Physics 294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Help-room hours: 12:40-2:40 Monday (note change); 3:00-4:00 PM Friday**
  - **36.73 hand-in problem for next Wed**
- Quizzes by iclicker (sometimes hand-written)
- Average on 2^{nd} exam (so far)=71/120
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

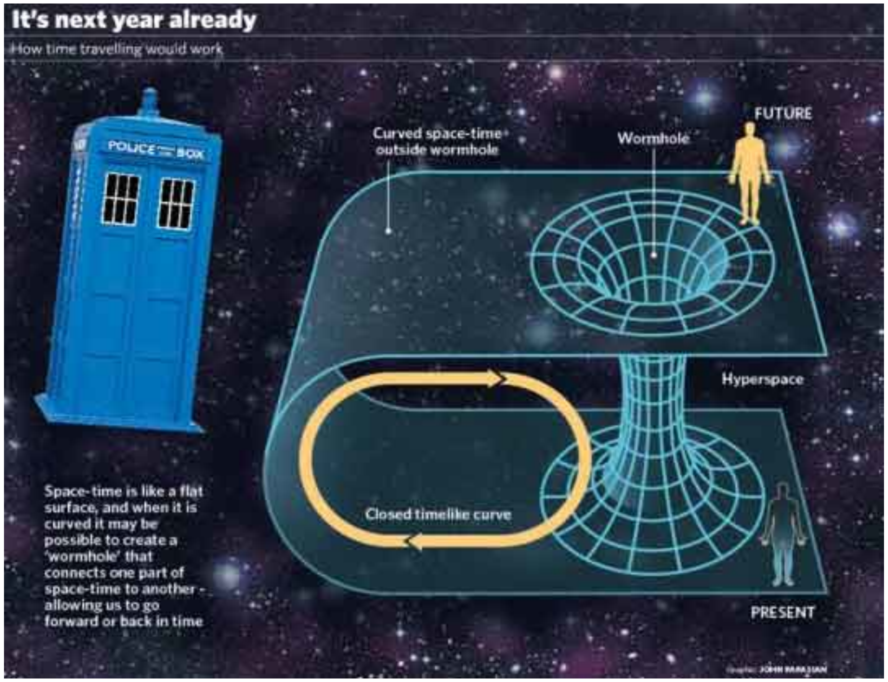
Einstein field equations

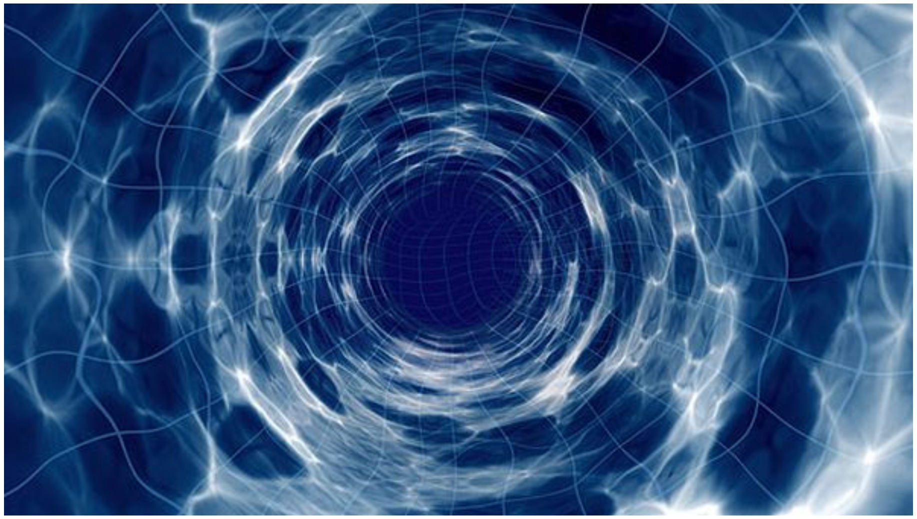
$$G_{ab} + \Lambda g_{ab} = \frac{8\pi G}{c^4} T_{ab}$$

- The Einstein field equations*:* The Einstein field equations (EFE) are the core of general relativity theory. The EFE describe how mass and energy (as represented in the stress-energy tensor) are related to the curvature of spacetime (as represented in the Einstein tensor). In abstract index notation, the EFE reads as shown above where G_{ab} is the Einstein tensor, Λ is the cosmological constant, c is the speed of light in a vacuum and G is the gravitational constant, which comes from Newton's law of gravity.The solutions of the EFE are metric tensors. The EFE, being non-linear differential equations for the metric, are often difficult to solve. The usual strategy is to start with an ansatz (or an educated guess) of the final metric, and refine it until it is specific enough to support a coordinate system but still general enough to yield a set of simultaneous differential equations with unknowns that can be solved for. Metric tensors resulting from cases where the resultant differential equations can be solved exactly for a physically reasonable distribution of energy-momentum are called exact solutions. Examples of important exact solutions include the Schwarzschild solution and the Friedman-Lemaître-Robertson-Walker solution.

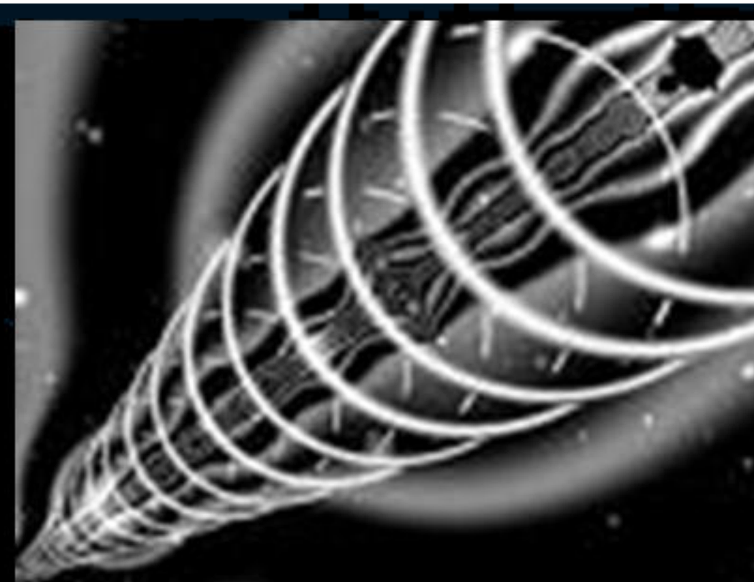
Einstein-Rosen bridge

- **A**lthough they may seem more the stuff of science fiction than science fact, physicists first dreamed up the idea of wormholes. In 1935, Albert Einstein and Nathan Rosen realized that general relativity allows the existence of "bridges," originally called Einstein-Rosen bridges but now known as wormholes. These space-time tubes act as shortcuts connecting distant regions of space-time. By journeying through a wormhole, you could travel between the two regions faster than a beam of light would be able to if it moved through normal space-time. As with any mode of faster-than-light travel, wormholes offer the possibility of time travel.





Tipler Cylinder



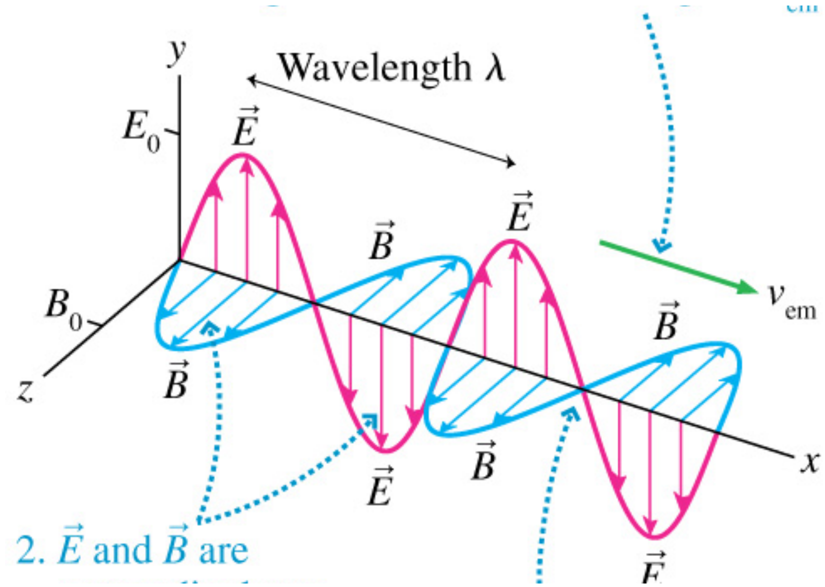
Tipler Cylinder: uses a massive and long cylinder spinning around its longitudinal axis.

The rotation creates a frame-dragging effect and fields of closed timelike curves traversable in a way to achieve subluminal time travel to the past.

