

# Charge and Mass of the Electron

## *Motivation for the Experiment*

The aim of this experiment is to measure the charge and mass of the electron. The charge will be measured directly using a variant of the Millikan oil drop experiment while the mass will be deduced from a measurement of the charge to mass ratio,  $e/m$ , combined with the charge measurement. The two separate measurements can be done in either order with the combined analysis performed at the end. Table I gives the values for the mass and charge of the electron for comparison purposes. They are known to much higher accuracy than we can hope to measure in our experiments and, therefore, they will be considered to be exact.

**Table I**  
**Electron Charge and Mass**

<u>Quantity</u>	<u>Symbol</u>	<u>Value</u>	<u>Units</u>
Mass	$m_e$	$9.109 \times 10^{-31}$	kg
Charge	$e$	$1.602 \times 10^{-19}$	C

## *I. The Charge to Mass Ratio ( $e/m$ ) for the Electron*

### **Introduction**

In 1897 J. J. Thompson discovered the first "elementary particle", the electron, by measuring the ratio of its charge to mass in a manner similar to the experiment that we will perform. Given the mass and charge in Table I we expect to measure a value that is close to:

$$\frac{e}{m_e} = \frac{1.602 \times 10^{-19} \text{ C}}{9.109 \times 10^{-31} \text{ kg}} = 1.759 \times 10^{11} \frac{\text{C}}{\text{kg}} \quad (\text{I-1})$$

### **Theory**

Both conceptually and experimentally this part of the experiment is quite simple. A hollow glass sphere is evacuated to a high vacuum and filled with a small amount of Neon gas at a pressure of about 1 millitorr. Inside the sphere, a heated wire (a "filament") emits a large number of electrons which are accelerated by a potential difference  $V$ , to acquire kinetic energy:

$$\frac{1}{2} m_e v^2 = e \cdot V \quad (\text{I-2})$$

and, therefore, a velocity:

$$v = \left[ 2V \cdot \frac{e}{m_e} \right]^{\frac{1}{2}} \quad (\text{I-3})$$

The moving electrons strike and ionize the Ne atoms which give off light when they recombine, producing a visible beam along the electron track. The sphere is placed in a

region where a magnetic field,  $\mathbf{B}$ , is produced by a current in two coils of wire.  $\mathbf{B}$  is applied perpendicular to the velocity vector of the electrons resulting in a magnetic force:

$$\mathbf{F} = e\mathbf{v} \times \mathbf{B} \quad (\text{I-4})$$

The force is perpendicular to both the velocity and the magnetic field vectors and produces an acceleration  $\mathbf{a}$ , whose magnitude is given by:

$$|a| = \frac{|F|}{m_e} = \frac{v^2}{r} \quad (\text{I-5})$$

where  $r$  is the radius of the circular path of the electron. (Recall that such a force is called a **centripetal** force and the corresponding acceleration a **centripetal** acceleration with both vectors directed toward the center of the circle.)

Therefore:

$$m_e \frac{v^2}{r} = e \cdot v \cdot B \quad (\text{I-6})$$

which yields:

$$\frac{e}{m_e} = \frac{v}{B \cdot r} \quad (\text{I-7})$$

Substituting for  $v$  from Eq. 3, we obtain:

$$\frac{e}{m_e} = \frac{1}{B \cdot r} \left[ \frac{2V \cdot e}{m_e} \right]^{\frac{1}{2}} \quad (\text{I-8})$$

which simplifies to:

$$\frac{e}{m_e} = \frac{2V}{(B \cdot r)^2} \quad (\text{I-9})$$

This equation is correct **IF**  $V$  and  $B$  are both constant in space and time so that all the electrons have the same velocity  $v$ , and follow the same circular path of radius  $r$ .

$V$  is applied between two conducting equipotential surfaces by a stable power supply and is therefore well defined.

Two coils of radius  $R$ , separated by a distance  $d$ , each having  $N$  turns with current  $i$  passing through them in the same sense, generate a magnetic field on their common axis, half way between them, equal to:

$$B_0 = \frac{\mu_0 Ni}{R} \left[ \frac{1}{1 + \left( \frac{d}{2R} \right)^2} \right]^{\frac{3}{2}} \quad (\text{I-10})$$

here  $\mu_0 = 4\pi \times 10^{-7}$  henries/per meter is the permeability constant. You should verify that when  $d = R$ , this expression reduces to the familiar equation for a Helmholtz pair. In our setup, the separation of the coils is somewhat larger than their radii, approximately in the ratio of 14/11. We need to know how the field varies as a function of the radius in the mid plane between the two coils. Figure 1 shows this variation as a function of the fractional radius,  $r/R$ . Note that in each case the field is normalized to the value at  $r = 0$ .

