NLO tools and MCFM
Lecture I

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Slides available from http://theory.fnal.gov/people/ellis/Talks/CTEQ07/

Bibliography

QCD and Collider Physics
(Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology)
by R. K. Ellis, W.J. Stirling and B.R. Webber

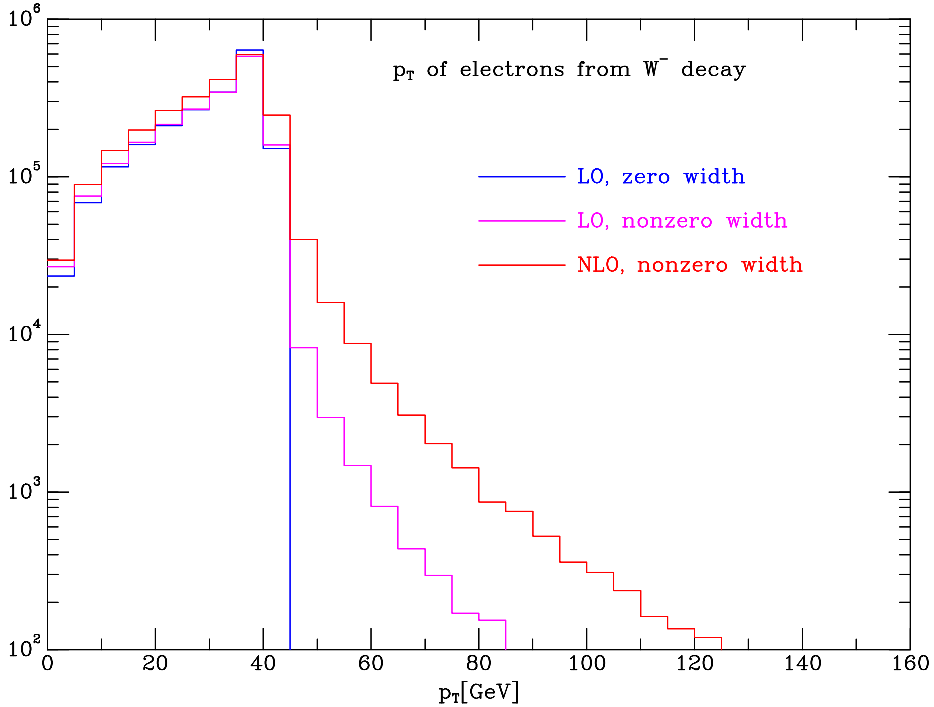
R. K. Ellis, D. A. Ross and A. E. Terrano, The Perturbative Calculation Of Jet Structure In e^+e^- Annihilation, Nucl. Phys. B **178**, 421 (1981).

S. Catani and M. H. Seymour, The Dipole Formalism for the Calculation of QCD Jet Cross Sections at Next-to-Leading Order,
Phys. Lett. B **378** (1996) 287 [arXiv:hep-ph/9602277].

Hard scattering cross sections

- Why NLO?
- Treatment of e^+e^- at NLO
- General method
- W production
- Subtraction method
- MCFM
  - ⋆ W + jet production
  - ⋆ W + charm jet production
  - ⋆ H + jet production

Why NLO?

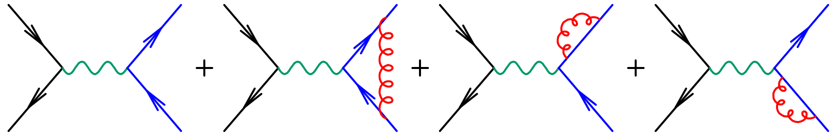


- Calculation of NLO corrections, give a better prediction for the rate. For example, changes in the renormalization scale lead to terms which are of the same order as NLO terms.

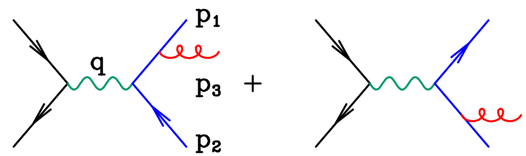
$$\alpha_s(Q^2) = \alpha_s(\mu^2) - b_0 \alpha_s^2(\mu^2) \ln(Q^2/\mu^2)$$

- Extra radiation can modify kinematic distributions.
- However NLO will ot be sufficient for all quantities

Jet structure in e^+e^-



(a)



(b)

- For simplicity, we shall consider only the photon-exchange contribution in detail. In addition we shall ignore the correlation with the initial state leptons; ie we shall sum over the polarization of the virtual photon $\sum_h \varepsilon_h^\mu(q)\varepsilon_h^\nu(q) = -g_{\mu\nu}$
- The result for the lowest order graph is (in $d = 4 - 2\epsilon$ dimensions)

$$|M_{q\bar{q}}|^2 = 4Q_q^2(1-\epsilon)N\ q^2$$

- The result with one gluon emission ($N = 3, C_F = 4/3$)

$$|M_{q\bar{q}g}|^2 = |M_{q\bar{q}}|^2\ \frac{2C_F g_s^2}{q^2}\ \Big[\frac{(1-\epsilon)[(p_1.p_3)^2 + (p_2.p_3)^2] + p_1.p_2 q^2}{p_1.p_3\ p_2.p_3} - 2\epsilon\Big]$$

Divergences

- Introduce the rescaled energies $2p_1.q = x_1 q^2, 2p_2.q = x_2 q^2$

$$|M_{q\bar{q}g}|^2 = |M_{q\bar{q}}|^2 \frac{2C_F g_s^2}{q^2} \left[\frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(x_1+x_2-1)}{(1-x_1)(1-x_2)} - 2\epsilon\right]$$

- The x_i integrals are divergent along the boundaries at $x_i = 1$. Since $1-x_1 = x_2 E_g(1-\cos\theta_{2g})/\sqrt{q^2}$ and $1-x_2 = x_1 E_g(1-\cos\theta_{1g})/\sqrt{q^2}$, where E_g is the gluon energy and θ_{ig} ($i=1,2$) the angles between the gluon and the quarks.
- The virtual emission graphs also contain singularities revealed as poles in ϵ

$$|2M^*_{q\bar{q}V} M_{q\bar{q}}| = |M_{q\bar{q}}|^2 \frac{C_F \alpha_s}{2\pi} \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + O(\epsilon)\right]$$

- For sufficiently inclusive quantities, the divergences will cancel. We need to find a method to include these cancellations in a Monte Carlo program.

