Experiment 2

Reaction Time

2.1 Objectives

- Make a series of measurements of your reaction time.

- Use statistics to analyze your reaction time.

2.2 Introduction

The purpose of this lab is to demonstrate repeated measurements that do not yield identical results; but this variation can give us an uncertainty (δx) on the measurements. In science, we often take measurements of the same thing multiple times and want to know how these measurements relate to each other. Today, we will be looking at your reaction time, and will try to find your average reaction time. After finding your reaction time you will find a measure of how confident you are in this value and place your reaction times into a predictable model.

2.3 Key Concepts

If you don't remember your Physics I lecture material, you may need to refer to the chapters in an introductory physics textbook. Alternatively, you can find a summary on-line at Hyperphysics.[1] Look for keywords: mean, standard deviation, gaussian distribution

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[1] http://hyperphysics.phy-astr.gsu.edu/hbase/hph.html

2.4   Theory

Two of the main purposes of this experiment are to familiarize yourself with the taking of experimental data and with the reduction of such data into a useful and quantitative form.

In any experiment, one is concerned with the measurement of some physical quantity. In this particular experiment it will be your reaction time. When you make repeated measurements of a quantity you will find that your measurements are not all the same, but vary over some range of values. As the spread of the measurements increases, the reliability or precision of the measured quantity decreases. If the measured quantity is to be of any use in further work, or to other people, it must be capable of being described in simple terms. One method of picturing measured values of a single quantity is to create a histogram.

The histogram is a diagram drawn by dividing the original set of measurements into intervals or "bins" of predetermined size, and counting the number of measurements within each bin. One then plots the frequency (the number of times each value occurs) versus the values themselves. A histogram has the advantage of visually presenting the distribution of readings or measurements. A typical histogram is shown in Fig. 2.1. It displays the number of measurements for a given reaction time. For example, the first bin has 2 measurements between 0.195 seconds and 0.200 seconds. When placing the values into bins, one systematically puts values that occur on the bin limits into the next higher bin.

When analyzing data using a histogram, the distribution often suggests that there is a "best" or most likely value, around which the individual measurements are grouped. Intuitively one might say that the best value is somehow related to the middle of the distribution, while the uncertainty is related to the spread of the distribution. The following formulas, which we will define, will in general only have significance for symmetrical distributions. Using mathematical statistical theory it turns out that the best value is nothing more than the arithmetic average or **mean** of our measurements, which we will denote with the symbol: $\overline{x}$.

$$\text{Best value} = \text{average} = \textbf{mean} = \overline{x} = \frac{\sum x_i}{N}$$

where

$$\sum x_i = x_1 + x_2 + x_3 + ... + x_N$$
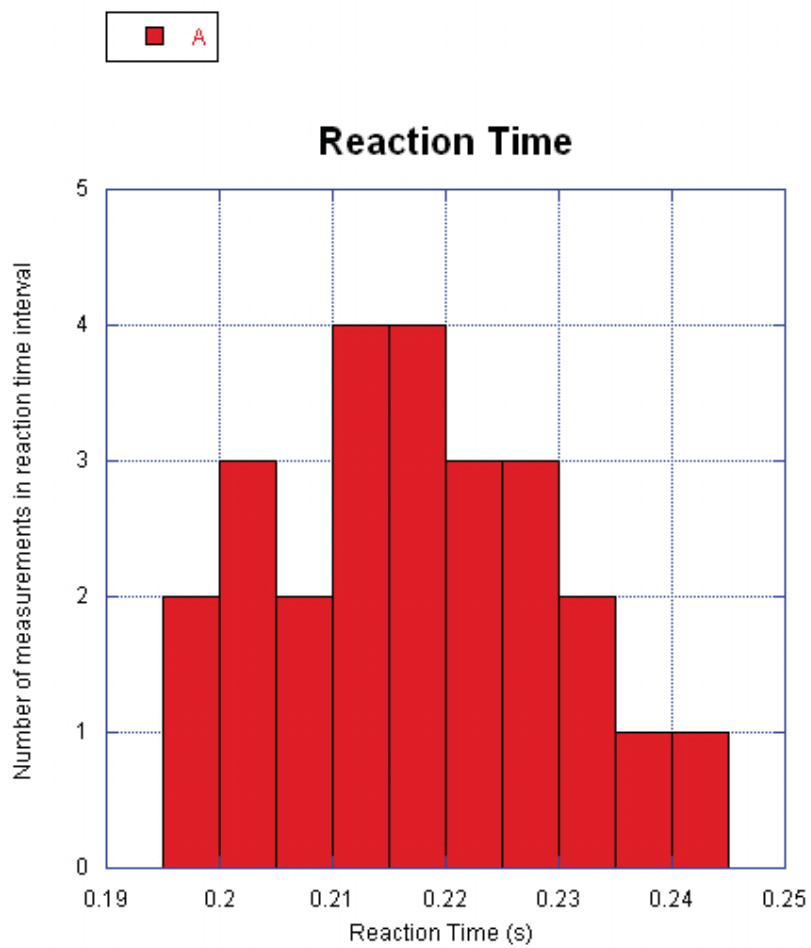
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Figure 2.1: A histogram showing 25 measurements of reaction time using bin sizes of 0.005 seconds. Note that if you add up the number of measurements (from the y-axis) in each bin you get a total of 25.