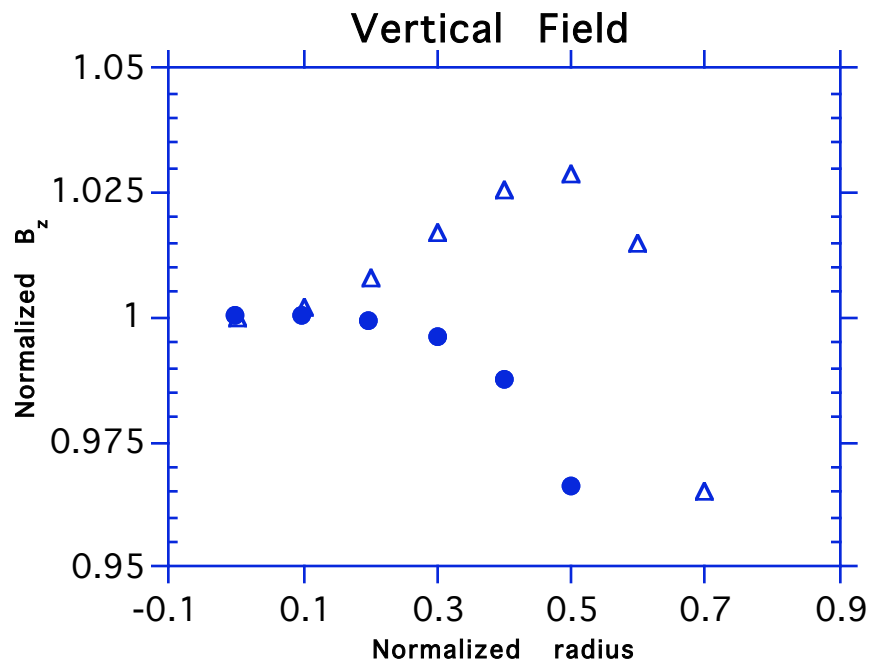


Fig. I-1: Normalized magnetic field in mid plane for Helmholtz pair (closed circles) and our configuration (open triangles).

To avoid large corrections, confine your measurements to radii less than about 70% of the radius of the coils thereby limiting corrections to 5% or less.

**Apparatus**

e/m tube

Magnetic field coils with 150 turns per coil:  $N = 150$ .