Reminder: wave equations

- Let's play around with these equations

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \frac{\partial B_z}{\partial x} = -\varepsilon_o \mu_o \frac{\partial E_y}{\partial t}$$



- Take extra derivative

$$\frac{\partial^2 B_z}{\partial t \partial x} = -\varepsilon_o \mu_o \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial t \partial x} = -\frac{\partial^2 E_y}{\partial x^2}$$

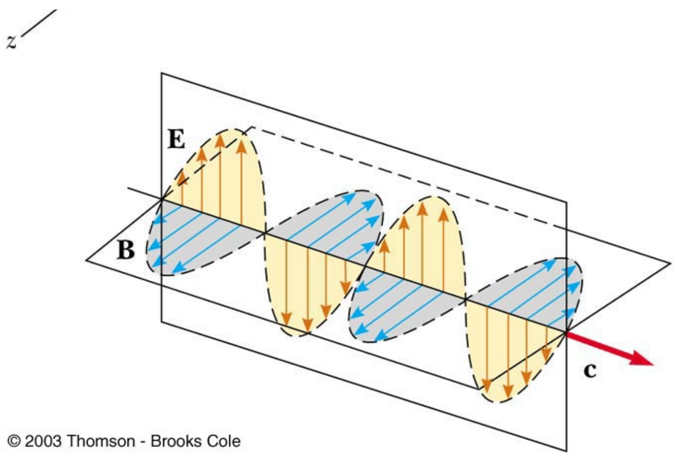
$$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_o \mu_o \frac{\partial^2 E_y}{\partial t^2}$$

wave equations for E and B fields

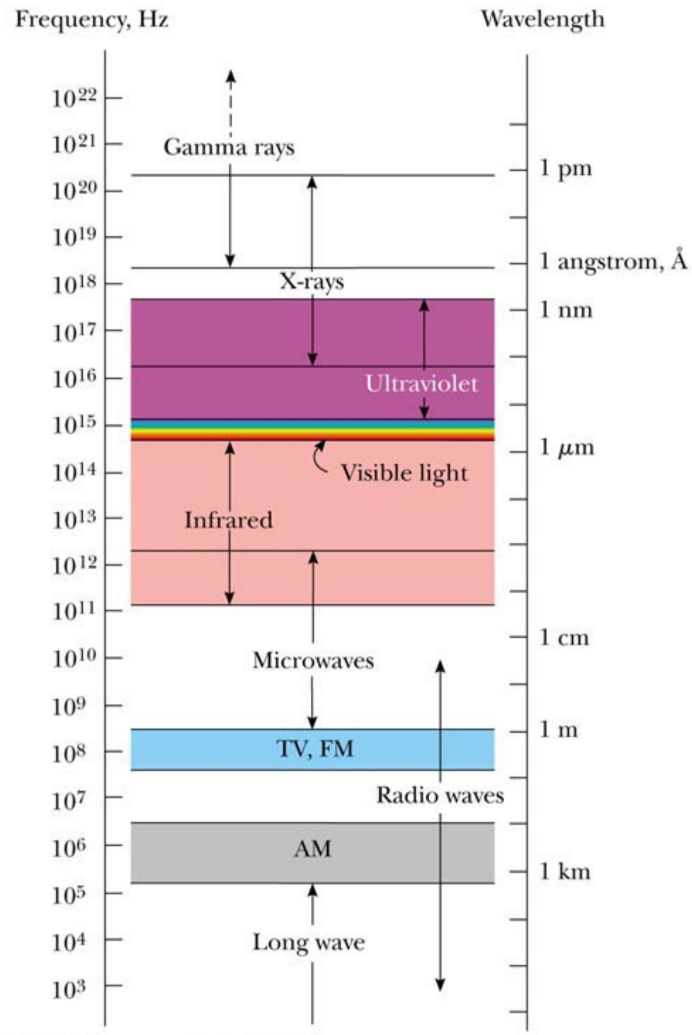
$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}} \qquad \boxed{\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 B_z}{\partial t^2}}$$

Electromagnetic spectrum

- All electromagnetic waves travel through vacuum with a speed c (3 X 10^8 m/s)
- For all EM waves, $c=\lambda f$ (true for any type of wave)
- $\lambda = c/f$



- The visible portion of the spectrum forms a tiny portion of the total EM spectrum



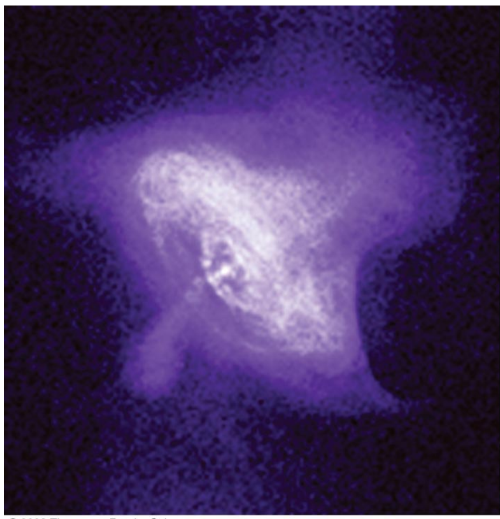
And now for something not so completely different

- Now that we've discovered that light is an electromagnetic wave moving at a speed of 3 X 10^8 m/s, we're ready to begin the study of optics, i.e. the behavior of light
- First, there's the question of the nature of light
  - is light a stream of particles, or is it a wave?
- Some people (Isaac Newton, for example) were fond of the particle theory of light
- Others, such as Christian Huygens thought that light had to be a wave (and we saw how Maxwell explained light as an EM wave)
- Who's right? Both. Light can behave both as a wave and as a stream of particles depending on what phenomenon you're looking at.
  - It's our problem that we insist on trying to classify it as one thing or another.
- In fact, we can divide optics into two branches
  - geometric optics: stream of particles idea works just fine
  - physical optics: definite wave-like properties, stream of particles concept doesn't work at all

Views of Crab Nebula

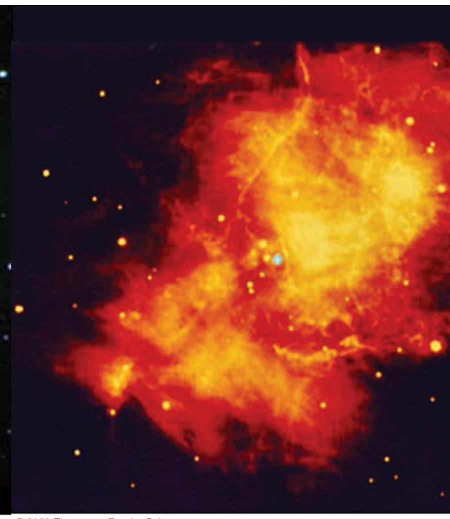
increasing energy ($E = hc/\lambda = hf$) (more particle-like)

