1^\mu \epsilon_2^\nu, (\epsilon_1 \epsilon_2) p_2^\mu p_4^\nu, (\epsilon_1 p_4)(\epsilon_2 p_3) g^{\mu \nu}, \ldots \right\} \left[ \bar{v}(p_3) \ldots \gamma_\mu \gamma_\rho \gamma_6 \ldots u(p_4) \right] \left[ \bar{v}(p_5) \ldots \gamma^\rho \gamma_\nu \gamma_2 \gamma_3 \ldots u(p_6) \right]
\]

- gluon polarization vectors
- $t\bar{t}$ chain
- $b\bar{b}$ chain

**Strategy B.1**: employ q\bar{q}-channel method ($\gamma^5$ and Chisolm-based identities) to reduce two fermion chains $\Rightarrow$ 502 SMEs

**Strategy B.2**: less-sophisticated 4-dim reduction based only on a relation of type

\[
\gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} = g^{\mu_1 \mu_2} \gamma^{\mu_3} \gamma^{\mu_4} \gamma^{\mu_5} + \ldots + g^{\mu_1 \mu_2} g^{\mu_3 \mu_4} \gamma^{\mu_5} + \ldots
\]

- can be derived from Chisolm identity but does not involve $\gamma^5$
- permits to eliminate all $\gamma$-products with more than 3 terms $\Rightarrow$ 970 SMEs
- factor 2 more SMEs but completely process-independent reduction

**Surprising result**: speed of codes based on B.1 and B.2 reduction almost identical. CPU efficiency not due to highly sophisticated process-dependent manipulations!
Stable reduction of tensor loop integrals (developed for $e^+e^- \to 4f$) [Denner/Dittmaier '05]

### 6- and 5-point tensor integrals

- reduction to lower-point integrals [Melrose '65; Denner/Dittmaier '02]
- simultaneous rank-reduction [Binoth/Guillet/Heinrich/Pilon/Schubert '05; Denner/Dittmaier '05]
- no Gram determinants $\Rightarrow$ no numerical problems in practice

### 4- and 3-point tensor integrals

- Passarino–Veltman reduction [Passarino/Veltman '79]
- alternative methods for small Gram determinants [Denner/Dittmaier '05] (analogies with techniques proposed by Ferroglia/Passera/Passarino/Uccirati '03; Binoth/Guillet/Heinrich/Pilon/Schubert '05; Ellis/Giele/Zanderighi '06)

### 2- and 1-point tensor integrals

- numerically stable analytic representations [Passarino/Veltman '79; Denner/Dittmaier '05]
(3) Real corrections (qq/gg/qg channels)

- Also for real corrections: 2 independent calculations

**Two types of matrix elements**

\[ \bar{q}q \rightarrow t\bar{t}b\bar{b}g \quad \text{and} \quad qg \rightarrow t\bar{t}b\bar{b}q \quad (64 \text{ diagrams}) \]

\[ gg \rightarrow t\bar{t}b\bar{b}g \quad (341 \text{ diagrams}) \]

- **Madgraph 4.1.33** [Alwall/Demin/deVisscher/Frederix/Herquet/Maltoni/Plehn/Rainwater/Stelzer'07] for all channels

- analytical calculation with **Weyl–van der Waerden spinors** [Dittmaier '98] for qq/qg channels

- in-house numerical algorithm based on **off-shell recursions** [Berends/Giele '88; Caravaglios/Moretti '95; Draggiotis/Kleiss/Papadopoulos '98] for gg channel
Treatment of soft and collinear singularities with dipole subtraction
Catani/Seymour '96; Dittmaier '99; Catani/Dittmaier/Seymour/Trócsányi '02

\[ \int d\sigma_{2\to5} = \int \left[ d\sigma_{2\to5} - \sum_{i,j=1 \atop i \neq j}^6 d\sigma_{\text{dipole},ij}^{2\to5} \right] + \sum_{i,j=1 \atop i \neq j}^6 F_{ij} \otimes d\sigma_{2\to4} \]

- numerically stable/efficient but non-trivial: 30 qq/gg (10 qg) subtraction terms
- in-house dipoles checked against MadDipole [Frederix/Gehrmann/Greiner '08] (gg/qg) and PS slicing [Giele/Glover '92; Giele et al. '93; Keller/Laenen '98; Harris/Owens '01] (qq)
- initial-state collinear singularities cancelled by $\overline{\text{MS}}$-redefinition of PDFs

Phase-space integration

- adaptive multi-channel Monte Carlo [Berends/Kleiss/Pittau '94; Kleiss/Pittau '94] as in RACOONWW [Denner/Dittmaier/Roth/Wackeroth'99]/PROFECY4f [Bredenstein/Denner/Dittmaier/Weber'06]
- $\mathcal{O}(1400)$ channels to map all peaks from propagators (300) and dipoles (1100)

11-dimensional phase space, many channels and dipoles $\Rightarrow$ CPU-time! (see later)
Numerical checks

(A) **LO checked against SHERPA** [Gleisberg/Hoche/Krauss/Schalicke/Schumann/Winter '03]

(B) **Precision checks for individual NLO components in single PS points**
(typical precision: 10 to 14 digits)

**Virtual corrections**
- UV, soft and collinear cancellations
- agreement between 2 independent implementations

**Real emission**
- agreement of $2 \rightarrow 5$ matrix elements
- agreement between two dipole implementations
- cancellations in soft and collinear regions

(C) **Integrated NLO cross section**
- qq channel: agreement between 2 dipole implementations and PS slicing
- gg/qg channels: two independent calculations based on dipole subtraction agree at 1-2 sigma level with statistical accuracy at $2 \times 10^{-3} \times \sigma_{\text{NLO}}$ level
Parton masses

- $m_t = 172.6 \text{ GeV}$
- g, q and b massless $\Rightarrow$ recombination!

