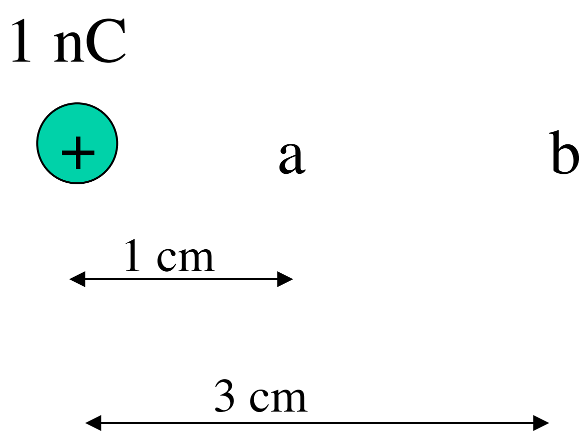
PHY294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Added problem 28.68 for 3rd MP assignment due Wed Feb. 3 as a hand-in problem**
  - **Help-room hours: 12:40-2:40 Tues; 3:00-4:00 PM Friday**
- Quizzes by iclicker (sometimes hand-written)
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

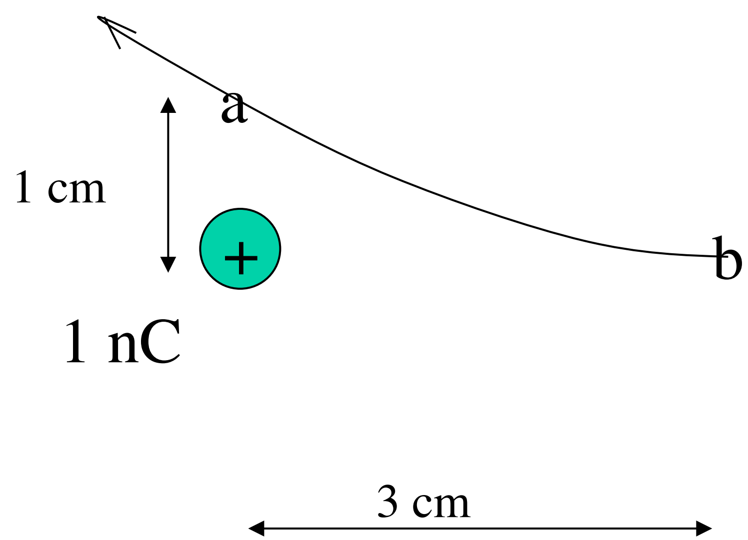
Another example before we leave this chapter

- What is:
  - the potential at points a and b
  - the potential difference between a and b
  - the potential energy of a proton at a and b
  - the speed at point b of a proton that was moving to the right at point a with a speed of 4 X 10^5 m/s
  - the speed at point a of a proton that was moving to the left at point b with a speed of 5.3 X 10^5 m/s



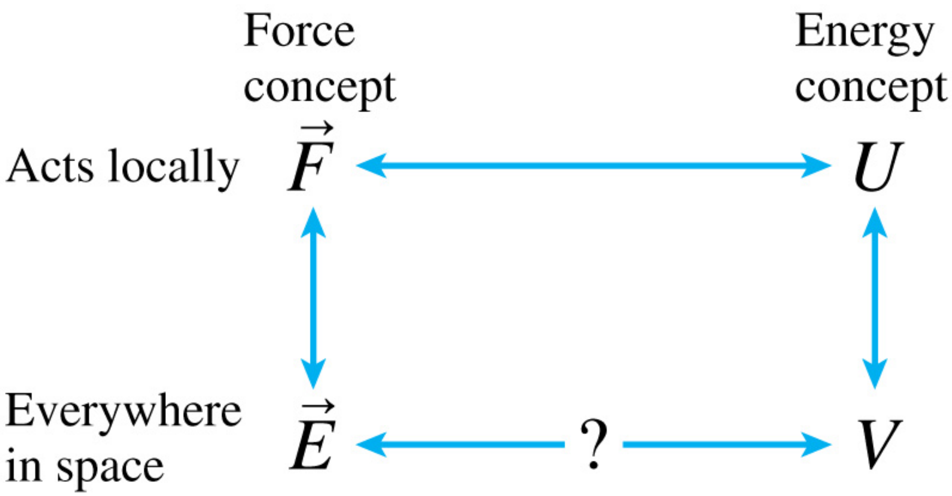
Another example before we leave this chapter

- Suppose the proton isn't heading towards the positive charge head-on, but at a small angle
- Then the path travelled will look like that on the right
- Suppose the proton's speed at point b is 5.3 X 10^5 /ms
- What is the speed at point a?