Kepco power supply

Heathkit power supply

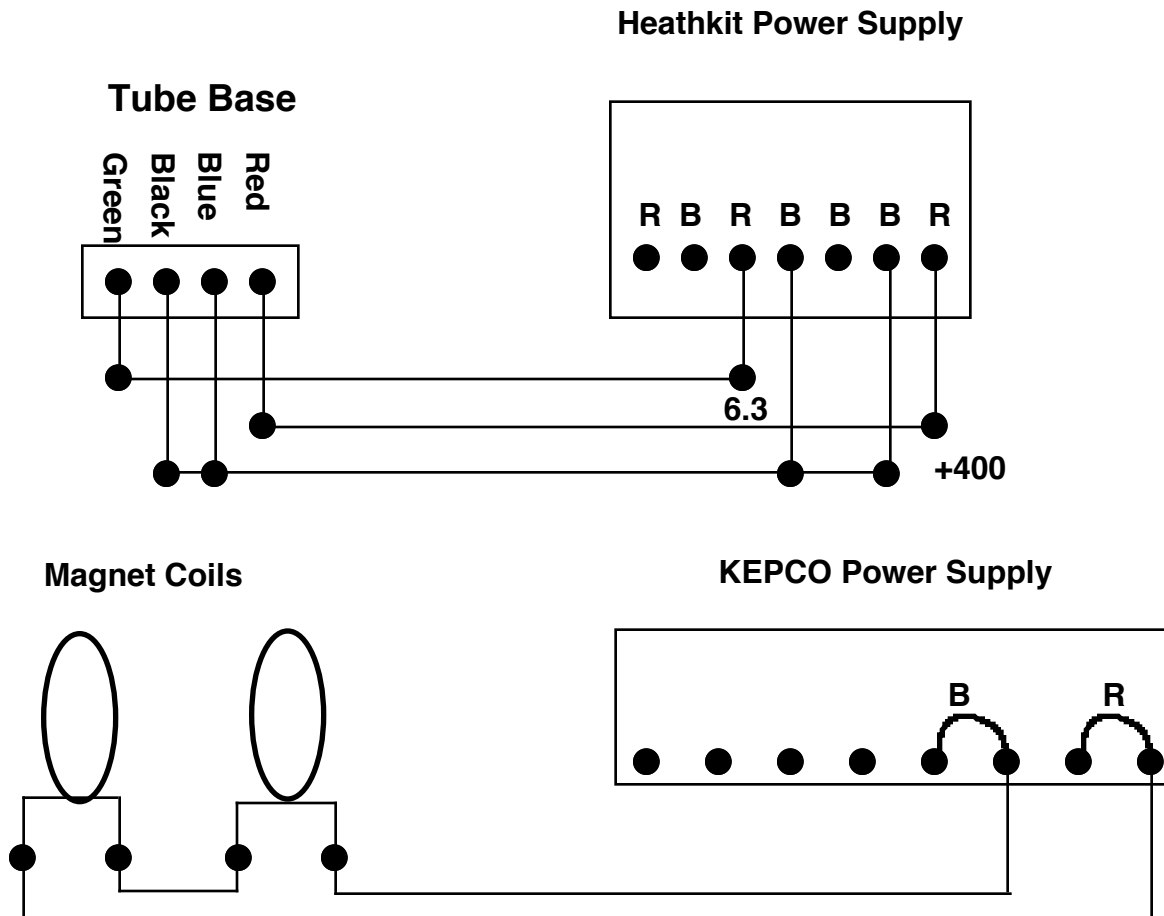
**The Experiment**

The e/m tube consists of the glass bulb, the hot wire filament, the accelerating electrodes and a set of reference marks to indicate the diameter of the electron path which is made visible by ionization of the Neon gas.

The Kepco power supply provides the current for the magnet coils while the Heathkit power supply provides the current to heat the filament and for the AC internal meters.

Connect the two power supplies as shown in the schematic (Fig. I-2). Turn on the heater voltage, let it warm up a few minutes and then apply about +200 V accelerating voltage. Turn on the Kepco supply and increase the current until the beam is deflected.

Collect data for several different voltages and magnet currents. Measure the radius of one of the coils as well as their separation in order to calculate the central magnetic field from Eq. I-10. Compute the actual magnetic field at each radius using Fig. I-1 and then use Eq. I-9 to calculate  $e/m_e$  for each value of radius. Plot your values for  $e/m_e$  as a function of the radius using different symbols for the different accelerating voltages. Do a simple statistical analysis to determine the errors in your measurement and put these as error bars on your plot. Do your data vary by more than the statistical error bars. If so, these variations may indicate the presence of a systematic error in your measurements. Do your data suggest the presence of such a systematic error? If they do, you will need to quote these systematic errors and take them into account when you compute your value for  $m_e$  at the conclusion of this lab.



Note: The coils are connected "in series", but the correct "sense" will be determined by how the beam deflects

Fig. I-2: Schematic Circuit Diagram

## II. The Charge of the Electron

### Introduction to the Measurement

The charge of the electron will be measured using a variation of the Millikan oil drop experiment.