X-ray

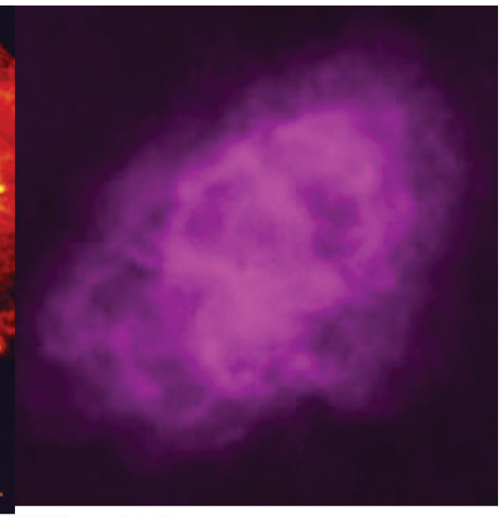


optical

infra-red



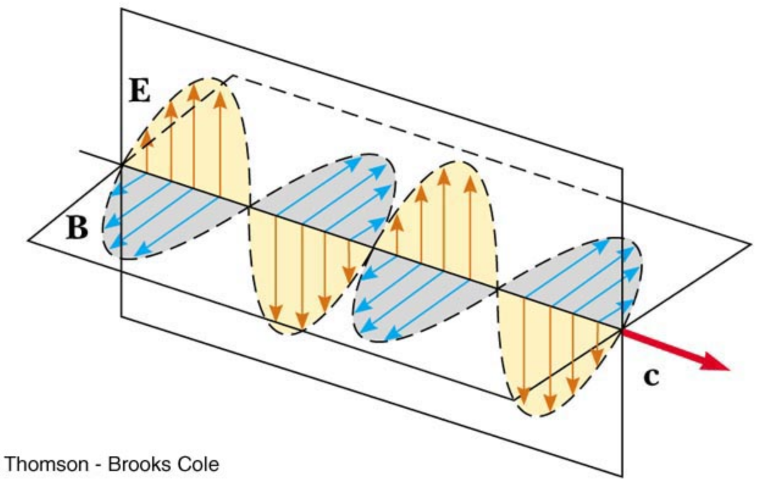
radio

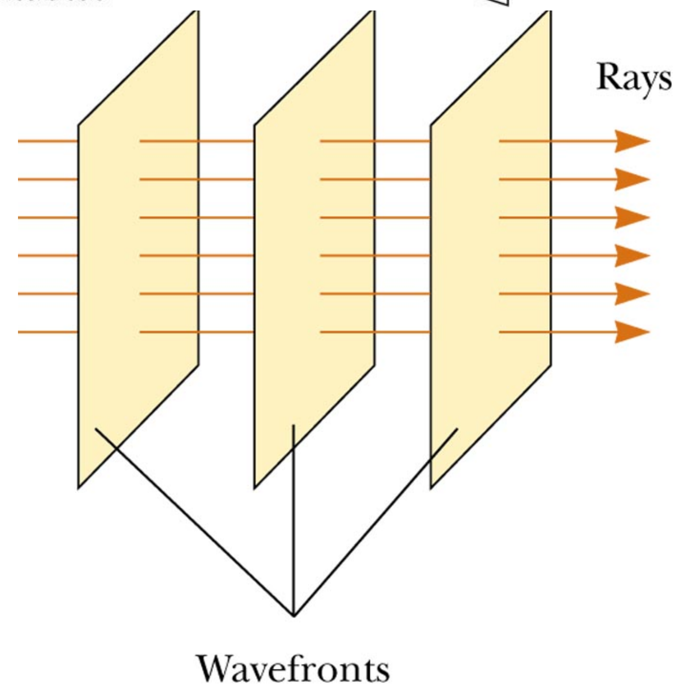


increasing wavelength (more wavelike)

Let’s start with geometric optics (it’s easier)

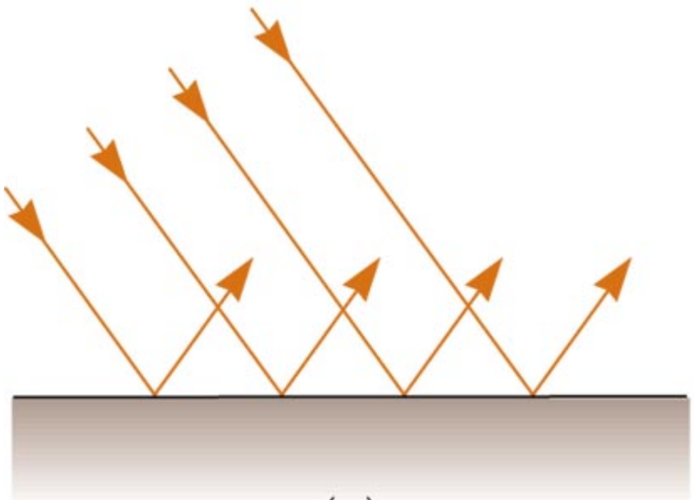
- Light travels in a straight line (as long as it’s travelling through a homogenous medium)
- I can represent a light wave by
  - (1) drawing the wavefronts (surfaces where the electric field has the same phase)
  - (2) drawing ray(s) perpendicular to the wavefronts that indicate the direction that the wave is travelling



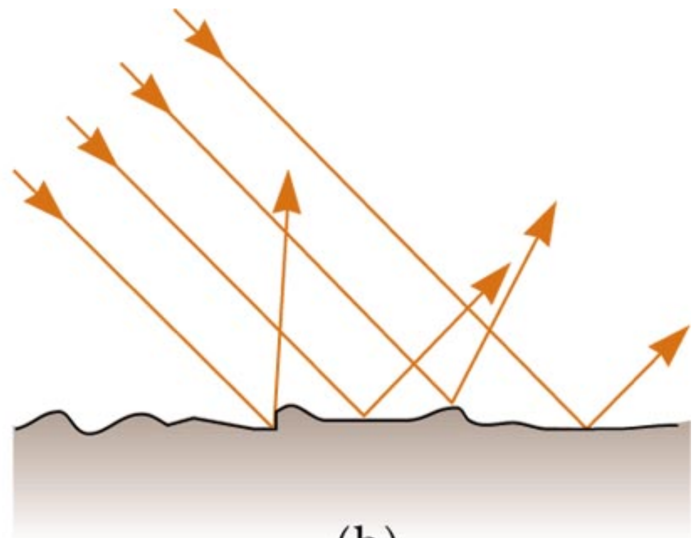


Reflection

When a light ray hits a surface of a medium with a different index of refraction, part of the light will be reflected.



(a)

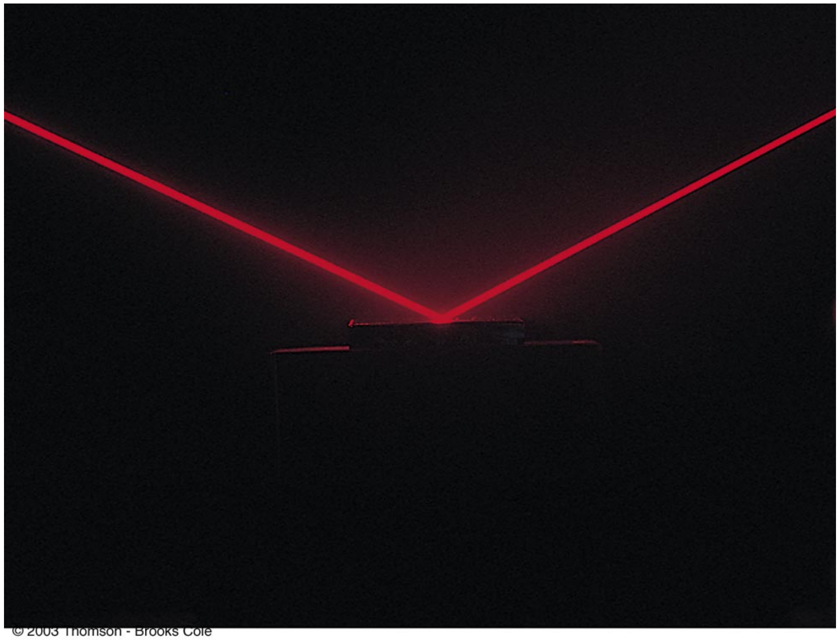


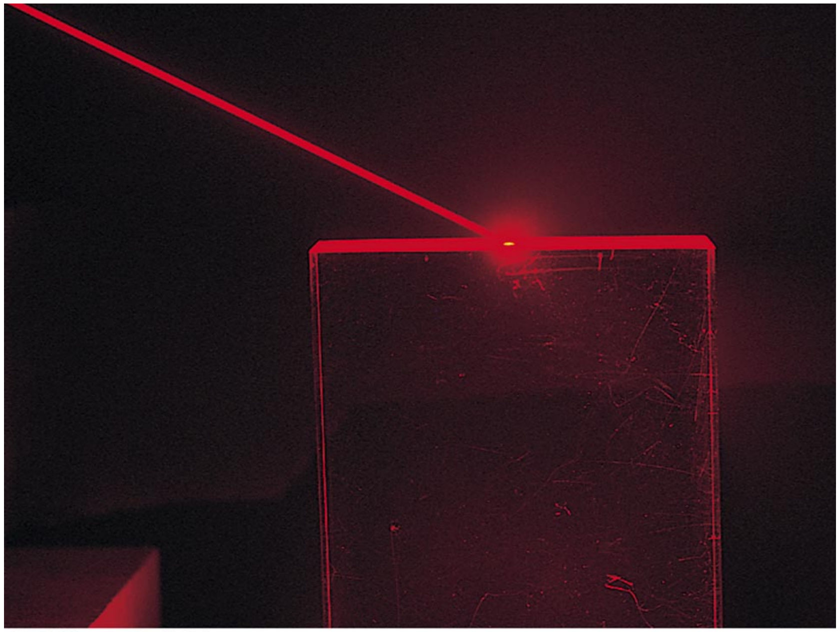
(b)