Electric potential and electric field

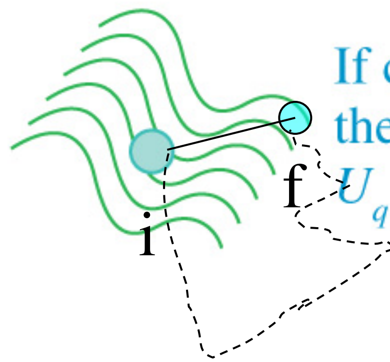
- We' ve talked so far about the connections between force and potential energy, between force and the electric field, and between the potential and the potential energy
- Now in this chapter, we' ll study the relationship between the electric field and the electric potential



Potential

- We can write the change in potential enegy in terms of the work done by the field

$$\Delta U = -W(i->f) = -\int_{si}^{sf} \vec{F} \cdot \vec{ds}$$



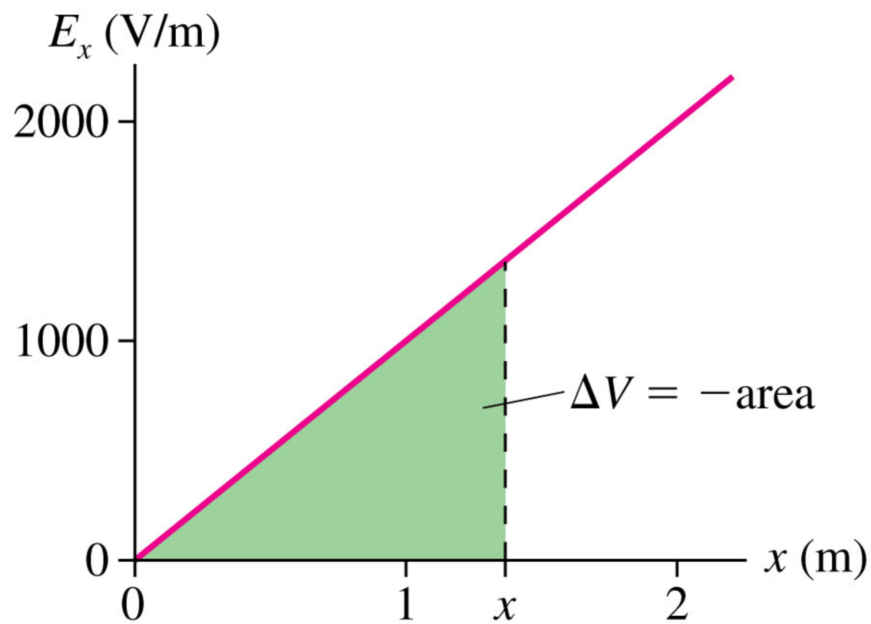
- But we can also write the potential energy as U=qV and the force as F=qE
- Thus, we can write the potential difference in terms of an integral over the electric field

Because electrostatic forces are conservative, the potential difference is independent of the path taken

$$\Delta V = V(s_f) - V(s_i) = -\int_{si}^{sf} \vec{E} \cdot \vec{ds} = -\int_{si}^{sf} E_s ds$$

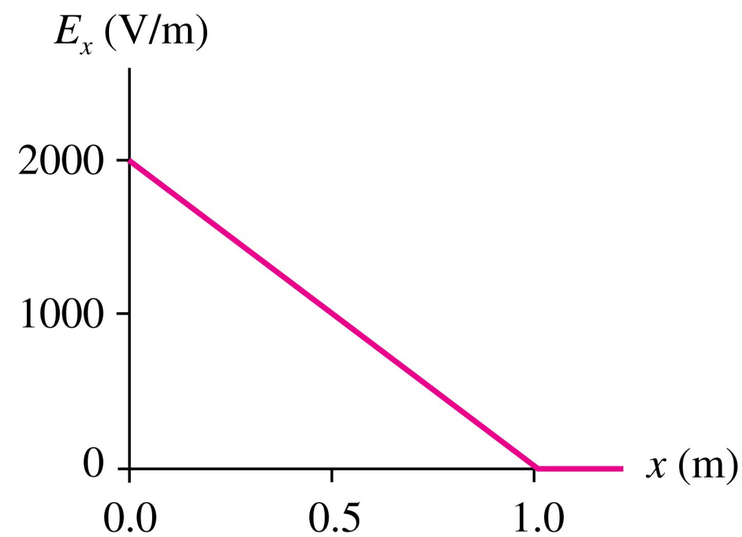
For a uniform electric field, we can write simply $\Delta V = -E_s \Delta s$

- The electric field is the negative of the area given in green (the integral of the electric field over x)



$$\Delta V = V(s_f) - V(s_i) = -\int_{s_i}^{s_f} E ds = -\int_{s_i}^{s_f} E_s ds$$

This is a graph of the x-component of the electric field along the x-axis. The potential is zero at the origin. What is the potential at x = 1m?



A. 2000 V.

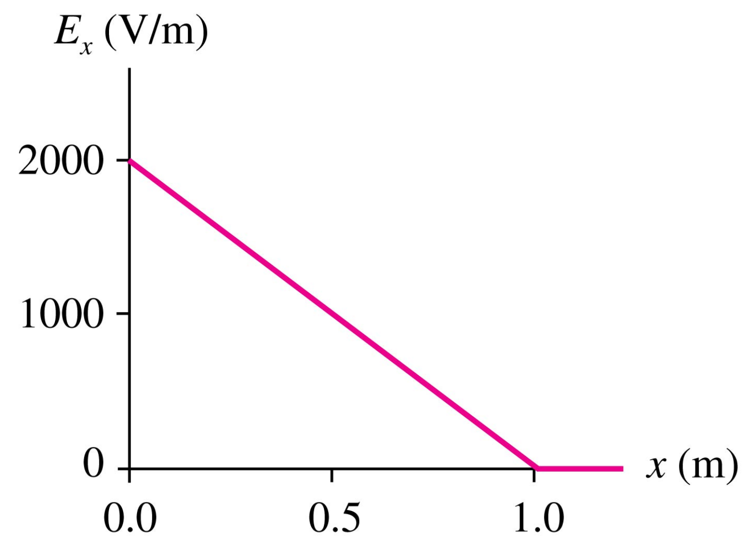
B. 1000 V.