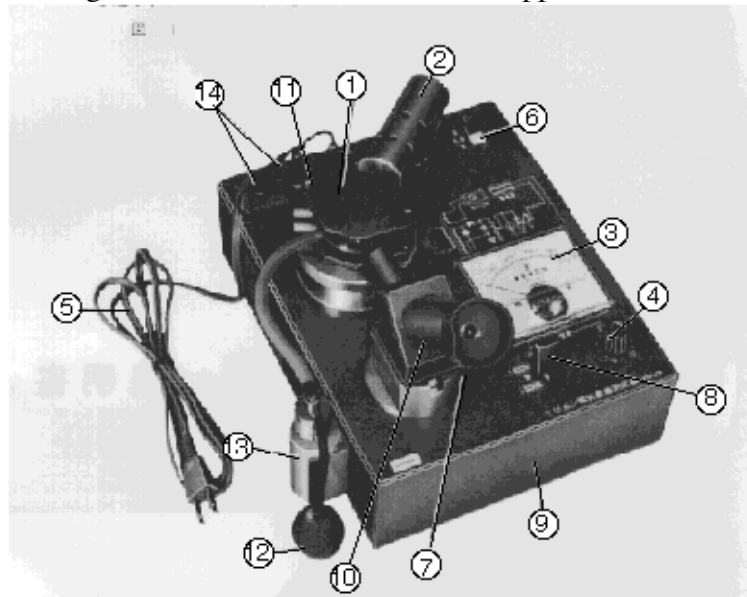
### Description of the Apparatus

The apparatus is designed so that all necessary components are contained in one unit. It is mounted on a metal base, which supplies all necessary power inputs when it is connected to a 110 V AC, 50/60 Hz outlet and consists of the following parts:

- A storage bottle with spray bulb pump for producing spheres of latex liquid (diameter  $\sim 1/1000$  mm).
- A 6V, 10W projector.
- A 30X scale microscope which has a resolution of 0.2 mm between the small divisions and 1 mm between the large divisions.
- An electrode assembly.

- Appropriate controls, including a polarity-reversing switch, a potentiometer for fine control of plate voltage, and a voltmeter indicating the plate voltage applied.

See Figure II-1 for an illustration of the apparatus with a key listing the parts.



- (1) Electrode Housing
- (2) Illuminator
- (3) Voltmeter
- (4) Potentiometer
- (5) Power Cord
- (6) On-Off Switch
- (7) Microsc. Adj. Knob
- (8) Polarity-Rev. Switch
- (9) Metal Base
- (10) Box Microscope
- (11) Electrode Housing Set Screw
- (12) Spray Bulb
- (13) Latex Storage Bottle
- (14) HV Electrode Leads

Fig. II-1: The Millikan Apparatus

### Theory

The experiment is named for R. A. Millikan, the American physicist who devised it. (Millikan's original experiment used drops of oil, while this apparatus uses spheres of latex liquid.) Millikan wanted to determine whether electrical charge occurred in discrete units and, if it did, whether there was such a thing as an elementary charge.

In the Millikan experiment, a small charged ball made of latex moves vertically between two metal plates. This sphere is too small to be seen by the naked eye, so the projector and microscope are used to enable the user to see the sphere as a small dot of light. When there is no voltage applied to the plates, the sphere falls slowly and steadily under the influence of gravity, quickly reaching its terminal velocity. When a voltage is applied to the plates, the terminal velocity of the sphere is affected not only by the force of gravity but also by the electric force acting on the sphere.

When the experimenter knows the density of the latex ball, the terminal velocity of the ball falling under the influence of gravity alone, and the charge on the plates of the Millikan apparatus, it is possible to find the force produced by the electric charge on the ball. A series of observations will produce a group of terminal velocity values which are seen to be multiples of a lowest value. From this data, it is possible to determine the elementary unit of charge.

Consider a latex sphere of mass  $m$  and charge  $q$ , falling under the influence of gravity between two horizontal plates. In falling, the sphere is subjected to an opposing force due to air resistance. The speed of the sphere quickly increases until a constant terminal speed is reached, at which time the weight of the sphere,  $mg$ , minus the buoyant force is exactly equal to the air resistance force. The value of the air resistance force on a sphere was first derived by Sir George Stokes and is given as  $6\pi\eta rs$  where  $\eta$  is the coefficient of viscosity of air,  $r$  is the radius of the sphere and  $s$  is its terminal speed. If the buoyant force of the air is neglected, the equation of motion of the sphere is:

$$mg - 6\pi\eta rs = 0 \quad (\text{II-1})$$

Now suppose that the metal plates are connected to a source of constant potential difference such that an electric field of intensity  $E$  is established between the plates and a latex sphere of charge  $q$  is made to move upwards. The direction of the electric field must depend on the sign of the charge  $q$ , which may be either positive or negative. The resultant upward force on the charge is  $Eq - mg$  and this force causes the sphere to move upwards with a terminal speed  $s^+$ . The equation of motion is:

$$Eq - mg = 6\pi\eta r s^+ \quad (11-2)$$

If now the polarity of the electric field is reversed, the sphere will move downward under the combined force of gravity and the electrostatic force. The equation of motion is:

$$Eq + mg = 6\pi\eta r s^- \quad (11-3)$$

Note that the forces are now additive and that the terminal speed is achieved in the opposite direction than in the previous case.