N is the total number of measurements and x_i are the values of individual measurements (i.e. x_1, x_2, x_3, etc.).

We now need to define a quantity that is connected with the width of the distribution curve. We use a quantity that tells us how the individual measurements deviate from the central (mean) value of the distribution. This is called the "**standard deviation**", denoted by "s", and is defined as follows:

$$s = \sqrt{\frac{\sum(x_i - \overline{x})^2}{N - 1}}$$

where

$$\sum(x_i - \overline{x})^2 = (x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + (x_3 - \overline{x})^2 + ... + (x_N - \overline{x})^2$$

We are also interested in the uncertainty of the mean $\overline{x}$. That is, for different sets of data, by how much does the mean ($\overline{x}$) for each set deviate from one another. This uncertainty is characterized by the width of the experimental distribution of values of the mean $\overline{x}$ called the "**standard deviation of the mean**", denoted by s_m, and is calculated by:

$$s_m = \frac{s}{\sqrt{N}}$$

Note: The larger the number (N) of measurements made of a quantity the smaller the random uncertainty associated (or s_m) with the mean value ($\overline{x}$).

If the number of readings is very high and the bins are small, the histogram approaches a continuous curve and is called a "distribution curve". Many theoretical distribution curves have been defined and their properties evaluated, but the one that is most significant in the theory of measurement is the Gaussian or "Normal" distribution. If all of the experimental data you obtain corresponds to one physical quantity, then for a very large number of measurements they will be described by the Gaussian distribution with a peak at the average (aka mean) value $\overline{x}$.

Some of the properties of this continuous distribution are that it is symmetric around a peak value and that it falls to zero on either side of the peak, giving it a "bell shaped" appearance (see Fig. 2.2). We use the Greek letter sigma "σ" to represent the standard deviation when referring to a
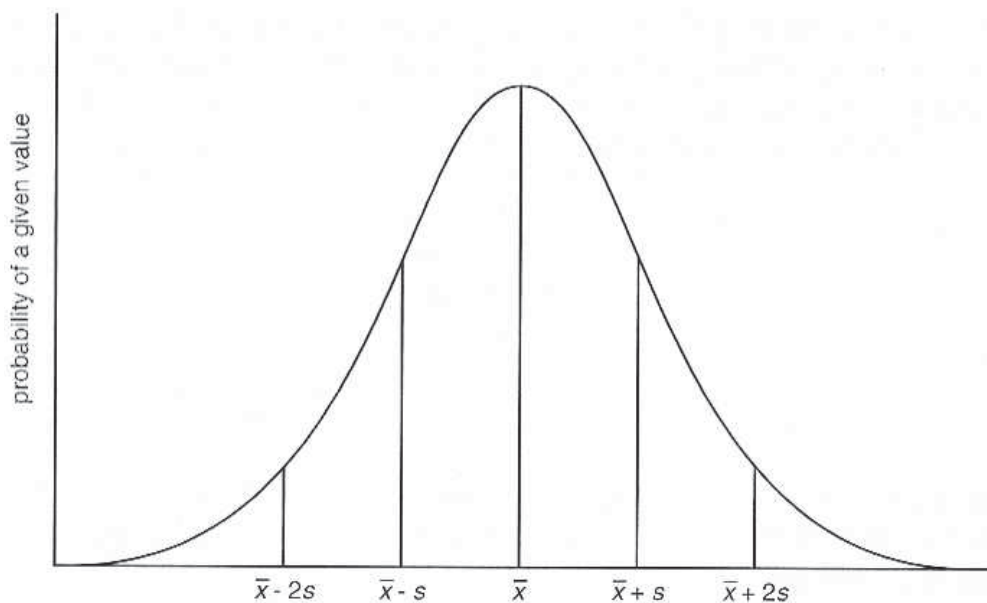
Figure 2.2: A Gaussian distribution curve with the mean $\overline{x}$, $\pm s$ (σ) and $\pm 2s$ (2σ) points labelled.