C. 0 V.

D. -1000 V.

E. -2000 V.

This is a graph of the x-component of the electric field along the x-axis. The potential is zero at the origin. What is the potential at x = 1m?



A. 2000 V.

B. 1000 V.

C. 0 V.

✔ D. **-1000 V.** ΔV = –area under curve

E. -2000 V.

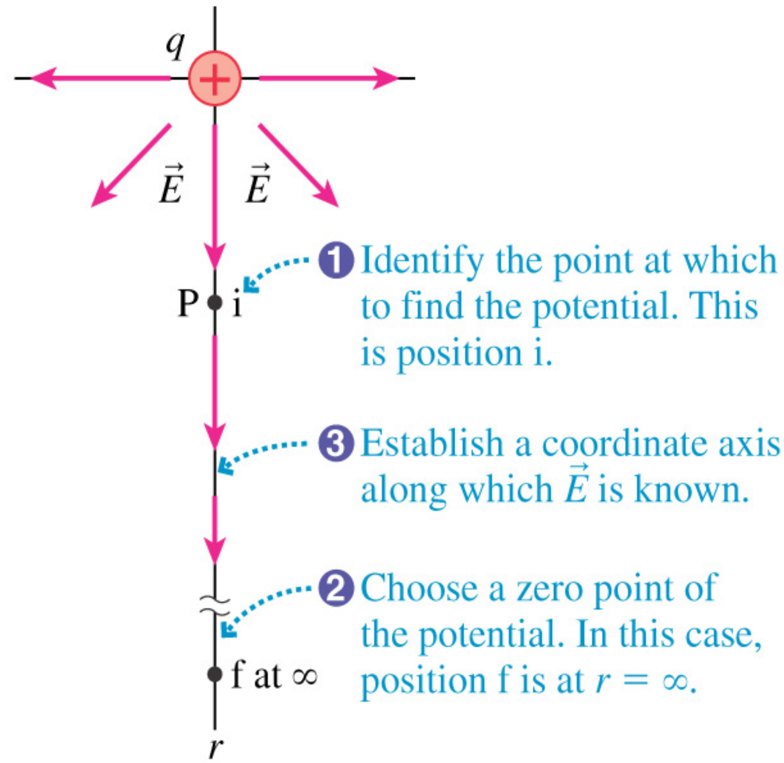
Potential of a point charge

- Let's first define the reference potential to be 0 at r=infinity
- And then write

$$\Delta V = V(infinity) - V(r) = -\int_{r}^{infinity} E_r dr$$

- We know the electric field from a point charge

$$E_r = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$



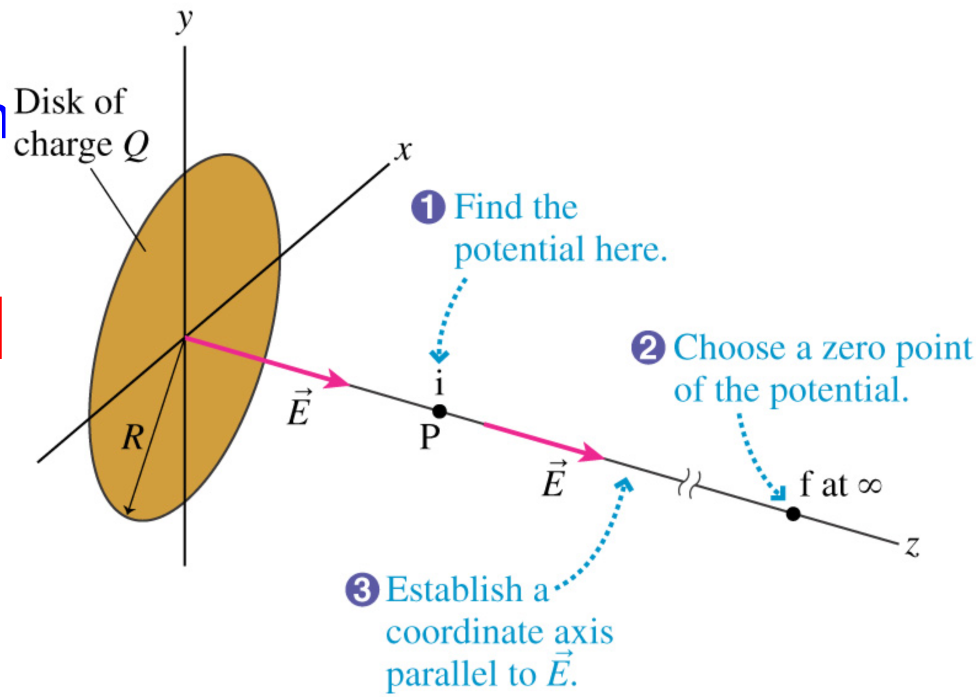
- So we can then write

$$V(r) = V(infinity) + \frac{q}{4\pi\varepsilon_o}\int_{r}^{infinity}\frac{1}{r^2}dr = V(infinity) + \frac{q}{4\pi\varepsilon_o}\left.\frac{-1}{r}\right|_{r}^{infinity} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$$

Let's go back to the charged disk

- We found that the electric field had the form of

$$E = \frac{Q}{2\pi R^2 \varepsilon_o}\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]$$