The effect of gravity can now be eliminated by adding the equations of motion yielding:

$$2Eq = 6\pi\eta r (s^+ + s^-). \quad (11-4)$$

If the terminal speeds are changed to velocities by incorporating the proper sign convention of upwards (+) and downwards (-), then one must change the symbols from  $s$  to  $v$  and add a minus (-) before the term for the downward velocity. We will not need to write that equation here because Eq. (11-4) will be sufficient for our purposes. (The corresponding equation with velocities can also lead to mistakes if one forgets that the downward velocity should have a negative sign.)

### Performing the Measurement

Begin by setting the apparatus on a level surface. Make sure all power to the unit is turned off whenever you are making any adjustments to it.

Make sure the electrode housing is set as in Figure II-2. This requires unscrewing the electrode housing set screws and removing the upper electrode housing plate. Next insert the atomizer ring between the upper and lower electrode housings and re-tighten the set screws.

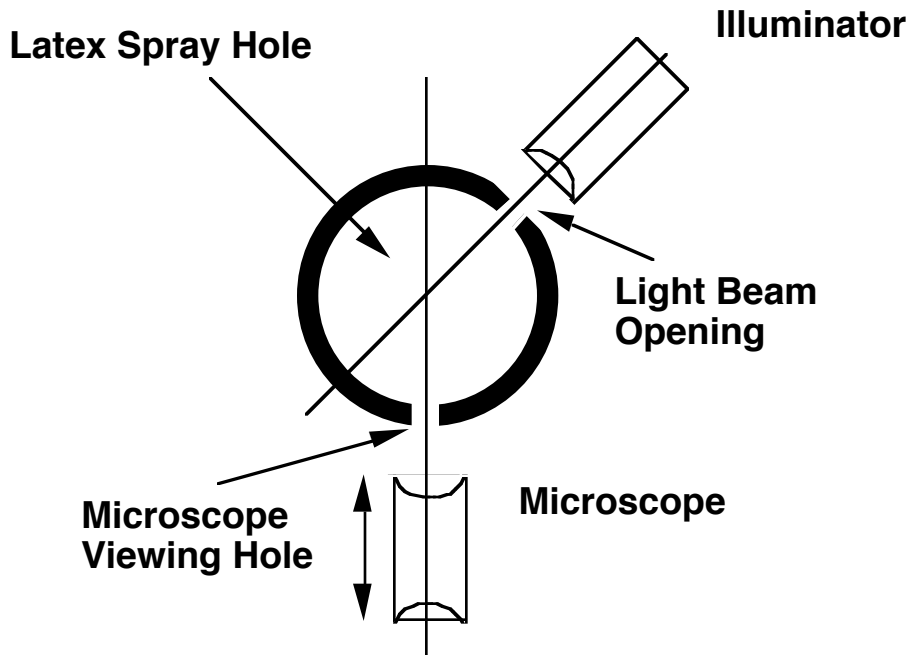


Fig. II-2: Millikan Apparatus Schematic Setup

If the light should be out of focus, it should be adjusted as follows: loosen the set screws at the top of the electrode housing and remove the housing: loosen the light socket set screw and move the socket so that an image of the bulb filament is on the screen (see Fig. II-3). The light was adjusted at the time the unit was assembled and, under normal conditions, no further adjustment should be necessary.