Subtraction method

- The method to cancel the divergences is the subtraction method, (ERT 1980)
- We want to invent a series of subtraction terms, which cancel the divergences of the real integral, but are sufficiently simply that they can be integrated analytically.
- This can be achieved by partial fractioning.

$$
\begin{aligned}
& \Big[\frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(x_1+x_2-1)}{(1-x_1)(1-x_2)} - 2\epsilon\Big] \\
= \quad & \frac{1}{1-x_1}\Big[\frac{2}{(2-x_1-x_2)} - 1 - x_2 - \epsilon(1-x_2)\Big] \\
+ \quad & \frac{1}{1-x_2}\Big[\frac{2}{(2-x_1-x_2)} - 1 - x_1 - \epsilon(1-x_1)\Big] - 2\epsilon
\end{aligned}
$$

- This has been called the dipole method by Catani and Seymour; a misleading name, since the essence of the method is to take the eikonal terms (dipoles?) and reduce them to single collinear poles by partial fractioning

$$
\frac{p_1.p_2}{p_1.p_3\ p_2.p_3} \rightarrow \frac{1}{p_1.p_3}\frac{p_1.p_2}{p_1.p_3+p_2.p_3} + \frac{1}{p_2.p_3}\frac{p_1.p_2}{p_1.p_3+p_2.p_3}
$$

Subtraction method II

There is considerable liberty in what we choose to subtract, since additional non-singular terms can be added. We will choose a standard form, following Catani and Seymour

$$
\begin{aligned}
& \Big[\frac{(1-\epsilon)(x_1^2+x_2^2)+2\epsilon(x_1+x_2-1)}{(1-x_1)(1-x_2)} - 2\epsilon\Big] \\
= \quad & \frac{1}{1-x_1}\Big[\frac{2}{(2-x_1-x_2)} - 1 - x_2 + \frac{(1-x_1)(1-x_2)}{x_1} - \epsilon\frac{(1-x_2)}{x_1}\Big] \\
+ \quad & \frac{1}{1-x_2}\Big[\frac{2}{(2-x_1-x_2)} - 1 - x_1 + \frac{(1-x_1)(1-x_2)}{x_2} - \epsilon\frac{(1-x_1)}{x_2}\Big] \\
- \quad & 2\epsilon - (1-\epsilon)\Big[\frac{(1-x_1)}{x_2} + \frac{(1-x_2)}{x_1}\Big]
\end{aligned}
$$

The first two terms are the subtraction pieces and the last term (which contains no singularities) is the part of the matrix element remaining after subtraction, which is extremely simple in this case.

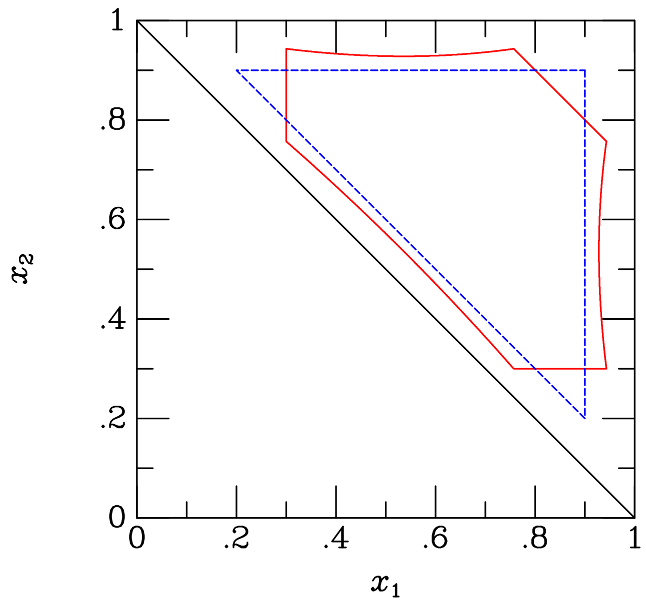
Phase space factorization

- The phase space can be factorized into a product of the phase space for the production of two on-shell particles times the phase space for the emission of the extra gluon.

$$d\Phi^{(3)}(Q;p_1,p_2,p_2) = d\Phi^{(2)}(Q;\tilde{p}_{13},\tilde{p}_2)[dp_i(\tilde{p}_{13},\tilde{p}_2)]$$

- We obtain an almost familiar form for the phase space, ($\int d\Omega^{(d-3)} = \frac{2\pi}{\pi^\epsilon \Gamma(1-\epsilon)}$)

$$[dp_i(\tilde{p}_{13},\tilde{p}_2)] = \quad = \frac{(q^2)^{1-\epsilon}}{16\pi^2}\,\frac{d\Omega^{(d-3)}}{(2\pi)^{1-2\epsilon}}\,dx_1\,dx_2\,\frac{\Theta(x_1+x_2-1)}{(1-x_1)^\epsilon(1-x_2)^\epsilon(1-x_3)^\epsilon}$$



★ Remember that three particle phase space is $dE_1 dE_2$. In four dimensions we have

$$d\Phi^{(3)} = \frac{q^2}{16\pi^2} dx_1 dx_2 \times \Theta(1-x_1)\Theta(1-x_2)\Theta(x_1+x_2-1).$$