- Can we calculate the potential from the field?

$$V(z) = V(\infty) + \int_z^{\infty} E_z(z)dz$$

$$= 0 + \frac{Q}{2\pi R^2 \varepsilon_o}\int_z^{\infty}\left[1 - \frac{z}{\left(z^2 + R^2\right)^{1/2}}\right]$$

let $u=z^2+R^2$

$$V(z) = \frac{Q}{2\pi R^2 \varepsilon_o}\left[\int dz - \int \frac{\frac{1}{2}du}{u^{1/2}}\right]_z^{\infty} = \frac{Q}{2\pi R^2 \varepsilon_o}\left[z - \sqrt{z^2 + R^2}\right]_z^{\infty}$$

$$= \frac{Q}{2\pi R^2 \varepsilon_o}\left[\sqrt{z^2 + R^2} - z\right]$$

Potential from a uniform line of charge

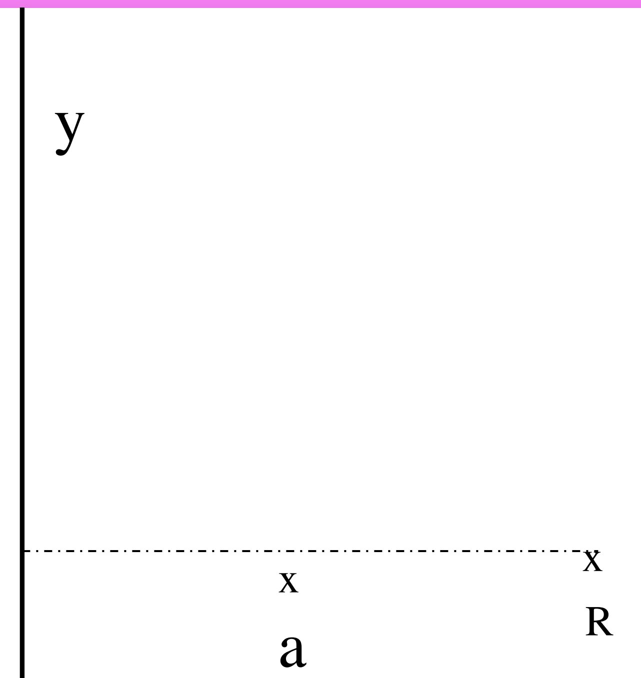
- What is the potential from an infinite line of charge with charge density λ as a function of R, the distance perpendicular to the line of charge?
- We know the electric field from an infinite line of charge; it has only a radial component

$$E = \frac{\lambda}{2\pi\varepsilon_o R}$$

$$\Delta V = -\int E_r dr = -\frac{\lambda}{2\pi\varepsilon_o}\int \frac{dr}{r}$$

$$\Delta V = -\frac{\lambda}{2\pi\varepsilon_o}\int_a^R \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon_o}\ln r\Big|_a^R$$

$$\Delta V = -\frac{\lambda}{2\pi\varepsilon_o}\ln\frac{R}{a}$$



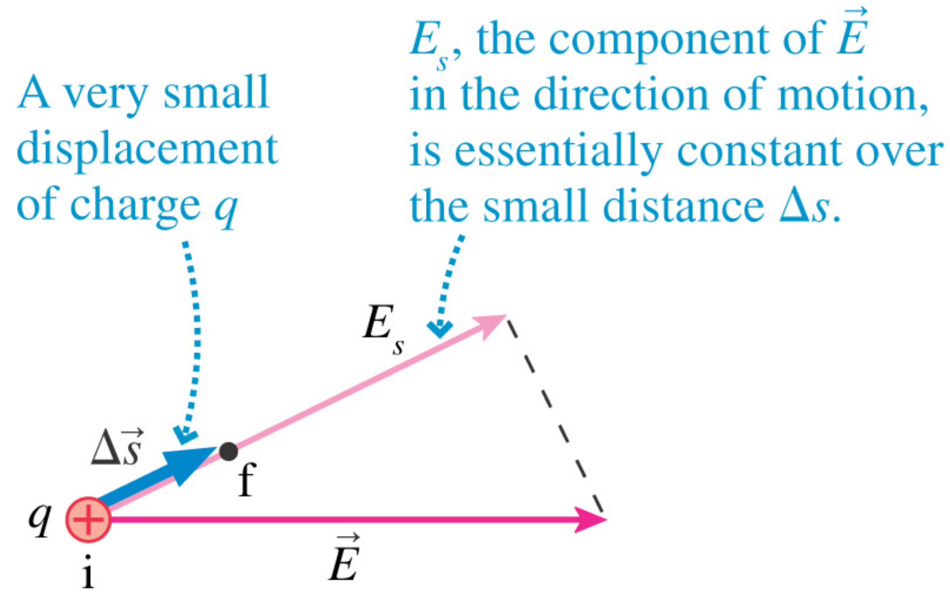
Let me choose the potential to be zero at r=a

Note that I can't set V to be zero at infinity since the log of infinity is infinite

Since the line is infinite, we can't get *far away* from it

Electric field from the potential

- We saw earlier that
  - $\Delta V = -E_s \Delta s$
  - Thus, $E_s = -\Delta V/\Delta s$
  - …in the s direction
- In the limit that Δs goes to zero, we can write
  - $E_s = -\, dV/ds$
  - so for s = r, we can write for a point charge



