PHY294H

- Professor: Joey Huston
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- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Help-room hours: 12:40-2:40 Monday (note change); 3:00-4:00 PM Friday**
  - **No hand-in problem for tomorrow; for next Wed 31.79**
- Quizzes by iclicker (sometimes hand-written)
- Average on exam is around 65; will pass back tomorrow
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

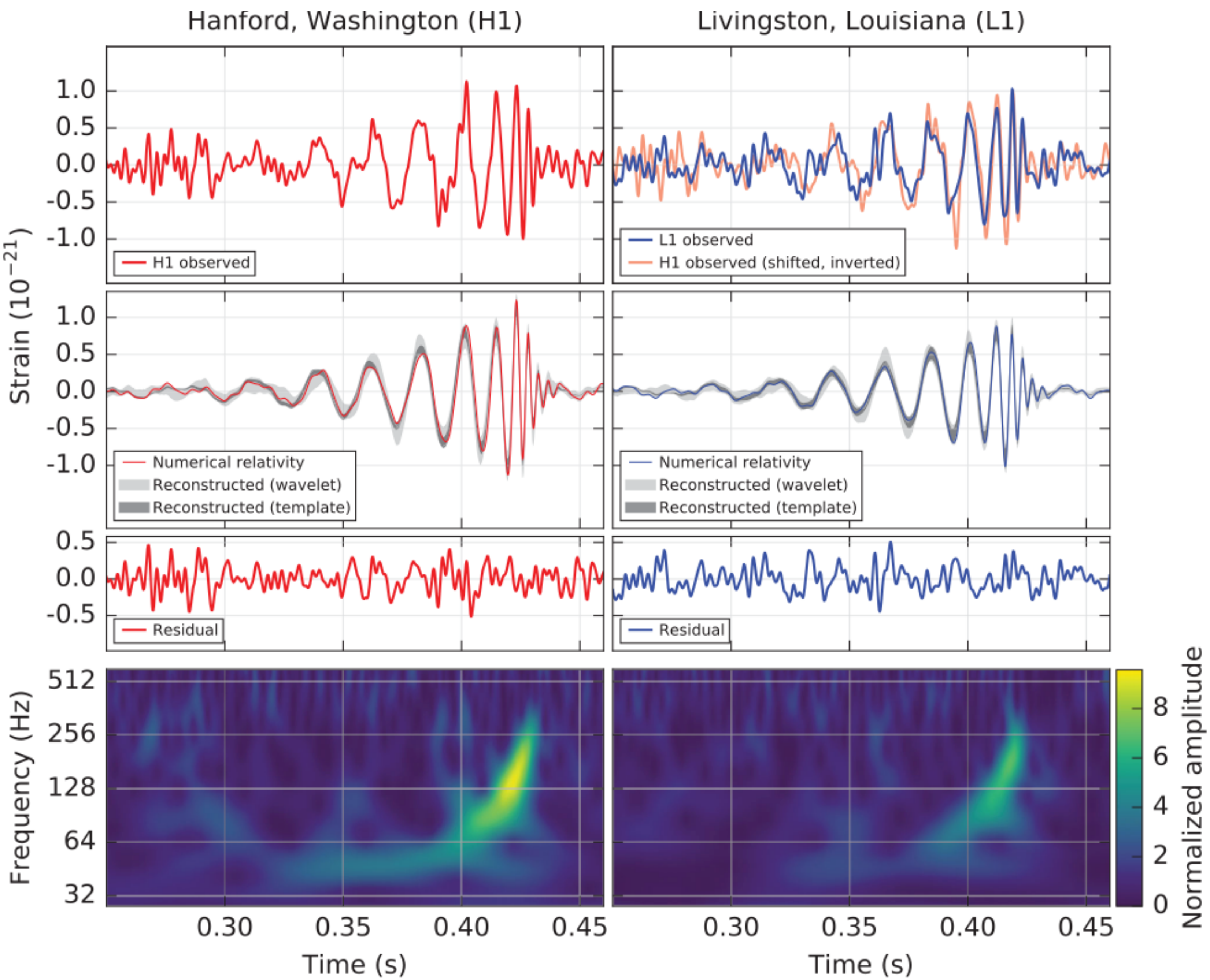


FIG. 1. The gravitational-wave event GW150914 observed by the LIGO Hanford (H1, left column panels) and Livingston (L1, right column panels) detectors. Times are shown relative to September 14, 2015 at 09:50:45 UTC. For visualization, all time series are filtered

propagation time, the events have a combined signal-to-noise ratio (SNR) of 24 [45].

Only the LIGO detectors were observing at the time of GW150914. The Virgo detector was being upgraded, and GEO 600, though not sufficiently sensitive to detect this event, was operating but not in observational mode. With only two detectors the source position is primarily determined by the relative arrival time and localized to an area of approximately 600 deg^2 (90% credible region) [39,46].