Integrating subtraction terms

Defining the reduced phase space integral as follows,

$$\langle\langle f(x_1,x_2)\rangle\rangle = \int_0^1 \, dx_1 \, (1-x_1)^{-\epsilon} \int_{(1-x_1)}^1 \, dx_2 \, (1-x_2)^{-\epsilon} \, (x_1+x_2-1)^{-\epsilon} \, f(x_1,x_2)$$

$$\langle\langle \frac{1}{1-x_1}[-1-\frac{x_2+x_1-1}{x_1}-\epsilon\frac{1-x_2}{x_1}]\rangle\rangle = \frac{3}{2\epsilon}+5+O(\epsilon)$$

$$\langle\langle \frac{2}{(1-x_1)(2-x_2-x_2)}\rangle\rangle \equiv \langle\langle \frac{1}{(1-x_1)(1-x_2)}\rangle\rangle = \frac{1}{\epsilon^2}-\frac{\pi^2}{2}+O(\epsilon)$$

To calculate the second quantity, perform the substitution $(1-x_2)=zx_1$

$$\int_0^1 \, dx_1(1-x_1)^{-1-\epsilon}\int_{1-x_1}^1 \, dx_2(1-x_2)^{-1-\epsilon}(x_1+x_2-1)^{-\epsilon}$$

$$= \int_0^1 \, dx_1(1-x_1)^{-1-\epsilon}x_1^{-2\epsilon}\int_0^1 \, dz(1-z)^{-1-\epsilon}z^{-\epsilon} = \frac{\Gamma(-\epsilon)\Gamma(1-2\epsilon)}{\Gamma(1-3\epsilon)} \, \frac{\Gamma(-\epsilon)\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

$\int_0^1 \, dz \, z^{a-1}(1-z)^{b-1} = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, $\Gamma(1+\epsilon) = 1-\gamma_E\epsilon+(\frac{\gamma_E^2}{2}+\frac{\pi^2}{12})\epsilon^2+O(\epsilon^3)$

Subtracted answer

$$\sigma^{NLO\,\{3\}} = \int_3 \left[d\sigma^R_{\epsilon=0} - d\sigma^A_{\epsilon=0} \right]$$

$$= \quad |\mathcal{M}_2|^2 \frac{C_F \alpha_S}{2\pi} \int_0^1 dx_1\, dx_2\, \Theta(x_1+x_2-1) \left\{ \frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} F_J^{(3)}(p_1,p_2,p_3) \right.$$
$$- \left[\frac{1}{1-x_2} \left(\frac{2}{2-x_1-x_2} - 1 - x_1 \right) + \frac{1-x_1}{x_2} \right] F_J^{(2)}(\widetilde{p}_{13},\widetilde{p}_2)$$
$$\left. - \left[\frac{1}{1-x_1} \left(\frac{2}{2-x_1-x_2} - 1 - x_2 \right) + \frac{1-x_2}{x_1} \right] F_J^{(2)}(\widetilde{p}_{23},\widetilde{p}_1) \right\} \quad .$$

Since the four-dimensional three-parton matrix element can be written as follows

$$\frac{x_1^2+x_2^2}{(1-x_1)(1-x_2)} = \left[\frac{1}{1-x_2} \left(\frac{2}{2-x_1-x_2} - 1 - x_1 \right) \right] + [x_1 \leftrightarrow x_2] \quad ,$$

F_J is a jet definition function; the integration over x_i is unrestricted by $F_J^{(2)}(\widetilde{p}_{23},\widetilde{p}_1)$. For any infrared safe observable (implying that $F_J^{(3)} \to F_J^{(2)}$ as $x_i \to 1$) this expression is finite.

Subtracted answer II

- Integration of subtraction term, $z = \frac{(x_1+x_2-1)}{x-2}$

$$|M_{q\bar{q}}|^2 \frac{\alpha_S C_F}{2\pi} \langle\langle \Big[\frac{1}{1-x_1}\Big(\frac{2}{(1-x_1 z)} - 1 - z - \epsilon(1-z)\Big)\Big] + \Big[1 \leftrightarrow 2\Big]\rangle\rangle$$
$$= \quad |M_{q\bar{q}}|^2 \frac{C_F \alpha_s}{2\pi}\Big[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + 10 - \pi^2 + O(\epsilon)\Big]$$

- Contribution of virtual diagrams

$$|2M^*_{q\bar{q}V} M_{q\bar{q}}| = |M_{q\bar{q}}|^2 \frac{C_F \alpha_s}{2\pi}\Big[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + O(\epsilon)\Big]$$

- sum is finite

$$\sigma^{NLO\,\{2\}} = \int_2 \Big[d\sigma^A + d\sigma^V\Big]$$
$$= \quad \frac{2C_F \alpha_S}{2\pi} \int_2 \int_0^1 dx_1\ dx_2\ \delta(1-x_1)\delta(1-x_2) F_J^{(2)}(\widetilde{p}_{23}, \widetilde{p}_1)$$

Removing the jet definition functions we recover the standard result for the correction to the total cross section $C_F \frac{3\alpha_s}{4\pi}$

General Method for NLO parton integrator

- We want to compute a jet cross section σ to NLO, namely

$$\sigma = \sigma^{LO} + \sigma^{NLO} \; .$$

- Born approximation involves m partons in the final state.

$$\sigma^{LO} = \int_m d\sigma^B \; ,$$

- At NLO we have the real cross section $d\sigma^R$ with $m+1$ partons in the final-state and the one-loop correction $d\sigma^V$ to the process with m partons in the final state:

$$\sigma^{NLO} \equiv \int d\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V \; .$$

- The two integrals are separately divergent in (four dimensions), although their sum is finite.