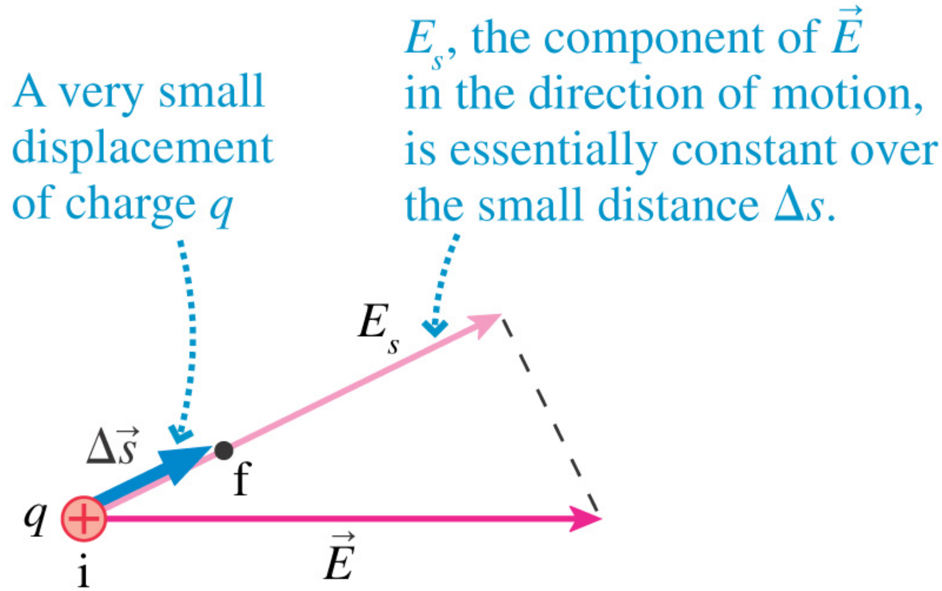
$$E_r = -\frac{dV}{dr} = -\frac{d}{dr}\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r}\right) = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}$$

Electric field from the potential

- In general…

$$d\vec{s} = dx\vec{i} + dy\vec{j} + dz\vec{k}$$

$$dV = -Eds = -E_x dx - E_y dy - E_z dz$$



- So we can write

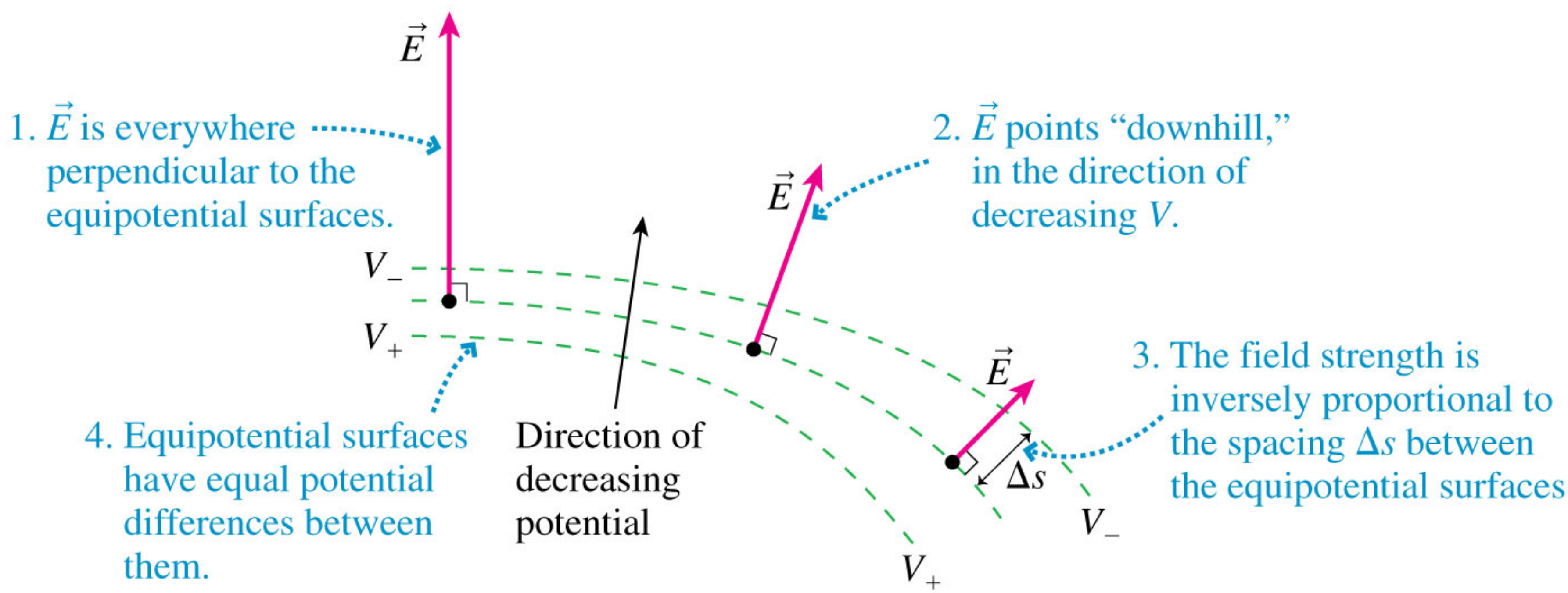
$$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

- and

$$E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}$$

In three dimensions, we can find the electric field from the electric potential as:

$$\vec{E} = E_x\hat{\imath} + E_y\hat{\jmath} + E_z\hat{k} = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$



Example

- Suppose $V = Axy^2 - Byz$
- What is E?