The basic features of GW150914 point to it being produced by the coalescence of two black holes—i.e., their orbital inspiral and merger, and subsequent final black hole ringdown. Over 0.2 s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150 Hz, where the amplitude reaches a maximum. The most plausible explanation for this evolution is the inspiral of two orbiting masses, m_1 and m_2, due to gravitational-wave emission. At the lower frequencies, such evolution is characterized by the chirp mass [11]

$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1+m_2)^{1/5}} = \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3}\dot{f}\right]^{3/5},$$

where f and $\dot{f}$ are the observed frequency and its time derivative and G and c are the gravitational constant and speed of light. Estimating f and $\dot{f}$ from the data in Fig. 1, we obtain a chirp mass of $\mathcal{M} \simeq 30 M_\odot$, implying that the total mass $M = m_1 + m_2$ is $\gtrsim 70 M_\odot$ in the detector frame. This bounds the sum of the Schwarzschild radii of the binary components to $2GM/c^2 \gtrsim 210$ km. To reach an orbital frequency of 75 Hz (half the gravitational-wave frequency) the objects must have been very close and very compact; equal Newtonian point masses orbiting at this frequency would be only $\simeq$350 km apart. A pair of neutron stars, while compact, would not have the required mass, while a black hole neutron star binary with the deduced chirp mass would have a very large total mass, and would thus merge at much lower frequency. This leaves black holes as the only known objects compact enough to reach an orbital frequency of 75 Hz without contact. Furthermore, the decay of the waveform after it

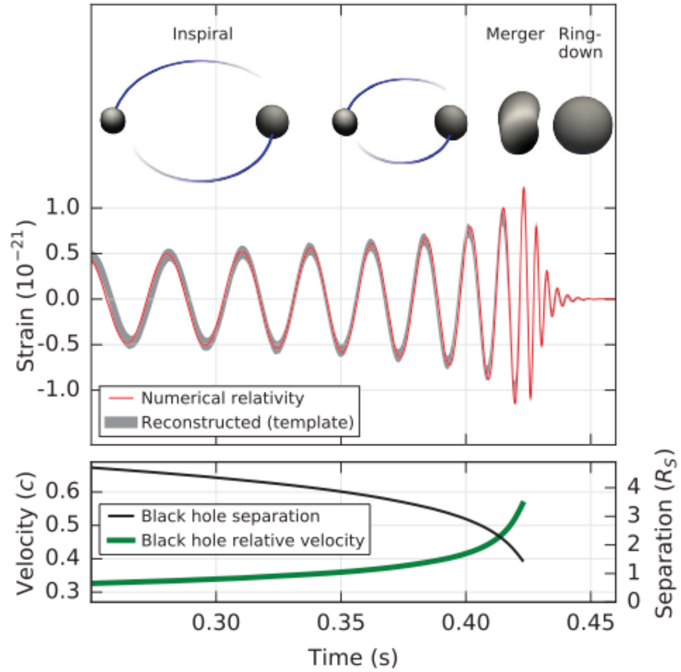
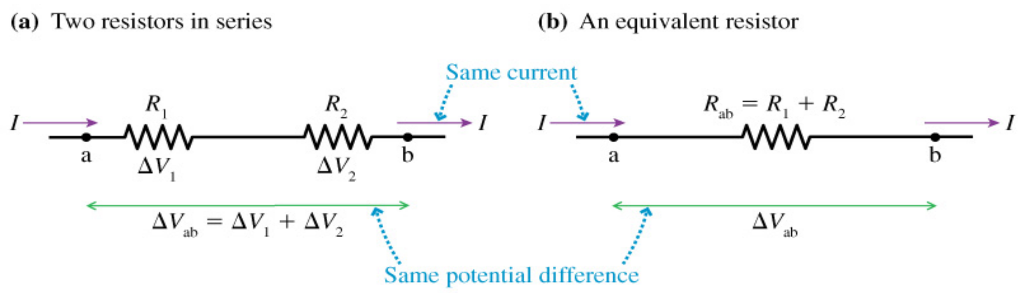


FIG. 2. *Top:* Estimated gravitational-wave strain amplitude from GW150914 projected onto H1. This shows the full bandwidth of the waveforms, without the filtering used for Fig. 1. The inset images show numerical relativity models of the black hole horizons as the black holes coalesce. *Bottom:* The Keplerian effective black hole separation in units of Schwarzschild radii ($R_S = 2GM/c^2$) and the effective relative velocity given by the post-Newtonian parameter $v/c = (GM\pi f/c^3)^{1/3}$, where f is the gravitational-wave frequency calculated with numerical relativity and M is the total mass (value from Table I).

detector [33], a modified Michelson interferometer (see Fig. 3) that measures gravitational-wave strain as a difference in length of its orthogonal arms. Each arm is formed by two mirrors, acting as test masses, separated by $L_x = L_y = L = 4$ km. A passing gravitational wave effectively alters the arm lengths such that the measured difference is $\Delta L(t) = \delta L_x - \delta L_y = h(t)L$, where h is the gravitational-wave strain amplitude projected onto the detector. This differential length variation alters the phase difference between the two light fields returning to the beam splitter, transmitting an optical signal proportional to