Finiteness

- The general idea of the subtraction method for writing a general-purpose Monte Carlo program is to use the identity

$$d\sigma^{NLO} = \left[d\sigma^{R} - d\sigma^{A}\right] + d\sigma^{A} + d\sigma^{V} \quad ,$$

where $d\sigma^{A}$ is a proper approximation of $d\sigma^{R}$ such as to have the same singular behaviour point-by-point as $d\sigma^{R}$ itself. Thus, $d\sigma^{A}$ acts as a local counterterm for $d\sigma^{R}$ and, introducing the phase space integration,

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^{R} - d\sigma^{A}\right] + \int_{m+1} d\sigma^{A} + \int_{m} d\sigma^{V} \quad ,$$

- one can safely perform the limit $\epsilon \longrightarrow 0$ under the integral sign in the first term on the right-hand side. The first term can be integrated numerically in four dimensions.

Jet definition

$$\sigma^{NLO} = \int_{m+1} \left[\left(d\sigma^{R} \right)_{\epsilon=0} - \left(d\sigma^{A} \right)_{\epsilon=0} \right] + \int_{m} \left[d\sigma^{V} + \int_{1} d\sigma^{A} \right]_{\epsilon=0} \quad ,$$

- and can be easily implemented in a ‘partonic Monte Carlo’ program, which generates appropriately weighted partonic events with $m+1$ final-state partons and events with m partons.

$$F_J^{(m+1)} \to F_J^{(m)} \quad ,$$

The Born-level cross section $d\sigma^B$ can be (symbolically) written as a function of the jet-defining function $F_J^{(m)}$ in the following way

$$d\sigma^B = d\Phi^{(m)} \, |\mathcal{M}_m|^2 \, F_J^{(m)} \quad ,$$

where $d\Phi^{(m)}$ and $\mathcal{M}_m$ respectively are the full phase space and the QCD matrix element to produce m final-state partons. The corresponding expression for the real cross section $d\sigma^R$ is:

$$d\sigma^R = d\Phi^{(m+1)} \, |\mathcal{M}_{m+1}|^2 \, F_J^{(m+1)} \quad .$$

Factorization of Phase space

- We start we two final state partons one of which is the emitter, and the other is the spectator
- The momenta of the emitter and the spectator are defined in differently

$$\widetilde{p}_k^\mu = \frac{1}{1-y_{ij,k}}\, p_k^\mu \quad , \qquad \widetilde{p}_{ij}^\mu = p_i^\mu + p_j^\mu - \frac{y_{ij,k}}{1-y_{ij,k}}\, p_k^\mu \quad ,$$

where the dimensionless variable $y_{ij,k}$ is given by

$$y_{ij,k} = \frac{p_i p_j}{p_i p_j + p_j p_k + p_k p_i} \quad .$$

Note that both the emitter and the spectator are on-shell $(\widetilde{p}_{ij}^2 = \widetilde{p}_k^2 = 0)$ and that, performing the replacement $\{i, j, k\} \to \{\widetilde{ij}, \widetilde{k}\}$, momentum conservation is implemented exactly:

$$p_i^\mu + p_j^\mu + p_k^\mu = \widetilde{p}_{ij}^\mu + \widetilde{p}_k^\mu \quad .$$

These are common features of all the subtraction formulas.

Phase space

- The above definition of the momenta is particularly useful because it allows us to exactly factorize the phase space of the partons i, j, k into the dipole phase space times a single-parton contribution.

$$
\begin{aligned}
d\phi(p_i, p_j, p_k; Q) \quad &= \quad \frac{d^d p_i}{(2\pi)^{d-1}} \, \delta_+(p_i^2) \, \frac{d^d p_j}{(2\pi)^{d-1}} \, \delta_+(p_j^2) \, \frac{d^d p_k}{(2\pi)^{d-1}} \, \delta_+(p_k^2) \\
&\times \quad (2\pi)^d \, \delta^{(d)}(Q - p_i - p_j - p_k) \; .
\end{aligned}
$$

- In terms of the momenta $\widetilde{p}_{ij}$, $\widetilde{p}_k$ and p_i, this phase-space contribution takes the factorized form:

$$
d\phi(p_i, p_j, p_k; Q) = d\phi(\widetilde{p}_{ij}, \widetilde{p}_k; Q) \; [dp_i(\widetilde{p}_{ij}, \widetilde{p}_k)] \quad ,
$$

  where

$$
[dp_i(\widetilde{p}_{ij}, \widetilde{p}_k)] = \frac{d^d p_i}{(2\pi)^{d-1}} \, \delta_+(p_i^2) \; \mathcal{J}(p_i; \widetilde{p}_{ij}, \widetilde{p}_k) \; ,
$$

  and the Jacobian factor is

$$
\mathcal{J}(p_i; \widetilde{p}_{ij}, \widetilde{p}_k) = \Theta(1 - \tilde{z}_i) \, \Theta(1 - y_{ij,k}) \; \frac{(1 - y_{ij,k})^{d-3}}{1 - \tilde{z}_i} \; .
$$

Phase space (continued)

In terms of the kinematic variables defined earlier, we have

$$
\begin{aligned}
[dp_i(\widetilde{p}_{ij}, \widetilde{p}_k)] &= \frac{(2\widetilde{p}_{ij}\widetilde{p}_k)^{1-\epsilon}}{16\pi^2} \frac{d\Omega^{(d-3)}}{(2\pi)^{1-2\epsilon}} \, d\tilde{z}_i \; dy_{ij,k} \; \Theta(\tilde{z}_i(1-\tilde{z}_i)) \; \Theta(y_{ij,k}(1-y_{ij,k})) \\
&\cdot \quad (\tilde{z}_i(1-\tilde{z}_i))^{-\epsilon} \; \left(1-y_{ij,k}\right)^{1-2\epsilon} y_{ij,k}^{-\epsilon} \; ,
\end{aligned}
$$

where $d\Omega^{(d-3)}$ is an element of solid angle perpendicular to $\widetilde{p}_{ij}$ and $\widetilde{p}_k$ and thus

$$
\int d\Omega^{(d-3)} = \frac{2\pi}{\pi^\epsilon \Gamma(1-\epsilon)} \quad .
$$