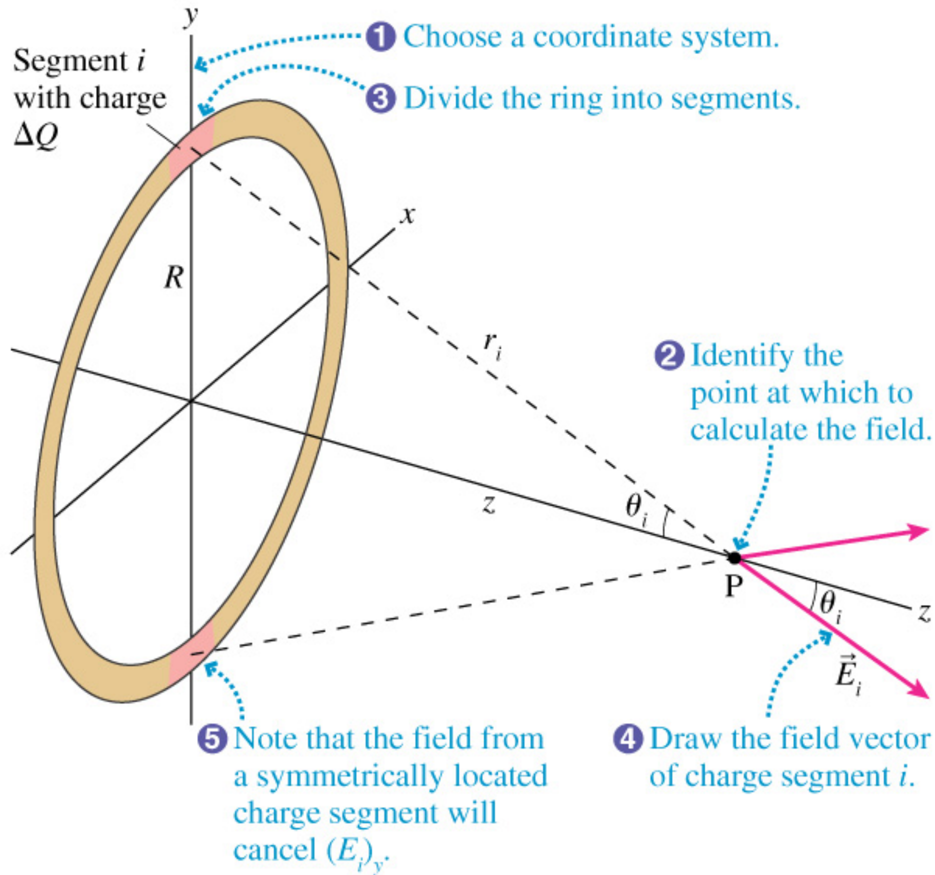
Electric field from a charged ring

- We know the potential is

$$V_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{z^2 + R^2}}$$

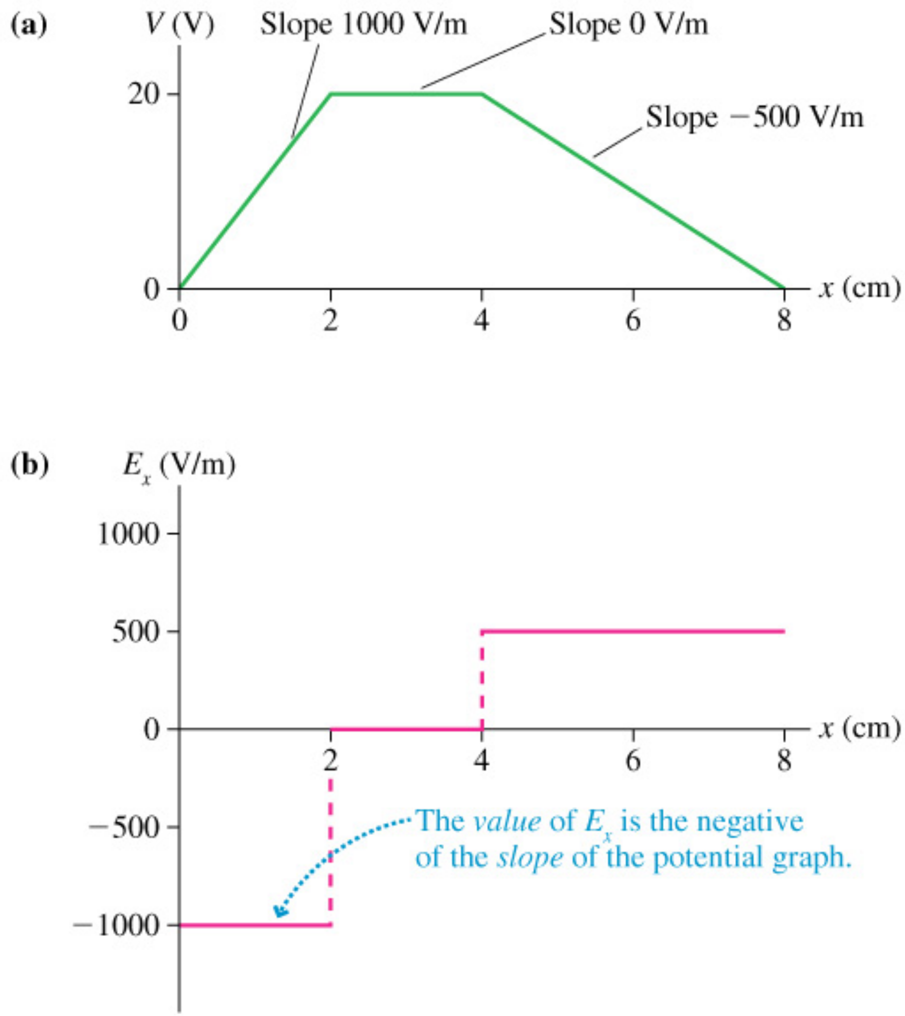
$$E = E_z = -\frac{dV}{dz} = -\frac{d}{dz}\left(\frac{1}{4\pi\varepsilon_o}\frac{Q}{\sqrt{z^2 + R^2}}\right)$$