Gaussian distribution and "s" for the standard deviation calculated from a **finite** (limited) sets of observations ("s" is the best estimate of "σ" for a finite set of observations). When considering Gaussian distributions, the area enclosed by the range $\pm\sigma$ around the peak will contain 68% of the area of the curve (or 68% of the measurements). This means that an individual measurement has a 68% chance of falling within a region $\pm\sigma$ around the peak, or "mean" value, of the distribution. An area bounded by the range $\pm 2\sigma$ will contain 95% of the area of the curve and therefore represent a 95% chance that an individual measurement will fall within this region of the distribution. This is illustrated in Fig. 2.2.

2.5 In today's lab

In today's lab, you will be measuring your own reaction time and will use the above statistical formulae to hopefully create a Gaussian distribution

of your reaction times. There should be sufficient time available to collect data and do the complete lab for both yourself and your partner.

2.6   Equipment

- Stop watch. - To run the stop watch, press START to start and press STOP or the red button to stop. In this lab, we will start the timer using the red remote start button. After getting your measurement, press the RESET button to return the timer to zero. It should be noted that you can increase the precision of the timer by holding the STOP button for 2 seconds. After increasing the precision of the timer, the smallest increment of measurement will go from 0.001 seconds to 0.0001 seconds. To go back, simply hold STOP again for 2 seconds.

Figure 2.3: The stopwatch used for this experiment.

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2.7 Procedure

Note: Before starting, please practice steps 1–3 a few times before recording your data.

1. Put your finger on the `STOP` button while your partner takes the red `START` button in the wired remote.

2. The partner with the `START` button will secretly start the timer.

3. Try and stop the clock as quickly as possible.

4. Record your time in the Time column of the ".xls" spreadsheet in the Reaction folder and reset the timer.

5. Repeat steps 1–4 25 times.

6. On a separate sheet of paper, calculate (**by hand**) the values $\bar{x}$, s, and s_m for $N = 5$ trials. Be sure to show your work!

7. On the spreadsheet, calculate the mean by putting the equation "`=SUM(B12:B16)/5`" in cell `C19`. Note that the "SUM" function can be used to find the sum of a group of numbers, and that B12:B16 will evaluate the sum from cell B12 to B16 (B12, B13, B14, B15, B16). The mean is simply the sum divided by the number of values in that sum (in this case we have 5 values). Make sure this value matches the number you calculated by hand.

8. Now fill in column C using the formula "`=(B12-$C$19)`" in the cell C12 and fill down. The use of $ in front of C and in front of 19 "locks" in the cell that has the mean value so that when you fill down, that cell will not change in the formula. For example, when using the fill down feature in excel, the next cell would have the equation "`=(B13-$C$19)`", and so on.

9. Fill in column D by putting "`=C12^2`" in cell D12 and fill down again. Note how this calculated value is used in the formula for standard deviation "s".