Specular reflection, from a smooth surface

Diffuse reflection, from an irregular surface

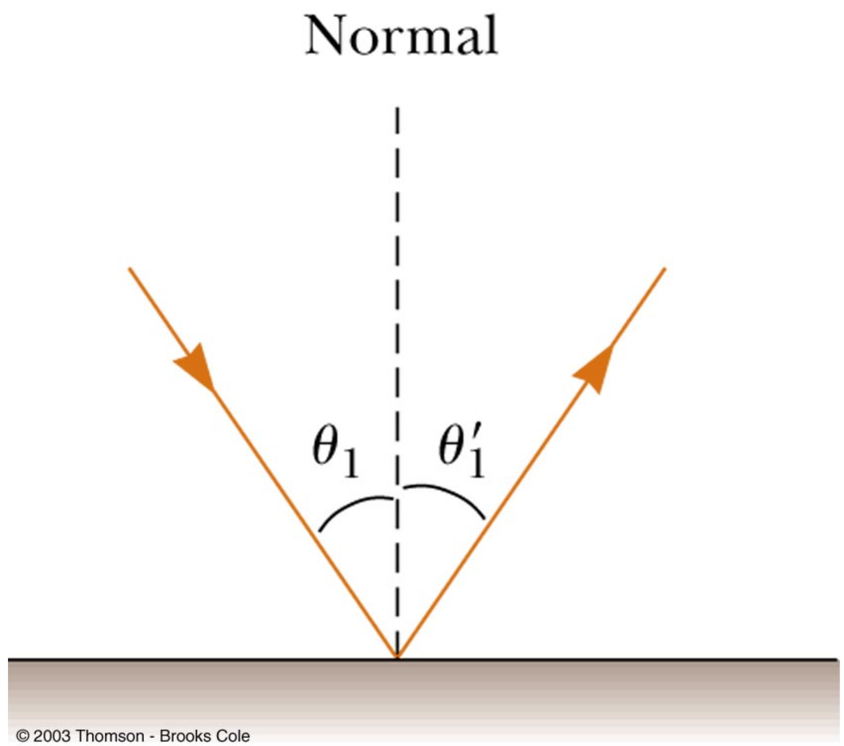
Specular and diffuse reflection



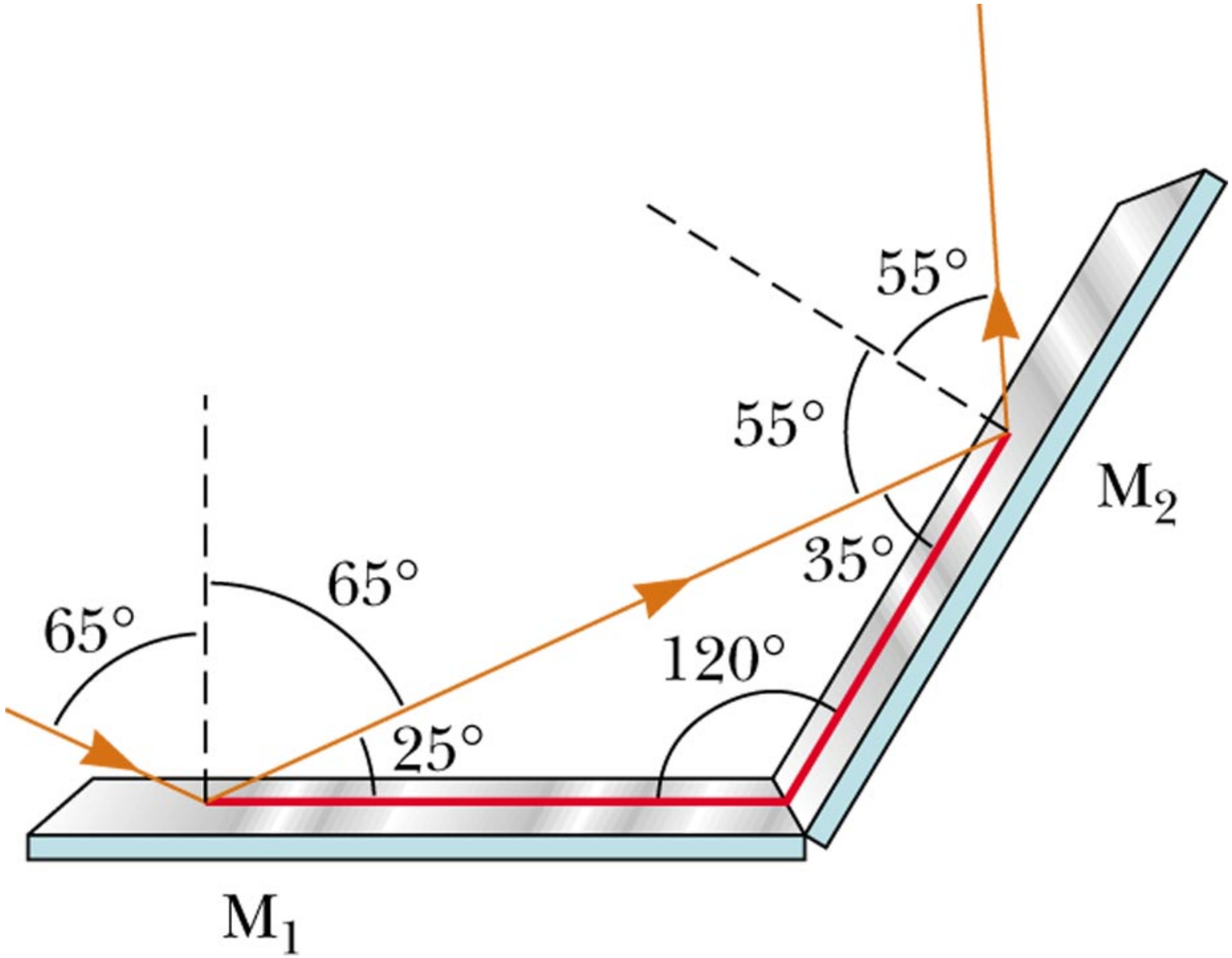


Reflection

- Draw a line normal (perpendicular) to the surface
- Consider the angle between the incident ray and the reflected ray
- $\theta_1 = \theta_1'$
- The angle of reflection is equal to the angle of incidence
  - just what you'd expect if light were a stream of particles

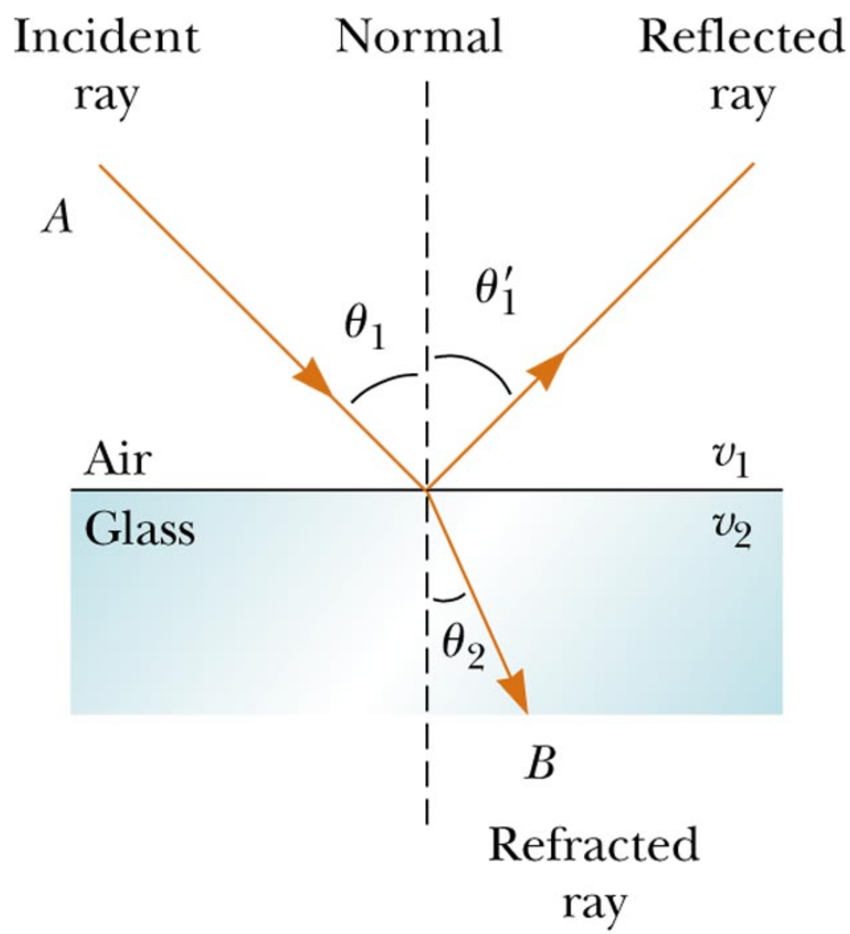


True, no matter what angle of surface



Reflection and refraction

- Not all of the light is reflected at the surface
- Some of it is transmitted (refracted) into the 2nd medium
- Note that the refracted angle is not equal to the incident angle
- In fact, the angles are related to the velocities of light in the two media
  - $\sin \theta_1 / \sin \theta_2 = v_2 / v_1$