$$E_{ring} = \frac{1}{4\pi\varepsilon_o}\frac{zQ}{(z^2 + R^2)^{3/2}}$$



Example

- $E_s = - dV/ds$
- …or in this case
- $E_x = - dV/dx$



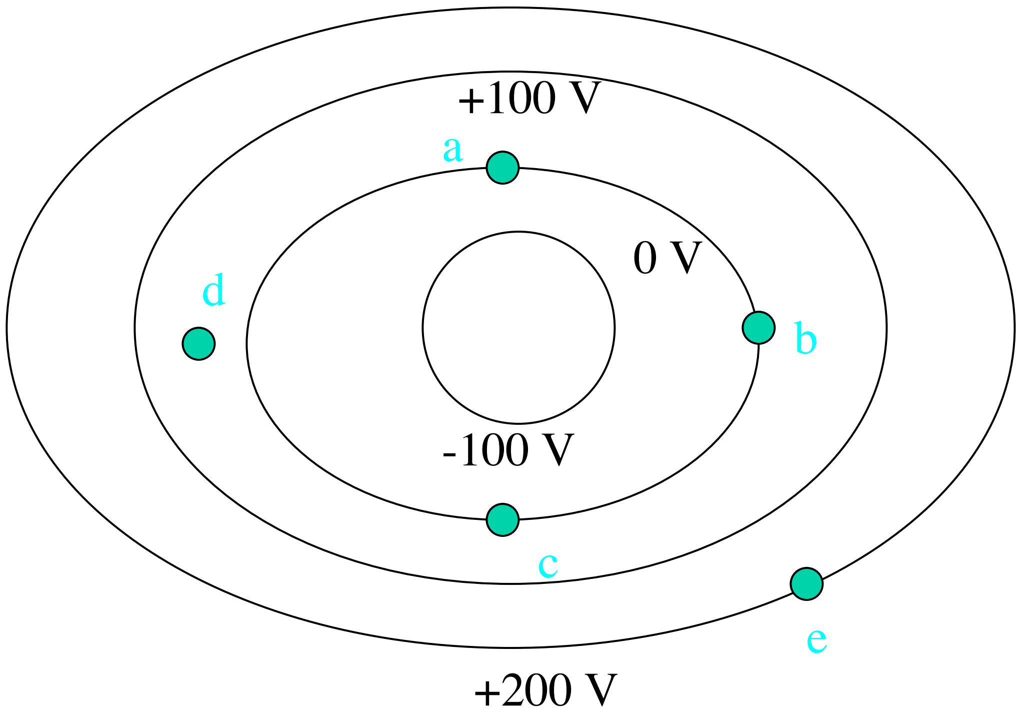
Useful table

ELECTRIC FIELDS AND POTENTIALS FOR VARIOUS CHARGE CONFIGURATIONS

Charge Configuration	*Magnitude of Electric Field*	*Electric Potential*	*Location of Zero Potential*
Point charge	$\frac{q}{4\pi\epsilon_0 r^2}$	$\frac{q}{4\pi\epsilon_0 r}$	∞
Infinite line of uniform charge density λ	$\frac{\lambda}{2\pi\epsilon_0 r}$	$-\frac{\lambda}{2\pi\epsilon_0}\ln\frac{r}{a}$	$r = a$
Parallel, oppositely charged plates of uniform charge density σ, separation d	$\frac{\sigma}{\epsilon_0}$	$\Delta V = -Ed = -\frac{\sigma d}{\epsilon_0}$	Anywhere
Charged disk of radius R, along axis at distance x	$\frac{Q}{2\pi\epsilon_0}\left(\frac{\sqrt{R^2+x^2}-x}{\sqrt{R^2+x^2}}\right)$	$\frac{Q}{2\pi\epsilon_0 R^2}(\sqrt{R^2+x^2}-x)$	∞
Charged spherical shell of radius R	$r \geq R$: $\frac{Q}{4\pi\epsilon_0 r^2}$	$r > R$: $\frac{Q}{4\pi\epsilon_0 r}$	∞
	$r < R$: 0	$r \leq R$: $\frac{Q}{4\pi\epsilon_0 R}$	∞
Electric dipole	Along bisecting axis only, far away: $\frac{p}{4\pi\epsilon_0 r^3}$	Everywhere, far away: $\frac{p\cos\theta}{4\pi\epsilon_0 r^2}$	∞
Charged ring of radius R, along axis	$\frac{Qx}{4\pi\epsilon_0(R^2+x^2)^{3/2}}$	$\frac{Q}{4\pi\epsilon_0\sqrt{R^2+x^2}}$	∞
Uniformly charged nonconducting solid sphere of radius R	$r \geq R$: $\frac{Q}{4\pi\epsilon_0 r^2}$	$r \geq R$: $\frac{Q}{4\pi\epsilon_0 r}$	∞
	$r < R$: $\frac{Qr}{4\pi\epsilon_0 R^3}$	$r < R$: $\frac{Q}{8\pi\epsilon_0}\left(3-\frac{r^2}{R^2}\right)$	∞