10. On the spreadsheet, calculate the standard deviation in the appropriate cell by using the formula "`=SQRT(SUM(D12:D16)/4)`". Here we have that $N = 5$, so our denominator $N - 1 = 4$. Make sure this value matches the number you calculated by hand!

11. Now calculate the standard deviation of the mean in the appropriate cell by using the formula "=C21/SQRT(5)". Make sure this value matches the number you calculated by hand.

12. Using the methods above and the equations from the Theory section of this lab, fill in the remaining cells on the excel sheet. You do **NOT** need to do hand calculations for N=10 and N=25! Your mean and standard deviation of the mean should change as you add more trials to its calculation, but the standard deviation should remain about the same.

13. Record your standard deviation in the box below for future reference. You will need them in a later lab.

$$s = \delta t = \underline{\hspace{2cm}}$$

14. Transfer your data from column B into KaleidaGraph and plot a histogram. Do this by going to `Gallery ▶ Stat` and select `Histogram`.

15. Adjust the range of values shown on the x-axis such that the minimum is a few hundredths lower than your lowest measured time and the maximum is a few hundredths greater than your greatest measured time. Do this by going to `Plot ▶ Axis Options ▶ Limits`, and enter the correct values in their respective boxes.

16. Change the number of bins such that your histogram looks similar to the one shown in Fig. 2.1. Do this by going to `Plot ▶ Plot Options ▶ Histogram ▶ Specifying the Number of Bins`, select `Fixed`, input an appropriate integer number, and press `OK`. Make sure most of the bins are filled in so that there are not many gaps in your histogram.

17. Make sure your histogram is properly labeled and print.

18. Please label **by hand** the positions of $\bar{x}$, $\bar{x} + s$, and $\bar{x} - s$ on your histogram.

2.8 Comparing Data

In just about every experiment in this course you will be asked if two quantities are compatible (or consistent). The following describes how to determine if two pieces of data are consistent (or compatible). Use this procedure to answer the last question of this lab and then use it as a reference whenever you are asked if two pieces of data are compatible or consistent.

Let us denote the pieces of data by d_1 and d_2. We'll arbitrarily set d_2 as our expected value, denoted by e, and d_1 as the data, denoted by d. If $d = e$ or $d - e = 0$, then they are clearly compatible but we want a mathematical test for compatibility. To do that we will use the formulas described below. We often use D to denote the difference between two quantities:

$$D = d - e \tag{2.1}$$

This comparison must take into account the uncertainties in the observation of both measurements. The data values are $d \pm \delta d$ and $e \pm \delta e$. To perform the comparison, we need to find δD. δD is the uncertainty for the difference between d and e as shown in Formula 2.1. The addition/subtraction rule for uncertainties is:

$$\delta D = \delta d + \delta e \tag{2.2}$$

We are interested in knowing if the difference (D) between the 2 data points is smaller than the uncertainty in the difference (δD). This is expressed by:

$$|D| \leq \delta D \tag{2.3}$$

In Fig. 2.4 we visually demonstrate three possible cases (A, B, and C) involving consistency checks. For all three cases, the value for $d \pm \delta d$ is 11 ± 3 ($d = 11$ and $\delta d = 3$) but the value for e changes ($e = 8, 6, 5$ respectively), while the value for δe remains the same ($\delta e = 3$). The values for d and e are represented by the circles while their uncertainties, δd and δe, are represented by the bars. We say the measurements d and e are compatible if their error bars touch or overlap. In case A the values are consistent as their error bars overlap. Case B is also consistent as their error bars touch. The values are inconsistent in Case C as their error bars do not overlap or touch.