General algorithm for subtraction terms

The calculation of the subtracted cross section involves the evaluation of two subtraction terms: $\mathcal{D}_{13,2}$ and $\mathcal{D}_{23,1}$.

$$\mathcal{D}_{13,2}(p_1, p_2, p_3) = \frac{1}{2p_1 p_3} V_{q_1 g_3,2} \, |\mathcal{M}_2|^2 \quad ,$$

with the following kinematics

$$\widetilde{p}_2^{\,\mu} = \frac{1}{x_2} \, p_2^\mu \; , \qquad \widetilde{p}_{13}^{\,\mu} = q^\mu - \frac{1}{x_2} \, p_2^\mu \quad .$$

$$V_{q_1 g_3,2} = C_F \Big[\frac{2}{(1-(1-y_{12,3})z)} - 1 - z - \epsilon(1-z) \Big]$$

The subtraction contribution $\mathcal{D}_{23,1}$ is obtained by the replacement $p_1 \leftrightarrow p_2$.

Integrating subtraction terms

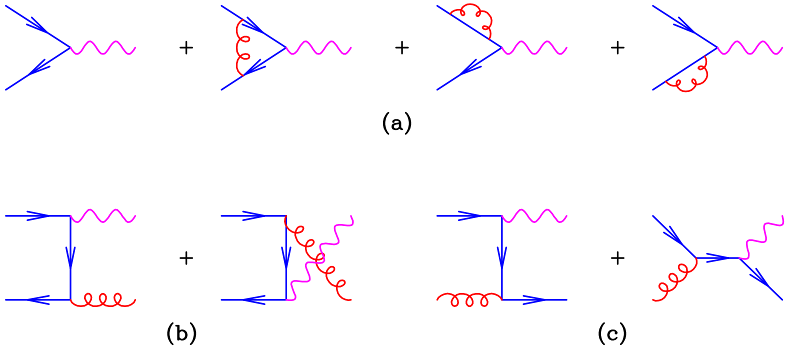
Defining the reduced phase space integral as follows,

$$\langle\langle f(y,z)\rangle\rangle = \int_0^1 dy\, y^{-1-\epsilon}(1-y)^{1-2\epsilon} \int_0^1 dz\, z^{-\epsilon}(1-z)^{-\epsilon}\, f(y,z)$$

We find, as before, that

$$\begin{aligned} \langle\langle -1-z-\epsilon(1-z)\rangle\rangle &= \frac{3}{2\epsilon}+5 \\ \langle\langle \frac{2}{(1-z(1-y))}\rangle\rangle &= \frac{1}{\epsilon^2}-\frac{\pi^2}{2} \end{aligned}$$

Next-to-leading order: Initial state



- The contribution of the real diagrams (in four dimensions) is

$$|M|^2 \sim g^2 C_F \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^2 s}{ut}\right] = g^2 C_F \left[\Big(\frac{1+z^2}{1-z}\Big)\Big(\frac{-s}{t} + \frac{-s}{u}\Big) - 2\right]$$

  where $z = Q^2/s$, $s + t + u = Q^2$.

- Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.
- The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.
- Ignore for simplicity the diagrams with incoming gluons.

- Control the divergences by continuing the dimensionality of space-time, $d = 4 - 2\epsilon$, (technically this is dimensional reduction). Performing the phase space integration, the total contribution of the real diagrams is

$$\begin{aligned}\sigma_R &= \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \Bigg[\Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \frac{\pi^2}{3}\Big) \delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) \\ &- 2(1-z) + 4(1+z^2) \Big[\frac{\ln(1-z)}{1-z}\Big]_+ - 2\frac{1+z^2}{(1-z)} \ln z \Bigg]\end{aligned}$$

  with $c_\Gamma = (4\pi)^\epsilon / \Gamma(1-\epsilon)$.
- The contribution of the virtual diagrams is

$$\sigma_V = \delta(1-z) \Bigg[1 + \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c'_\Gamma \Big(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 6 + \pi^2 \Big) \Bigg]$$

  $c'_\Gamma = c_\Gamma + O(\epsilon^3)$

- Adding it up we get in dim-reduction

$$\sigma_{R+V} = \frac{\alpha_S}{2\pi} C_F \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} c_\Gamma \left[\left(\frac{2\pi^2}{3} - 6\right)\delta(1-z) - \frac{2}{\epsilon} P_{qq}(z) - 2(1-z) \right.$$
$$\left. + \quad 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z \right]$$

- The divergences, proportional to the branching probability , are universal.
- We will factorize them into the parton distributions. We perform the mass factorization by subtracting the counterterm