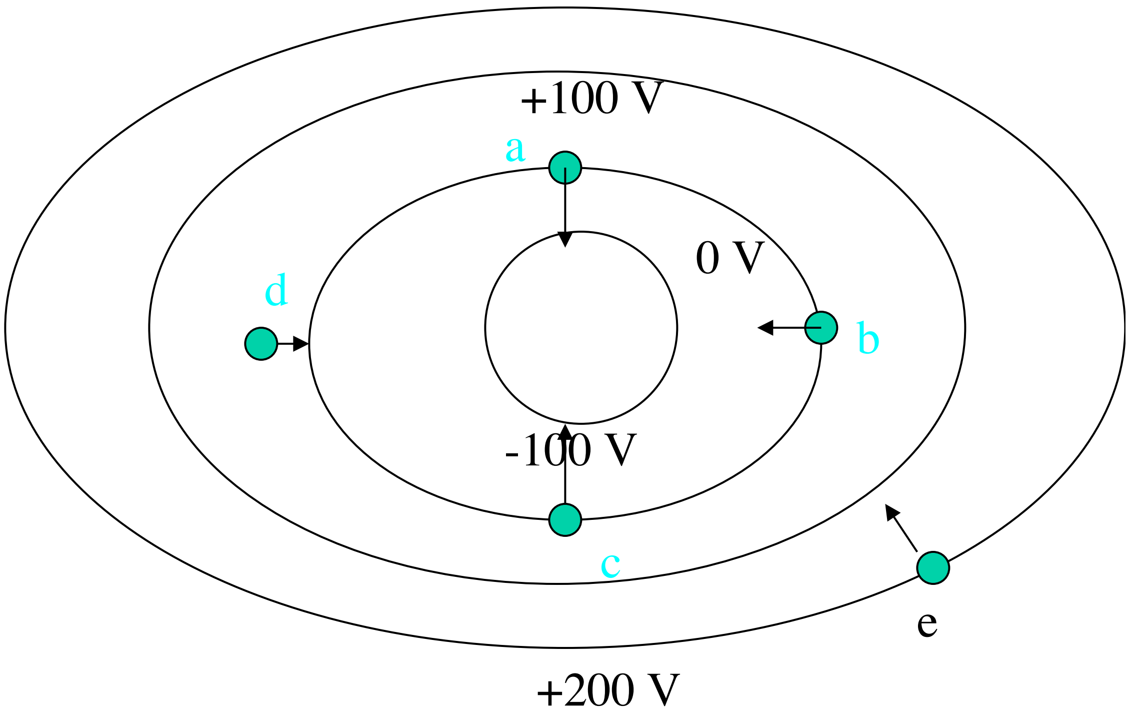
Equipotential surfaces

- Suppose I have the equipotential surfaces shown to the right
- Let me label some points
- What are the E field directions?
- Is $E_a = E_b$?
- What about E_c and E_d?



Equipotential surface

- E_a and E_c look to be about the same magnitude
- $E_b < E_{a,c}$
- $E_b > E_d$



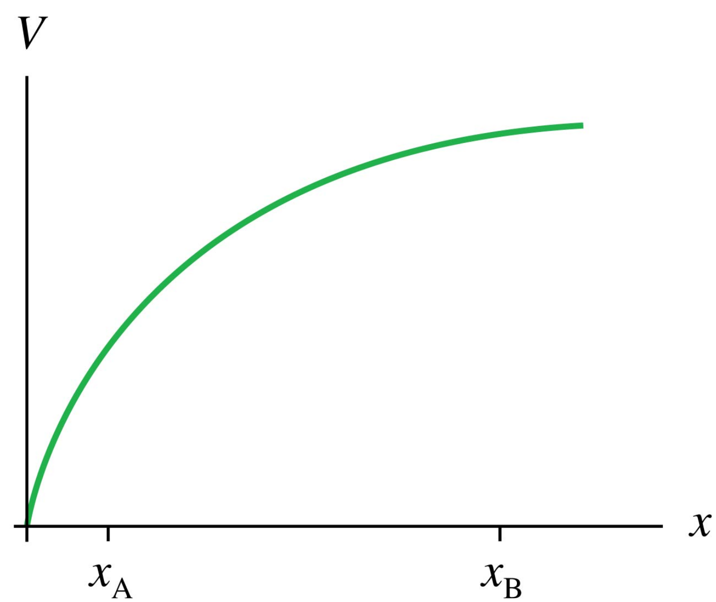
At which point is the electric field stronger?

A. At x_A.

B. At x_B.

C. The field is the same strength at both.

D. There’s not enough information to tell.



At which point is the electric field stronger?

✔ **A. At x_A.** $|E|$ = slope of potential graph

B. At x_B.

C. The field is the same strength at both.

D. There's not enough information to tell.