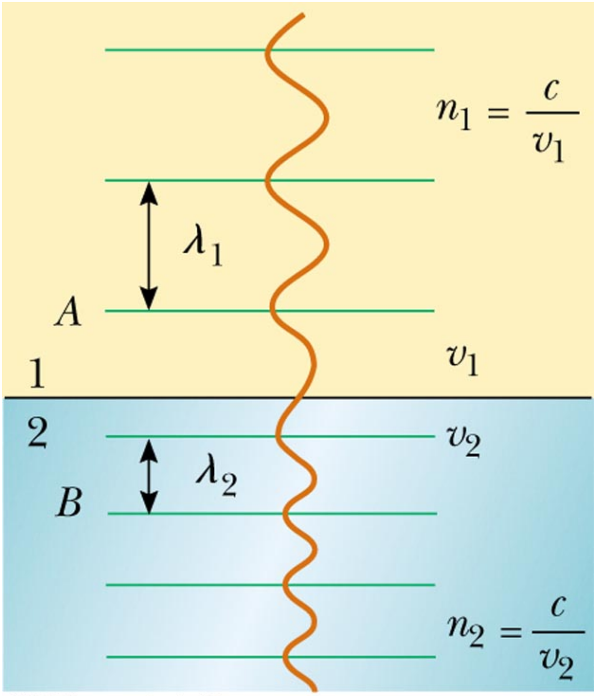


(a)

Snell's law

- Light travels at a speed c in vacuum, but slower in other media
- Define n = c/v, where v is the speed that light travels in a given medium (glass, water, etc)
- So, as light travels from air to glass, its speed changes; its frequency does not, so its wavelength must
  - $\lambda_1 = \lambda_o/n$
- What determines n?
  - a material with a dielectric constant κ has an index of refraction

$$n = \sqrt{\kappa} \qquad \varepsilon_o \rightarrow \kappa\varepsilon_o$$

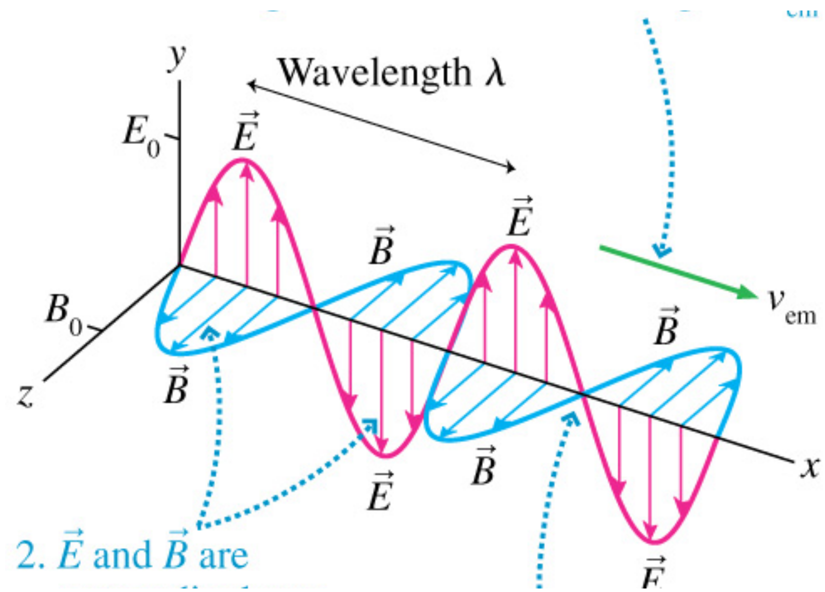


- the permittivity and permeability inside matter differ from those in free space
  - although we almost always talk about the permittivity changing and not the permeability changing

Back to wave equations: now in a dielectic medium

- Let's play around with these equations

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \frac{\partial B_z}{\partial x} = -\kappa\varepsilon_o\mu_o\frac{\partial E_y}{\partial t}$$



- Take extra derivative

$$\frac{\partial^2 B_z}{\partial t\partial x} = -\kappa\varepsilon_o\mu_o\frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial t\partial x} = -\frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \kappa\varepsilon_o\mu_o\frac{\partial^2 E_y}{\partial t^2}$$

wave equations for E and B fields

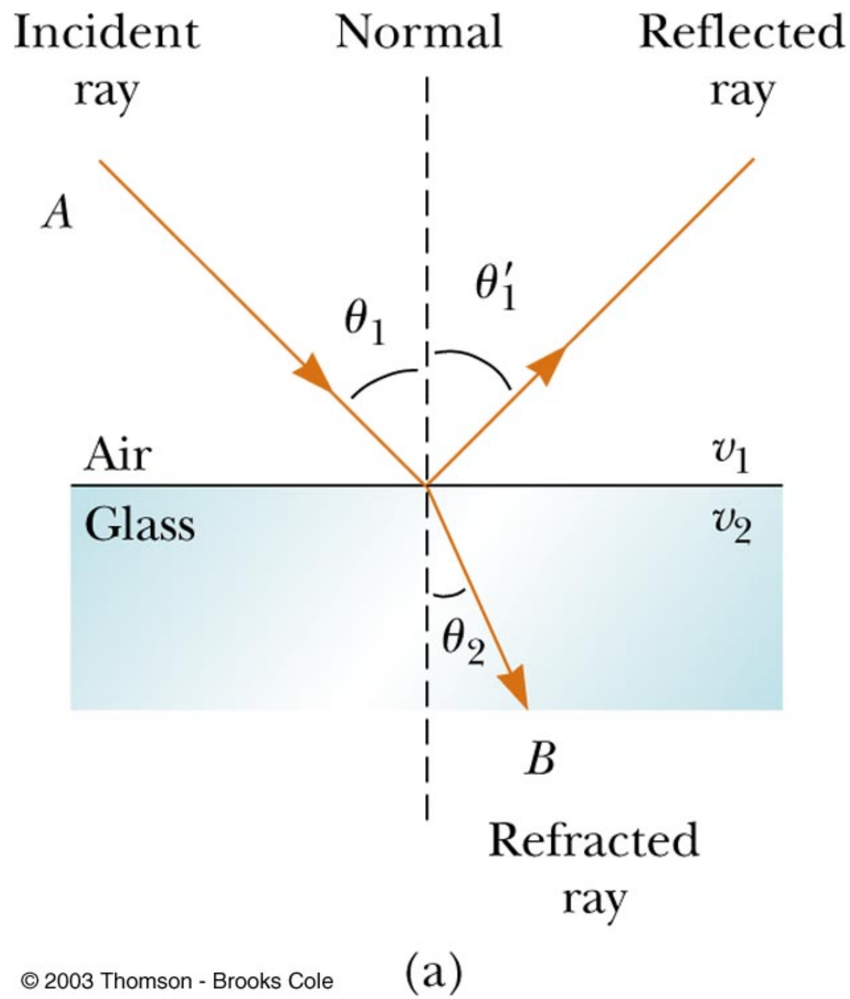
$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 B_z}{\partial t^2}$$