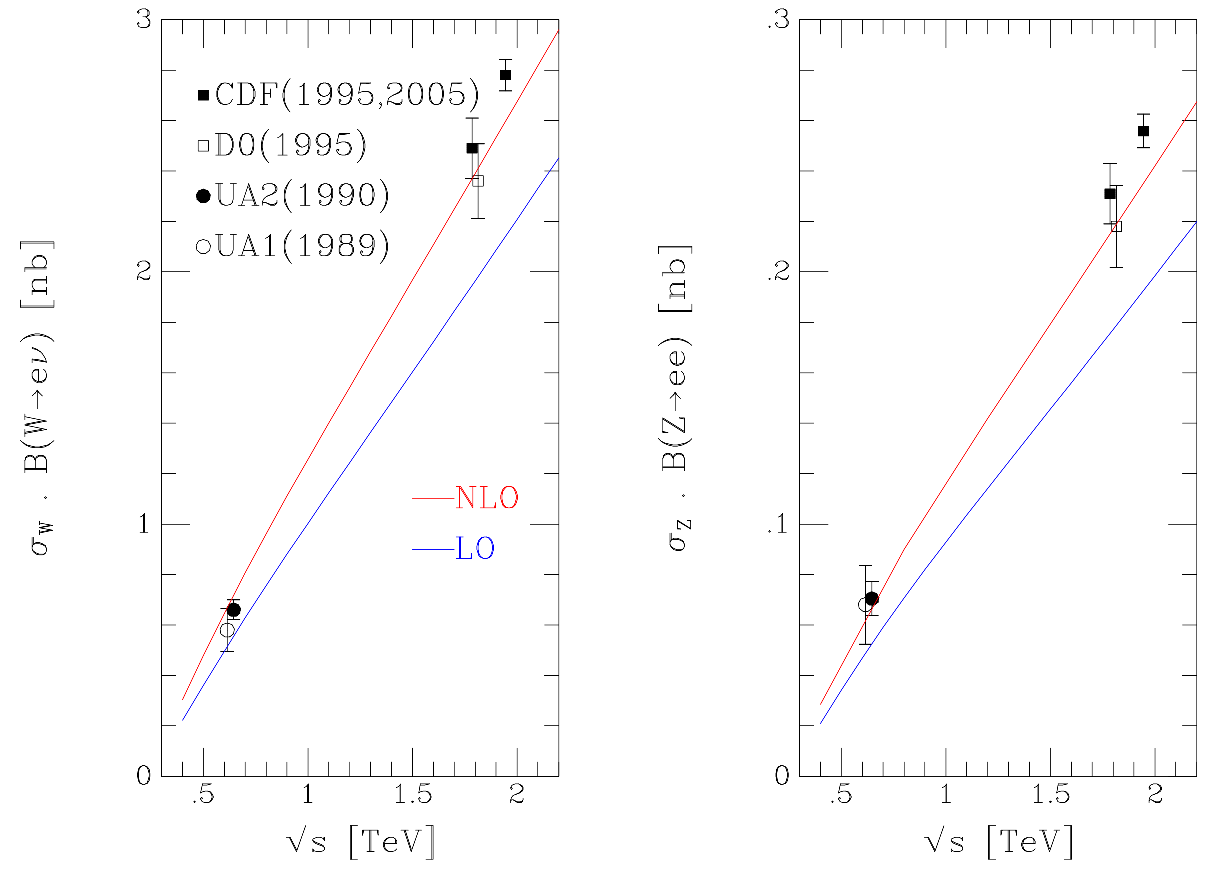
$$2\frac{\alpha_S}{2\pi} C_F \left[\frac{-c_\Gamma}{\epsilon} P_{qq}(z) - (1-z) + \delta(1-z) \right]$$

(The finite terms are necessary to get us to the $\overline{MS}$-scheme).

$$\hat{\sigma} = \frac{\alpha_S}{2\pi} C_F \left[\left(\frac{2\pi^2}{3} - 8\right)\delta(1-z) + 4(1+z^2)\left[\frac{\ln(1-z)}{1-z}\right]_+ - 2\frac{1+z^2}{(1-z)}\ln z + 2P_{qq}(z)\ln\frac{Q^2}{\mu^2} \right]$$

- Similar correction for incoming gluons.

Application to W, Z production



- Agreement with NLO theory is good.
- LO curves lie about 25% too low.
- NNLO results are also known and lead to a further modest (4%) increase at the Tevatron.

General calculational method for NLO

- Direct integration is good for the total cross section, but for differential distributions, (to which we want to apply cuts), we need a Monte Carlo method.
- We use a general subtraction procedure at NLO.
- at NLO the cross section for two initial partons a and b and for m outgoing partons, is given by

$$\sigma_{ab} = \sigma_{ab}^{LO} + \sigma_{ab}^{NLO}$$

where

$$\begin{aligned} \sigma_{ab}^{LO} &= \int_m d\sigma_{ab}^{B} \\ \sigma_{ab}^{NLO} &= \int_{m+1} d\sigma_{ab}^{R} + \int_m d\sigma_{ab}^{V} \end{aligned}$$

the singular parts of the QCD matrix elements for real emission, corresponding to soft and collinear emission can be isolated in a process independent manner

Calculational method (cont)

- One can use this the construct a set of counterterms

$$d\sigma^{ct} = \sum_{ct} \int_m d\sigma^B \otimes \int_1 dV_{ct}$$

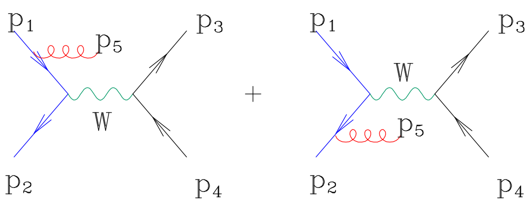
where $d\sigma^B$ denotes the appropriate colour and spin projection of the Born-level cross section, and the counter-terms are independent of the details of the process under consideration.

- these counterterm cancel all non-integrable singularities in $d\sigma^R$, so that one can write

$$\sigma_{ab}^{NLO} = \int_{m+1} [d\sigma_{ab}^R - d\sigma_{ab}^{ct}] + \int_{m+1} d\sigma_{ab}^{ct} + \int_m d\sigma_{ab}^V$$

The phase space integration in the first term can be performed numerically in four dimensions.