Equations 2.2 and 2.3 express in algebra the statement that "d and e are compatible if their error bars touch or overlap" (see Fig. 2.4). The combined length of the error bars is given by Eq. 2.2 and $|D|$ is the separation between
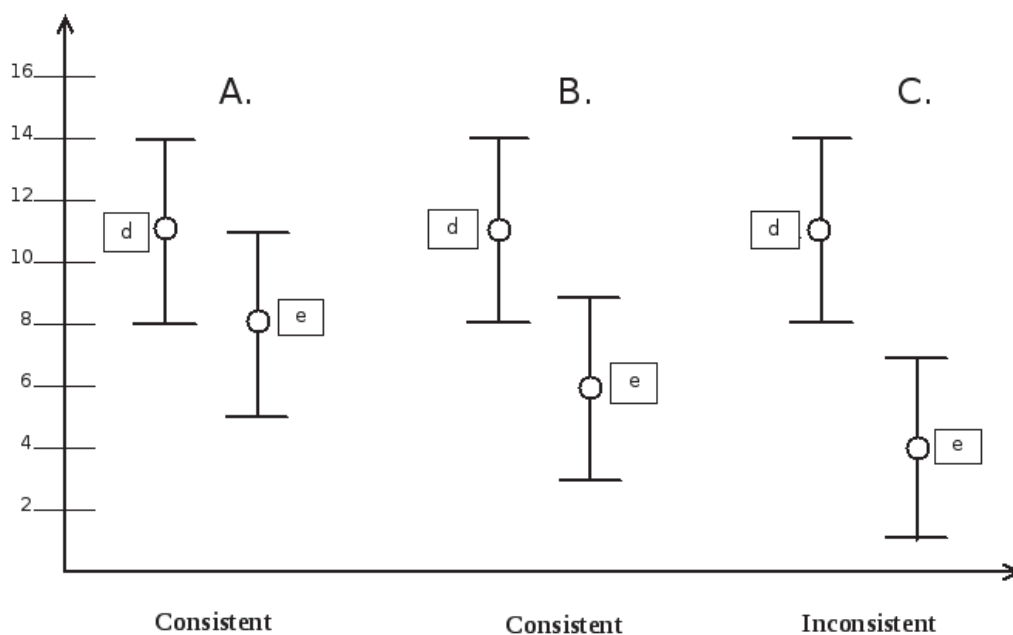
Figure 2.4: Visual representation of a consistency check.

d and e. The error bars will overlap if d and e are separated by less than the combined length of their error bars, which is what Eq. 2.3 says. Using Fig. 2.4 and the given equations, we can see that $D = 3, 5, 7$ respectively and $\delta D = 6$ for all cases. We can then see that $|D| \leq \delta D$ for cases A and B, so they are consistent. However, for case C, we can see that $|D| \not\leq \delta D$, so the values are not consistent. For more information on using uncertainties to compare data see Appendix B.

2.9   Checklist

1. The filled Excel spreadsheet plus the formula view.

2. The histogram.

3. Calculations done by hand for mean, standard deviation and standard deviation of the mean for N=5.

4. Answers to questions 1-5.

2.10 Questions

1. For all of your 25 measurements indicate on your spreadsheet whether or not each measurement lies between $\overline{x} \pm s$. How many trials would you have expected to be within that range for a pure Gaussian distribution? How many of your trials were in that range for your distribution? How does your distribution of measurements compare to a Gaussian distribution? If you made many sets of 25 trials of your reaction time, would there always be the same number of trials in that range?

2. Suppose your lab partner was talking to the students at an adjacent lab table when you started the timer. As a result, the time registered on the timer when it was stopped was 10 seconds. How many standard deviations (s) from your mean value does this represent? Should you include this data point with the rest of your data? Why or why not?

3. Compare the mean, the standard deviation and the standard deviation of the mean for $N = 10$ with those for $N = 5$ and $N = 25$. Are the values the same? Why or why not? Explain.

4. If you have already taken 25 measurements, how many more measurements of reaction time would you have to take to reduce s_m by a factor of two, assuming s does not change? Justify your response.

5. Two red blood cell counts are $(4.52 \pm 0.14) \times 10^6 \frac{\text{cells}}{\text{cm}^3}$ and $(4.84 \pm 0.18) \times 10^6 \frac{\text{cells}}{\text{cm}^3}$. Would you conclude that these measurements are consistent with being from the same human? Evaluate the difference and comment. (Use the formulas outlined in the Comparing Data section). Would your answer change if the second blood cell count is $(4.87 \pm 0.18) \times 10^6 \frac{\text{cells}}{\text{cm}^3}$?