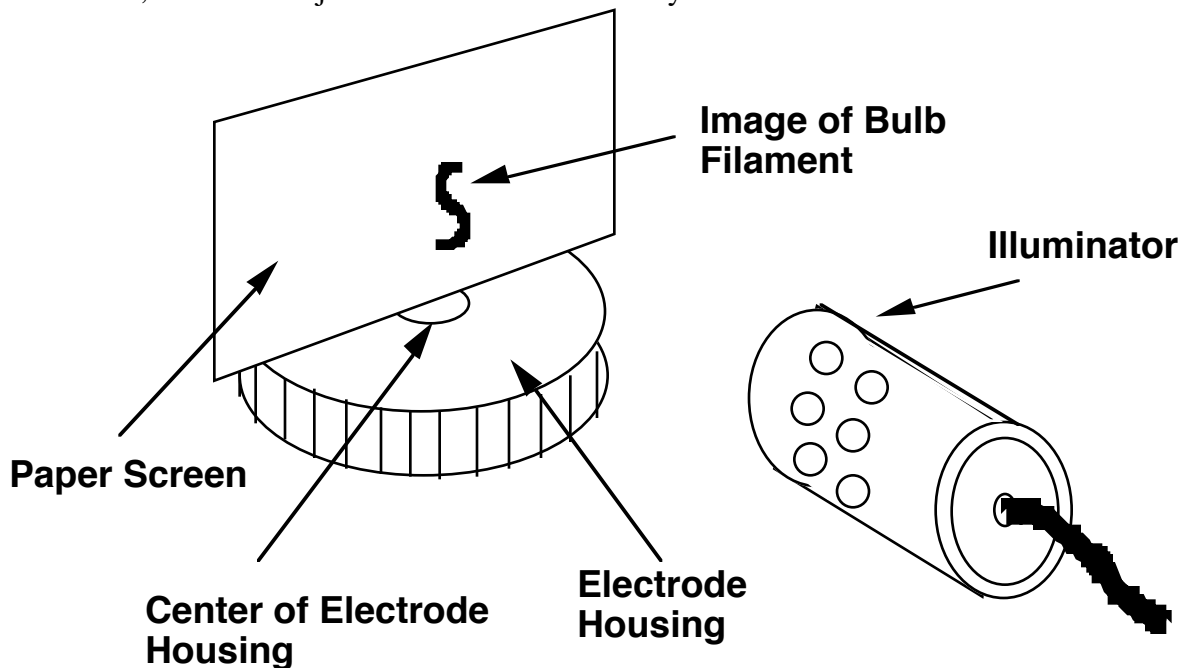


Fig. II-3: Arrangement for re-focusing light.

To use the latex spray, first loosen the set screws of the electrode housing to permit air to escape. Then inject latex spheres into the chamber by using the spray bulb pump. To



feed the latex spheres in, cover the air hole of the spray bulb pump with a finger and squeeze the bulb. Note that the spheres will not be injected unless the air hole is covered. Spraying is usually difficult the first few times. We recommend that you begin your measurements after making several test sprays.

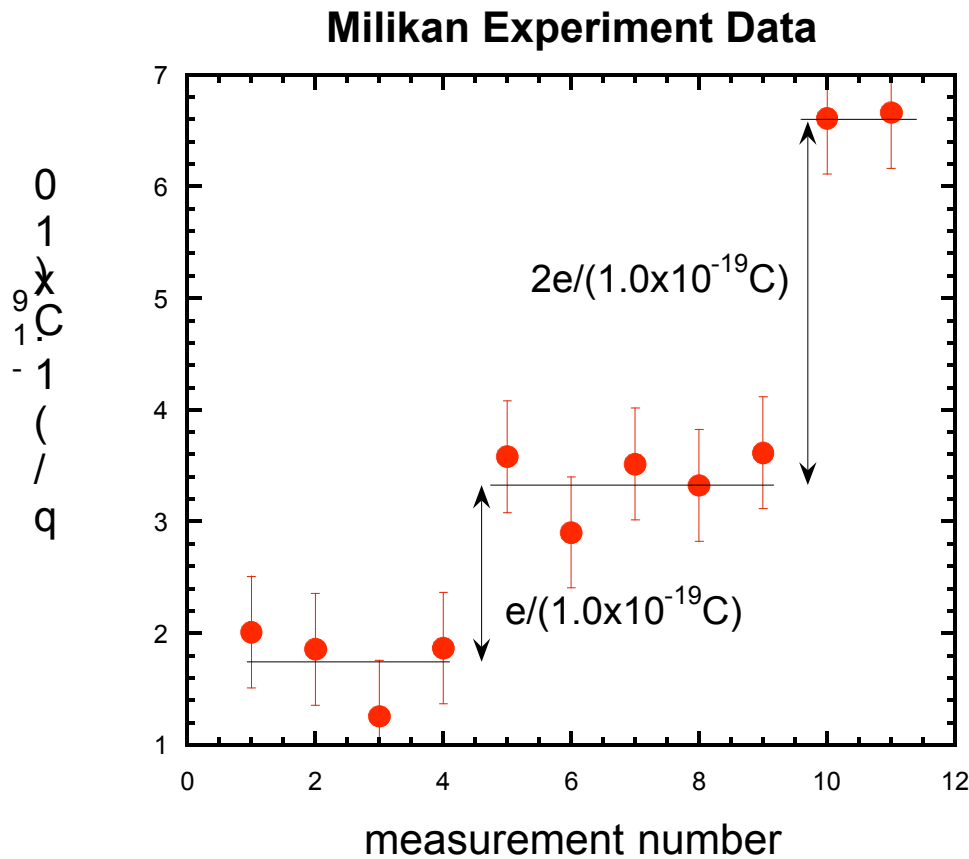
After using the apparatus, clean the spray tubing with water. If the spray is not cleaned after each use, the residue of latex in the spray tubing will harden, preventing smooth operation of the spray.