$k_T$-Jet-Algorithm [Run II Jet physics group: Blazey et al. hep-ex/0005012]

- Select partons with $|\eta| < 5$
- Reconstruct jets with $\sqrt{\Delta \phi^2 + \Delta y^2} > D = 0.8$

Cuts for b-jets

- require two b-jets with $p_{T,j} > 20 \text{ GeV}$ and $y_j < 2.5$
- $b\bar{b}$ invariant mass: $m_{b\bar{b}} > m_{b\bar{b},\text{cut}}$

Strong coupling, PDFs and central scale*

- CTEQ6M with $\alpha_S(M_Z) = 0.118$
- 2-loop running to $\mu = \mu_R = \mu_F$
- central scale $\mu_0 = m_t + m_{b\bar{b},\text{cut}}/2$

* LO obtained with LO $\alpha_S$, LO PDFs and 1-loop running
LO and NLO scale dependence (qq+gg+qg channels)

Reduction of scale dependence for $\sigma_{\text{tot}}$
- rescaling $\mu_R, F = m_t$ by common factor $\xi \in [0.5, 2]$
- 70% dependence at LO
- 34% dependence at NLO

Rescaling $\mu_F$ by $1/\xi$ (lower plot)
- qualitatively similar behaviour
- dominant dependence from $\alpha_S(\mu_R)^4$

Very large NLO correction
- LO and NLO curves do not cross around $\xi = 1$
- $K = 1.77$ at $\xi = 1$ (central scale too high!)
- completely different wrt $q\bar{q}$ channel ($K = 1.03$)
- bad news: strong $t\bar{t}H$-background enhancement!
How to reduce large NLO contribution?

Idea

- large $K$-factor due to hard jet emission
- underestimated by PDF evolution
  (collinear approximation)

Introduce a veto against jets with $p_{T}^{\text{jet}} \geq p_{T}^{\text{veto}}$

- realistic choice $p_{T}^{\text{veto}} \sim 50 - 100 \text{ GeV}$
- $\sigma_{NLO}^{\text{tot}}$ quite sensitive to $p_{T}^{\text{veto}}$ in this region
  - can reduce $K$-factor up to roughly 1.2

Important issues remain to be studied

- impact of $p_{T}^{\text{veto}}$ on scale dependence?
- resulting systematic uncertainties?
- impact on distributions?
$m_{b\bar{b}}$-dependence of $\sigma_{LO}$ and $\sigma_{NLO}$

Plotted curve

- $m_{b\bar{b}}$-distribution integrated over $m_{b\bar{b}} > m_{b\bar{b},\text{cut}}$
- no jet veto
- LO and NLO with uncertainty bands around $\mu_0 = m_t + m_{b\bar{b},\text{cut}}/2$

Similar behaviour as for $\sigma_{tot}$

- scale dependence reduced by factor 2
- bands overlap but NLO central value slightly above LO band (large $K$-factor)

$K$-factor quite insensitive to $m_{b\bar{b}}$
Statistical precision and speed of the calculation

Single 3GHz Intel Xeon processor & pgf77 Portland compiler

<table>
<thead>
<tr>
<th></th>
<th>$\sigma/\sigma_{LO}$</th>
<th># events (after cuts)</th>
<th>$\langle \Delta \sigma \rangle_{stat}/\sigma$</th>
<th>runtime</th>
<th>time/event</th>
</tr>
</thead>
<tbody>
<tr>
<td>NLOtree (gg)</td>
<td>85%</td>
<td>$5.8 \times 10^6$</td>
<td>$0.4 \times 10^{-3}$</td>
<td>2h</td>
<td>1.4ms</td>
</tr>
<tr>
<td>virtual (gg)</td>
<td>10%</td>
<td>$0.46 \times 10^6$</td>
<td>$0.7 \times 10^{-3}$</td>
<td>20h</td>
<td>160ms</td>
</tr>
<tr>
<td>real + dipoles (gg/qg)</td>
<td>87%</td>
<td>$16.5 \times 10^6$</td>
<td>$2.6 \times 10^{-3}$</td>
<td>47h</td>
<td>10ms</td>
</tr>
</tbody>
</table>

- couple of days on single CPU $\Rightarrow O(10^7)$ events and $O(10^{-3})$ stat. accuracy

- for same precision $\langle \Delta \sigma \rangle_{stat}$ virtual corrections require less CPU-time than real corrections (scale-dependent statement!)

- speed of virtual corrections is remarkably high: 160 ms/event
  (including colour and polarization sums!)
Some (process-dependent) remarks about CPU efficiency

• Speed of diagrammatic method in striking contrast to pessimistic expectations based on generic arguments (factorial complexity of Feynman diagrams)

• Is it possible to speed up the virtual corrections beyond 160ms/event?

⇒ Comparison with unitarity-based algorithms would be very instructive

⇒ Looking at method-independent (and simple) ingredient:

    CPU-time for master integrals $\sim 10$ ms/event

suggests that there is not much room for further dramatic improvement
Conclusions

NLO QCD calculation for $pp \rightarrow t\bar{t}b\bar{b}$ at the LHC

- very important for $t\bar{t}H$ measurement
- $2 \rightarrow 4$ reaction with highest priority in the 2005 Les Houches wish list

Phenomenological result

- strong enhancement of $t\bar{t}H$-background ($K \simeq 1.8$)
- might be reduced with a jet veto (to be studied)

Technical considerations

- First complete $2 \rightarrow 4$ NLO calculation at hadron colliders
- Excellent testing ground for NLO multi-leg methods
- Remarkably high CPU efficiency of diagrammatic tensor-reduction approach