Matrix element counter-event for W production



In the soft limit $p_5 \to 0$ we have

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = g^2 C_F \frac{p_1 \cdot p_2}{p_1 \cdot p_5 \; p_2 \cdot p_5} |M_0(p_1,p_2,p_3,p_4)|^2$$

- Eikonal factor can be associated with radiation from a given leg by partial fractioning

$$\frac{p_1 \cdot p_2}{p_1 \cdot p_5 \; p_2 \cdot p_5} = [\frac{p_1 \cdot p_2}{p_1 \cdot p_5 + p_2 \cdot p_5}][\frac{1}{p_1 \cdot p_5} + \frac{1}{p_2 \cdot p_5}]$$

- including the collinear contributions, singular as $p_1 \cdot p_5 \to 0$, the matrix element for the counter event has the structure

$$|M_1(p_1,p_2,p_3,p_4,p_5)|^2 = \frac{g^2}{x_a p_1 \cdot p_5} \hat{P}_{qq}(x_a) |M_0(x_a p_1, p_2, \tilde{p}_3, \tilde{p}_4)|^2$$

where $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$ and $\hat{P}_{qq}(x_a) = C_F(1+x^2)/(1-x)$

Subtraction method for NLO

- For event $q(p_1) + \bar{q}(p_2) \to W^+(\nu(p_3) + e^+(p_4)) + g(p_5)$ with $p_1 + p_2 = \sum_{i=3}^5 p_i$
- generate a counter event $q(x_a p_1) + \bar{q}(p_2) \to W^+(\nu(\tilde{p}_3) + e^+(\tilde{p}_4))$ and $x_a p_1 + p_2 = \sum_{i=3}^4 \tilde{p}_i$ with $1 - x_a = (p_1 \cdot p_5 + p_2 \cdot p_5)/p_1 \cdot p_2$.
- A Lorentz transformation is performed on all j final state momenta $\tilde{p}_j = \Lambda^\mu_\nu p^\nu_j, j = 3, 4$ such that $\tilde{p}^\mu_j \to p^\mu_j$ for p_5 collinear or soft.
- The longitudinal momentum of p_5 is absorbed by rescaling with x.
- The other components of the momentum, p_5 are absorbed by the Lorentz transformation.
- In terms of these variables the phase space has a convolution structure,

$$d\phi^{(3)}(p_1, p_2; p_3, p_4, p_5) = \int_0^1 dx\ d\phi^{(2)}(p_2, xp_1; \tilde{p_3}, \tilde{p_4})[dp_5(p_1, p_2, x)]$$

where

$$[dp_5(p_1, p_2, x_a)] = \frac{d^d p_5}{(2\pi)^3} \delta^+(p_5^2)\Theta(x)\Theta(1-x)\delta(x - x_a)$$

- If k_i is the emitted parton, and p_1, p_2 are the incoming momenta, define the shifted momenta

$$\widetilde{k}_j^\mu = k_j^\mu - \frac{2k_j \cdot (K+\widetilde{K})}{(K+\widetilde{K})^2}\,(K+\widetilde{K})^\mu + \frac{2k_j \cdot K}{K^2}\,\widetilde{K}^\mu\ ,$$

where the momenta K^μ and $\widetilde{K}^\mu$ are,

$$K^\mu = p_1^\mu + p_2^\mu - p_i^\mu\ \ ,\widetilde{K}^\mu = \widetilde{p}_{1i}^\mu + p_2^\mu\ \ .$$

- Since $2\sum_j k_j \cdot K = 2K^2$ and $2\sum_j k_j \cdot (K+\widetilde{K}) = 2K^2 + 2K\cdot\widetilde{K} = (K+\widetilde{K})^2$, $K^2 = \widetilde{K}^2$, the momentum conservation constraint in the $m+1$-parton matrix

$$p_1^\mu + p_2^\mu - \sum_j k_j^\mu - p_i^\mu = 0\ \ .$$

implies

$$\widetilde{p}_{1i}^\mu + p_2^\mu - \sum_j \widetilde{k}_j^\mu = 0\ \ .$$

- Note also that the shifted momenta can be rewritten in the following way:

$$\widetilde{k}_j^\mu = \Lambda^\mu{}_\nu(K,\widetilde{K})\, k_j^\nu \quad ,$$

$$\Lambda^\mu{}_\nu(K,\widetilde{K}) = g^\mu{}_\nu - \frac{2(K+\widetilde{K})^\mu (K+\widetilde{K})_\nu}{(K+\widetilde{K})^2} + \frac{2\widetilde{K}^\mu K_\nu}{K^2} \quad ,$$

- the matrix $\Lambda^\mu{}_\nu(K,\widetilde{K})$ generates a proper Lorentz transformation on the final-state momenta.
- If the emitted parton has zero transverse momenta, the Lorentz transformation reduces to the identity.