$$v = \frac{1}{\sqrt{\kappa\mu_o\varepsilon_o}} = \frac{c}{\sqrt{\kappa}}$$

$$n = \sqrt{\kappa}$$

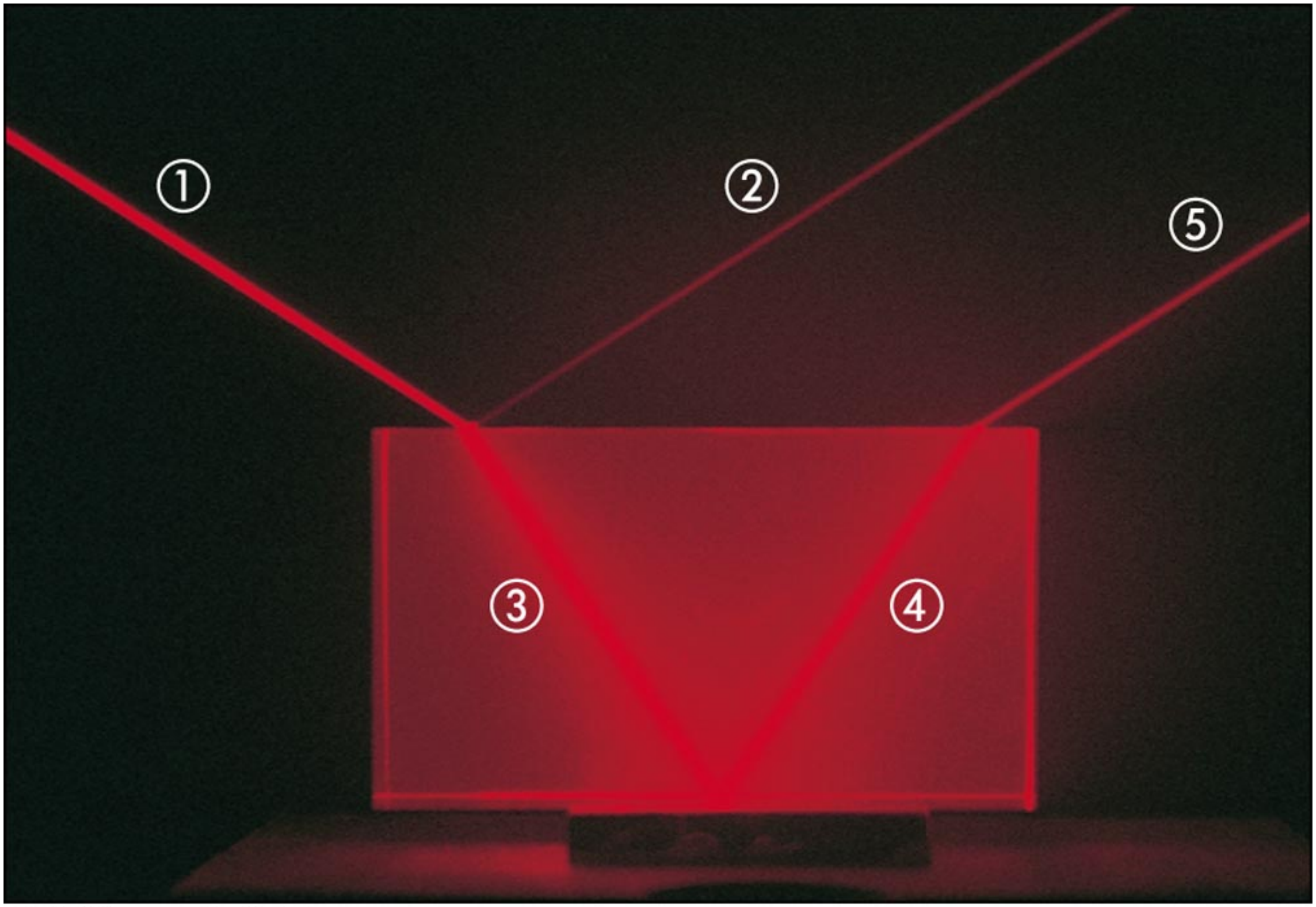
Snell's law

- Light travels at a speed c in vacuum, but slower in other media
- Define $n = c/v$, where v is the speed that light travels in a given medium (glass, water, etc)
- So, as light travels from air to glass, its speed changes; its frequency does not, so its wavelength must
  - $\lambda_1 = \lambda_o/n$
- I can re-write the proportionality that we had before
  - $\sin\theta_1/\sin\theta_2 = v_2/v_1$
- as
  - $\sin\theta_1/\sin\theta_2 = n_2/n_1$
  - $n_1 \sin\theta_1 = n_2 \sin\theta_2$