1. Ensure that the D. C. voltage leads are plugged into their respective color terminal plugs.
2. Position the 3-way polarity switch in its mid position, establishing a “no charge” condition. Turn the “on/off” switch to “on”. The illuminating lamp should light.
3. Adjust the microscope by rotating the focus-adjusting knob until an approximate mid position is established. The eyepiece divisions should be distinguishable from the background.
4. Adjust the electrode voltage to 450 volts using the voltage-adjusting knob, but keep the polarity switch in the mid position.
5. Spray latex into the apparatus and carefully look for the “dots of light” in the microscope. If, after several pumps on the atomizer bulb the latex spheres are still not visible, adjust the microscope carefully to attempt to focus in on the spheres.

Obtaining a suitable drop may require patience, for drops continue to enter the region between the plates for several seconds after spraying has stopped. Select a small drop which takes about 30 seconds to move between two large divisions in the eyepiece. Though one person can do this experiment alone, it is helpful if two work together, one taking the readings of the times of rise and fall of the latex sphere and the other recording these readings.

Record the rise time  $t^+$  and the fall time  $t^-$  of the latex sphere between two large divisions in the eyepiece (1 mm). Alongside, calculate the corresponding speeds  $s^+$  and  $s^-$ , which are merely the distance between the two large divisions divided by the time the sphere took to travel that distance.

Continue to take similar data on as many different spheres as possible but, no less than three with different charges.



Calculate the charge. In terms of the plate voltage,  $V$ , and distance between the plates,  $d$ , the charge from Eq. (II-4) becomes:

$$q = 3\pi\eta r(s^+ + s^-)d/V \quad (\text{II-5})$$

For the latex spheres in air,  $\eta = 1.8 \times 10^{-5}$ , the latex sphere radius is  $r = 5.5 \times 10^{-7} \text{ m}$ , the plate voltage is  $V$ , and the distance between the plates,  $d = 5 \times 10^{-3} \text{ m}$ .

Divide the charge on each measurement by  $1.0 \times 10^{-19} \text{ C}$ , and plot the values for  $q/(1.0 \times 10^{-19} \text{ C})$  on a graph similar to Fig. II-4. The data points should fall into groups, each group representing a different charge on the spheres. If the charge is an integer number times  $1.6 \times 10^{-19} \text{ C}$ , the groups will be bunched on the graph about that integer value times 1.6.

Draw a line through each group. Average the data in each group and use the differences between the average values for the groups to calculate  $e$ .

#### Analysis of the charge measurements

As discussed above, determine the charge on the electron,  $e$ , from your data. How do you know that this is the electron's charge and not some multiple of it? Assuming that the given constants,  $r$ ,  $\eta$ ,  $d$  have no uncertainty, perform an error analysis to determine the uncertainty in your value of  $e$ . Does it agree with the known value? In our discussion we neglected the effect of the buoyant force of air on the latex spheres. Was this justified?

**The Cleanup**

When the experiment is completed, clean the electrode housing. Turn it off and unplug the unit. Remove the electrode DC power cord from the terminal. Remove the pipe from the latex container. Loosen the setscrew of the electrode housing and remove the housing. Disassemble into electrode boards above and below rings. Wipe off any water and latex with a soft cloth. Clean the latex spray tube and put it back in place. Reassemble the housing and set it in the designated position (when assembling the intermediate ring, carefully align the objective lens of the microscope with the peep window).

***The Combined analysis***

Using the best estimate for  $e/m$  from part I of the experiment and your best estimate of  $e$  from part II, compute the mass of the electron,  $m_e$ , and the uncertainty in the mass,  $\delta m_e$ . Indicate whether or not your measurement agrees with the known value.

***The Write-up***

Include a brief statement of the purpose of the experiment, a description of the procedure and a summary of the raw data. Give examples of all calculations and intermediate results, using tables and graphs where appropriate. Give a summary of your final results for the charge and mass of the electron including an error analysis. Compare with the given values and discuss discrepancies, if any.