MCFM overview

John Campbell and R.K. Ellis

- Parton level cross-sections predicted to NLO in α_S

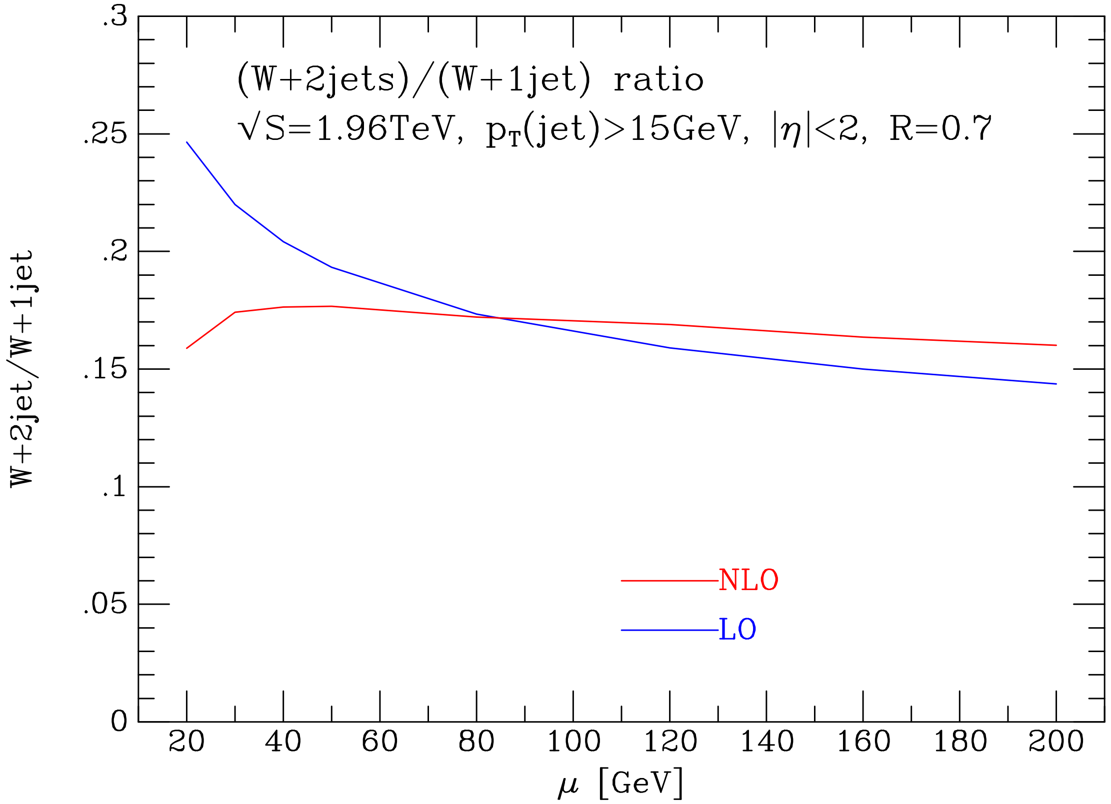
$p\bar{p} \to W^\pm/Z$	$p\bar{p} \to W^+ + W^-$
$p\bar{p} \to W^\pm + Z$	$p\bar{p} \to Z + Z$
$p\bar{p} \to W^\pm + \gamma$	$p\bar{p} \to W^\pm/Z + H$
$p\bar{p} \to W^\pm + g^\star\ (\to b\bar{b})$	$p\bar{p} \to Zb\bar{b}$
$p\bar{p} \to W^\pm/Z +$ 1 jet	$p\bar{p} \to W^\pm/Z +$ 2 jets
$p\bar{p}(gg) \to H$	$p\bar{p}(gg) \to H +$ 1 jet
$p\bar{p}(VV) \to H +$ 2 jets	$p\bar{p} \to t + X$
$pp \to t + W$	

$\oplus$ less sensitivity to μ_R, μ_F, rates are better normalized, fully differential distributions.

$\ominus$ low particle multiplicity (no showering), no hadronization, hard to model detector effects

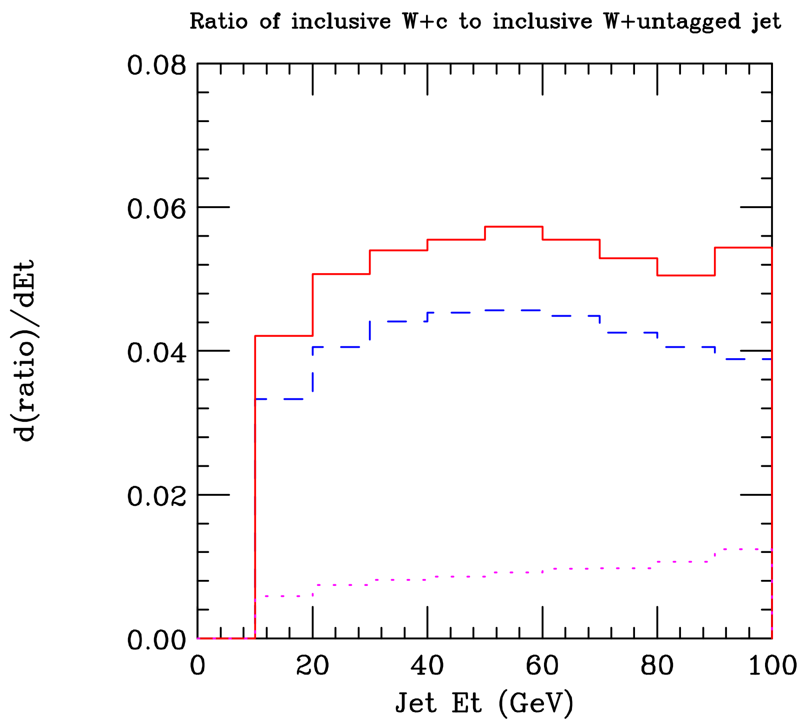
MCFM:examples

- (W+2 jet)/(W+1 jet)



W + charm jet

- The subtraction method can be applied with massive particles in the final state.
- Consider the case of a charm quark produced in association with a W.
- Leading order processes are $s + g \to W^- + c$ and Cabibbo suppressed $d + g \to W^- + c$ (dashed blue).
- NLO processes are gluonic dressings of the LO processes (solid red) as well as $q + \bar{q} \to W^+ c\bar{c}$ (shown blue dotted in figure)
- Curves are shown for $\sqrt{s} = 1.8$ TeV and $m_c = 1.7$ GeV.



MCFM examples

- Production of a $m_H = 120$ GeV Higgs, using effective Lagrangian $HG^{\mu\nu}G_{\mu\nu}$, obtained in heavy top limit.
- Cross sections for Higgs+anything or Higgs+1 jet+anything are the same.
- Radiation probability is one, and NLO is clearly inadequate.
- what is needed is a combination of NLO and shower Monte-Carlo, (MC@NLO)