Snell's law

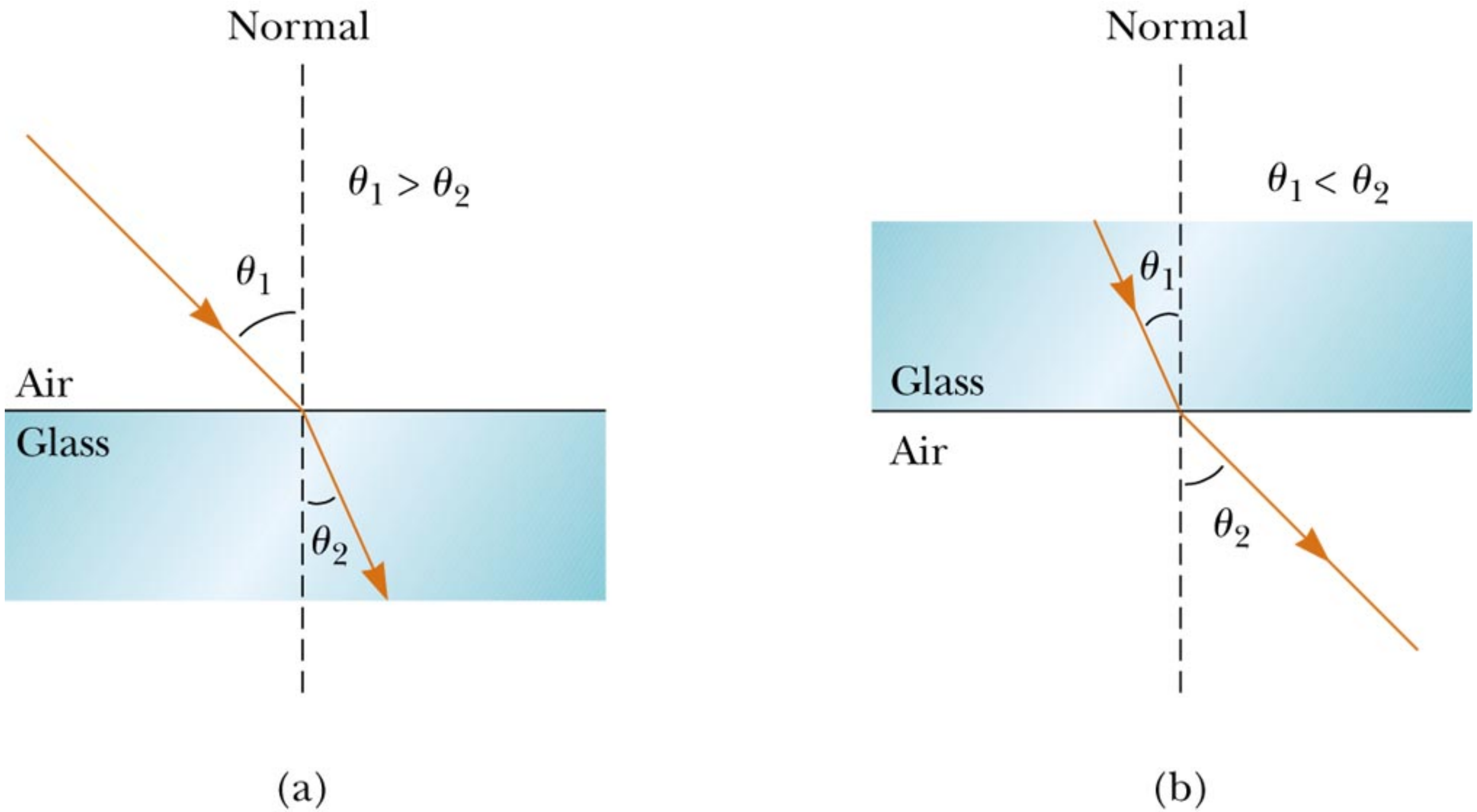
Reflection and refraction



Refraction

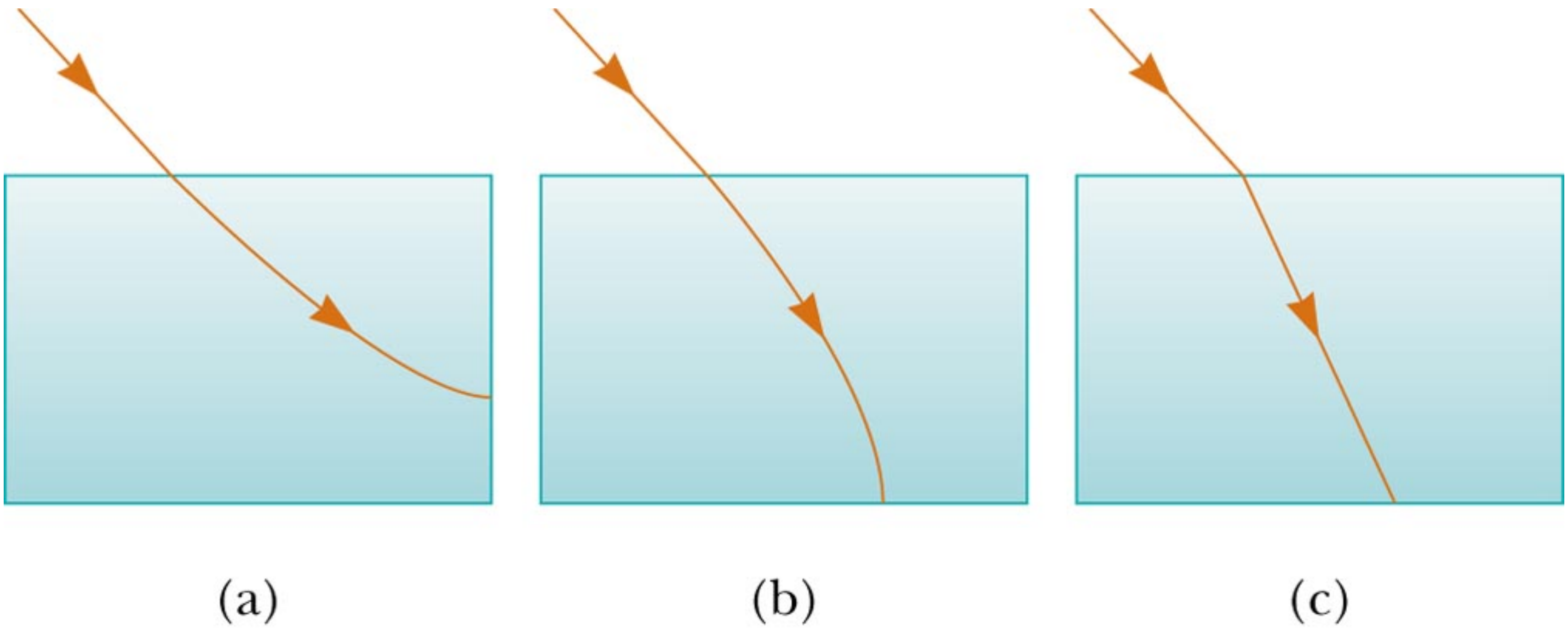
…bend in

…bend out



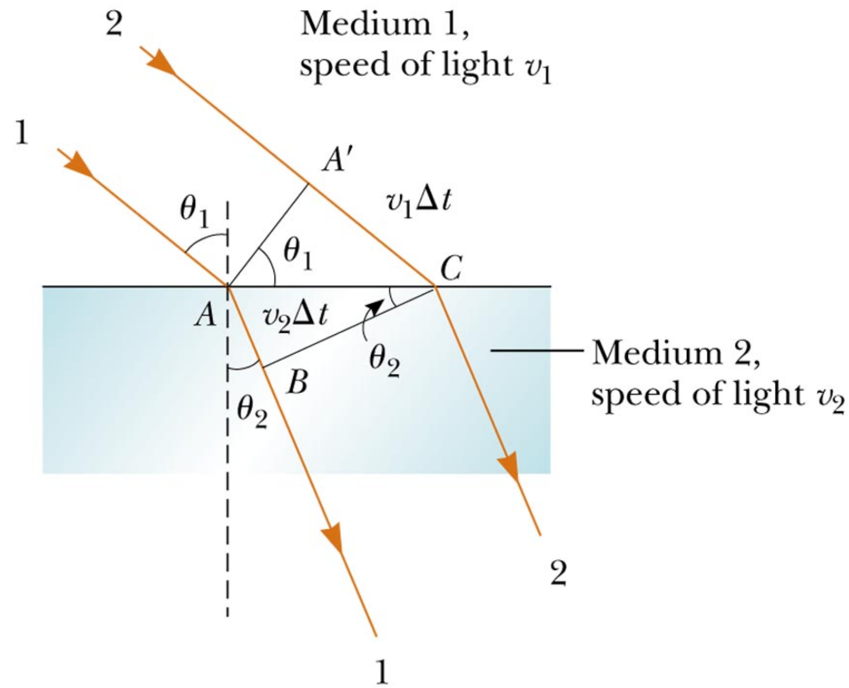
iclicker question

Which one of the following represents a light wave travelling through a medium in which n gets larger as the depth increases?

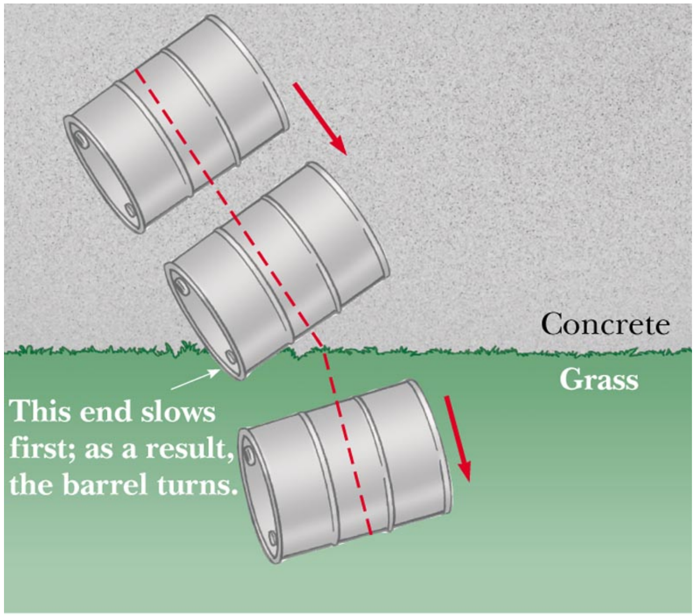


Refraction

- When light goes from air to glass, it bends towards the normal
  - because it travels more slowly in glass than in air



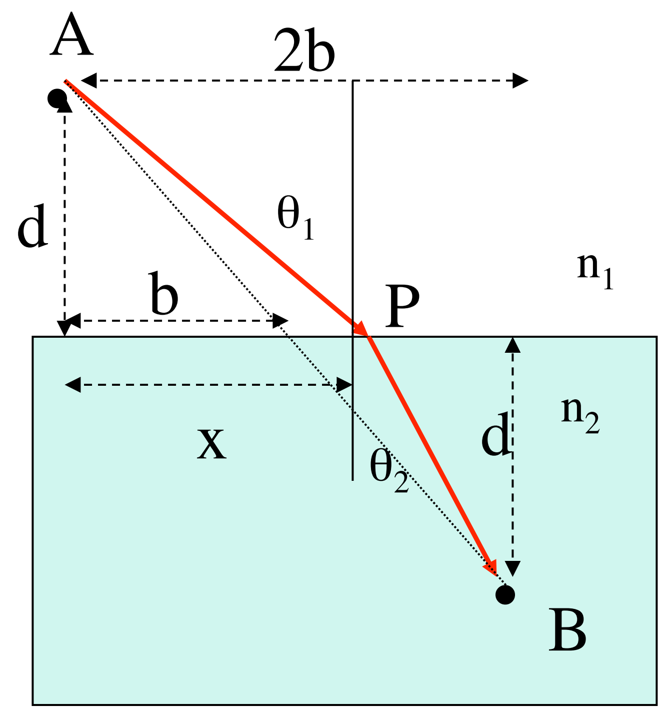
(a)



(b)

Snell's law from Fermat's principle

- Fermat's principle
  - the path of a ray of light between two points is the path that minimizes the travel time
  - Point A is in a medium with index of refraction n_1 and B is in a medium with index of refraction n_2
  - what path between A and B takes the least amount of time?
- A is a distance d above the interface and B is a distance d below
  - A and B are a horizontal distance 2b apart
  - x is the horizontal distance from A to where the light enters the 2nd medium



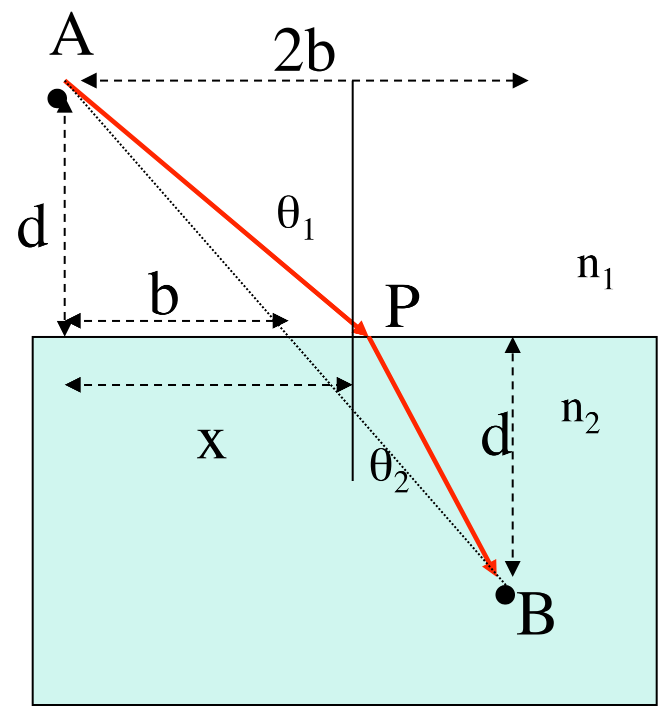
Snell's law from Fermat's principle

- The distance from A to P is given by
  $$\sqrt{d^2 + x^2}$$
- The distance from P to B is given by
  $$\sqrt{d^2 + (2b - x)^2}$$
- The total travel time for the light is

$$t_{AB} = t_{AP} + t_{PB} = \frac{n_1\sqrt{d^2 + x^2} + n_2\sqrt{d^2 + (2b - x)^2}}{c}$$

- Set the derivative of t_{AB} to be zero
  $$\frac{dt_{AB}}{dx} = 0$$
- Thus

$$\frac{dt_{AB}}{dx} = \left(\frac{1}{c}\right)\left[\frac{n_1 x}{\sqrt{d^2 + x^2}} - \frac{n_2(2b - x)}{\sqrt{d^2 + (2b - x)^2}}\right] = 0$$



Snell's law from Fermat's principle

- From the last slide, the condition for minimum time is

$$\frac{dt_{AB}}{dx} = \left(\frac{1}{c}\right)\left[\frac{n_1 x}{\sqrt{d^2 + x^2}} - \frac{n_2(2b - x)}{\sqrt{d^2 + (2b - x)^2}}\right] = 0$$

- Note from the figure that

$$\frac{x}{\sqrt{d^2 + x^2}} = \sin\theta_1$$

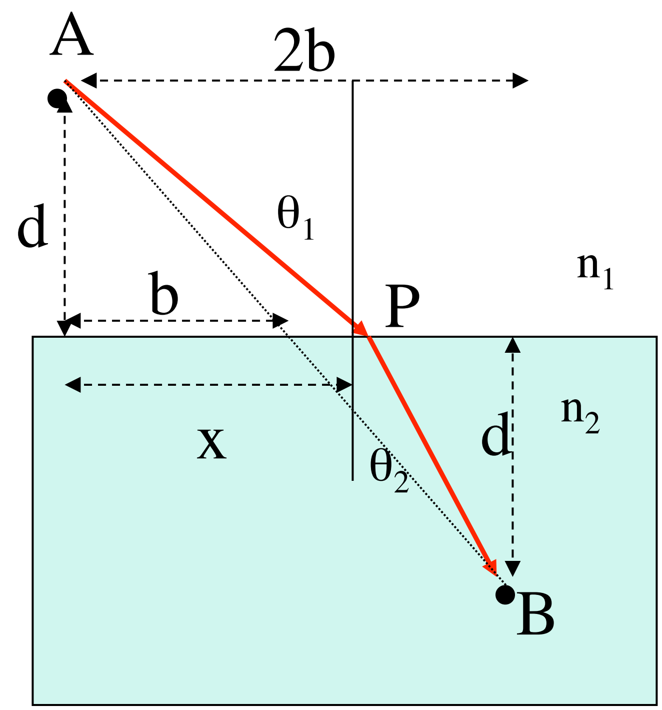
- And also that

$$\frac{2b - x}{\sqrt{d^2 + (2b - x)^2}} = \sin\theta_2$$

- Thus

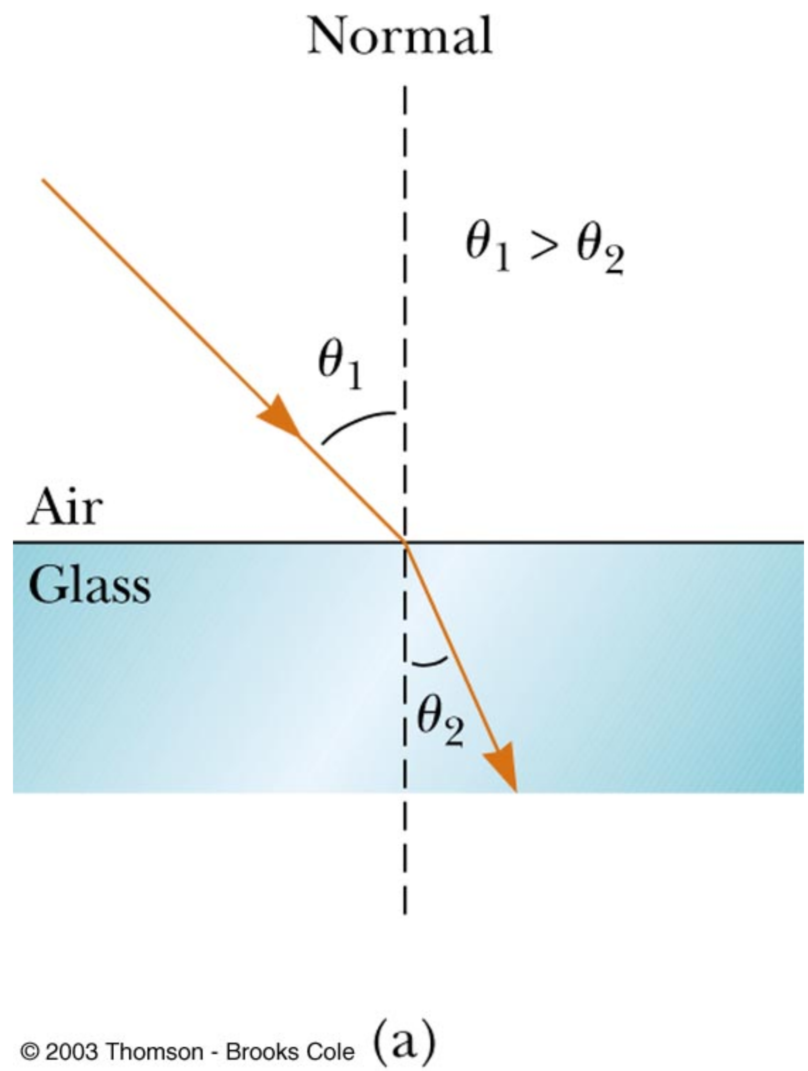
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

- Can also derive using Maxwell's equations

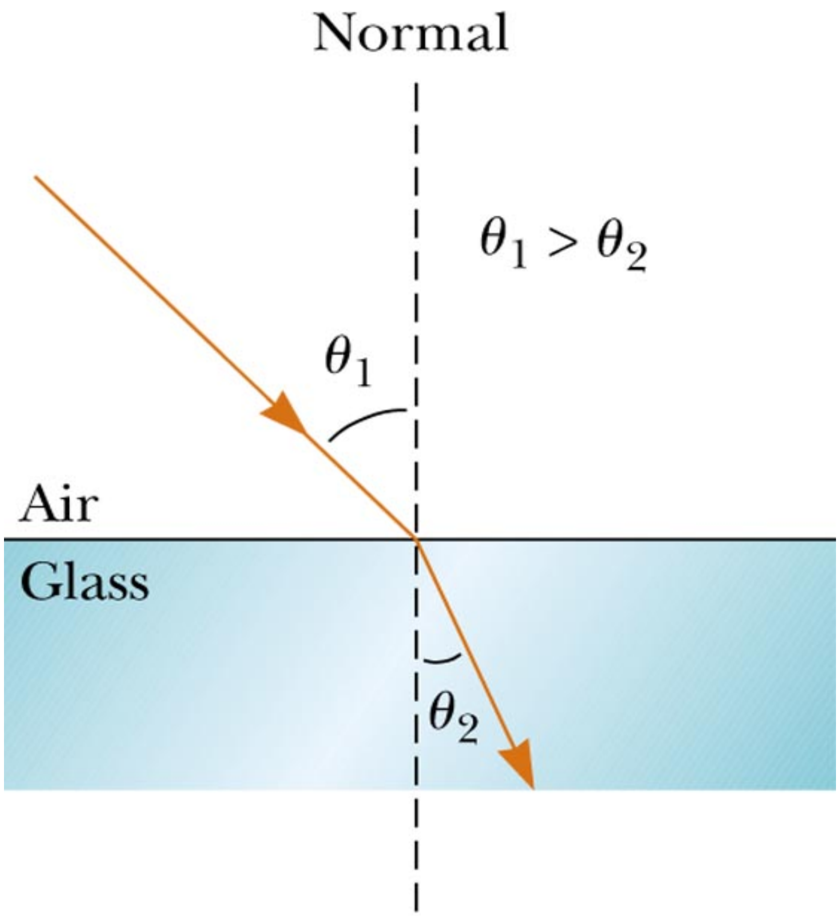


Something for the summertime

- When light goes from air to glass, it bends towards the normal
  - because it travels more slowly in glass than in air
- Why do waves in Florida (or anywhere else for that matter) come in parallel to the shore?
  - because of refraction

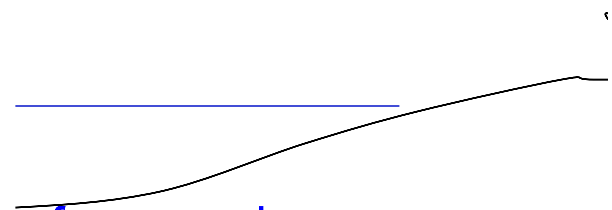


Something for the summertime



 (a)

- What happens near the shoreline?
  - The water gets shallower



- The velocity of a wave is proportional to the depth of the water
  - v α depth
- So as the wave approaches the shore, v decreases (like n increasing) and the wave bends towards the shore, no matter what its original direction