Resistors in series

- Consider two (or more) resistors in series
- The same current passes through both resistors
- The total voltage drop across the two resistors is the sum of the voltage drops across each resistor
- We'd like to find an equivalent resistance for which the current would be the same given the same voltage drop



$$\Delta V_{ab} = \Delta V_a + \Delta V_b = IR_1 + IR_2$$

$$I = \frac{\Delta V_{ab}}{R_1 + R_2} = \frac{\Delta V_{ab}}{R_{eq}}$$

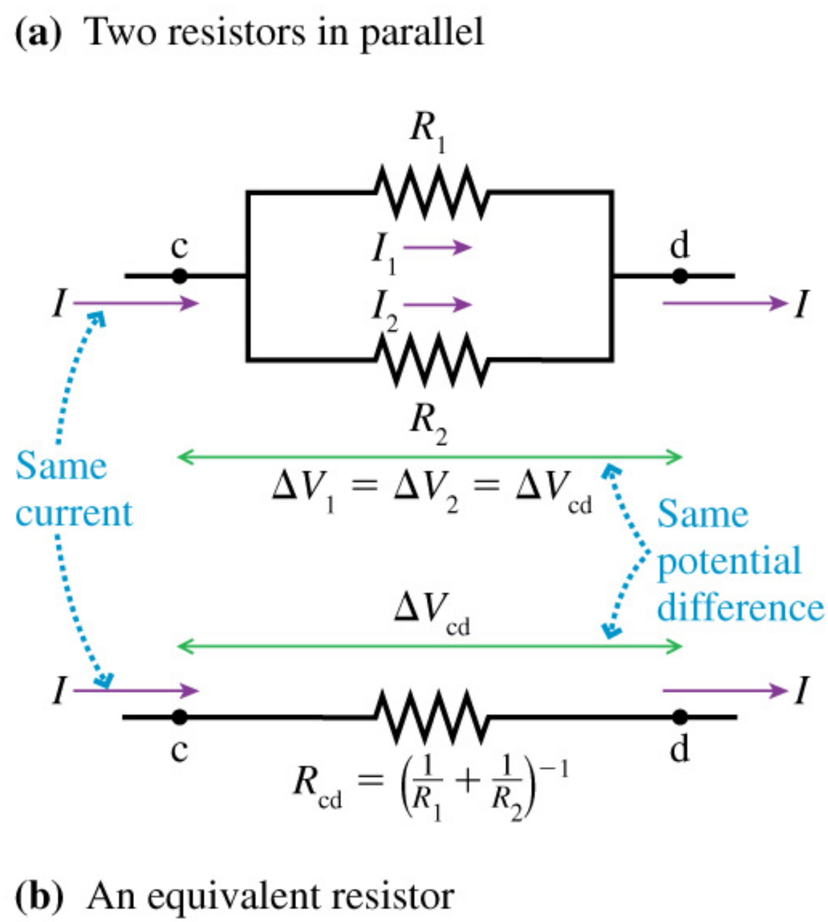
$$R_{eq} = R_1 + R_2 (+R_3 + \ldots)$$

Resistors in parallel

- In this case, the current splits up when going through R_1 and R_2, but we know that the voltage across R_1 and R_2 are the same
- Replace by an equivalent resistance

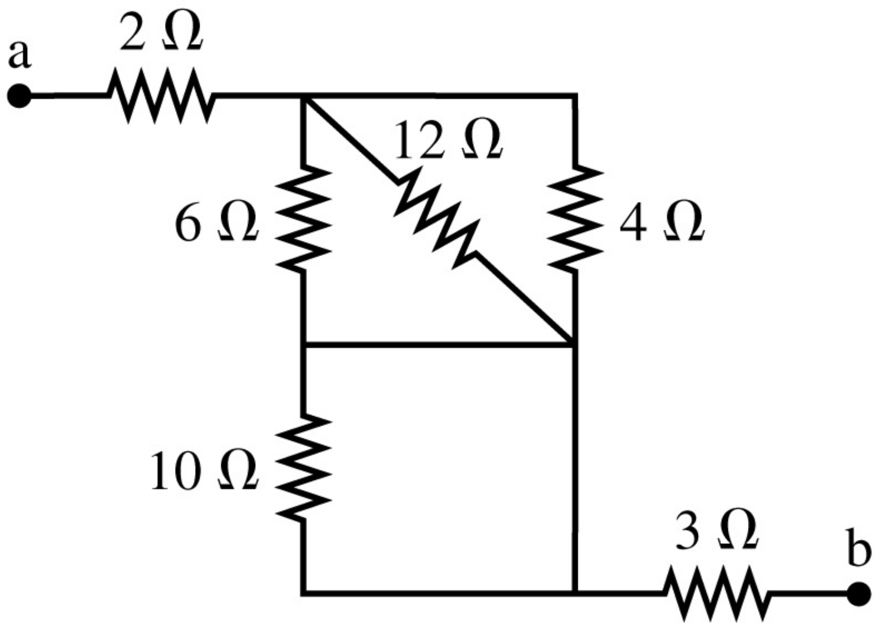
$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \frac{\Delta V}{R_{eq}}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \left(\frac{1}{R_3} + \ldots\right)$$



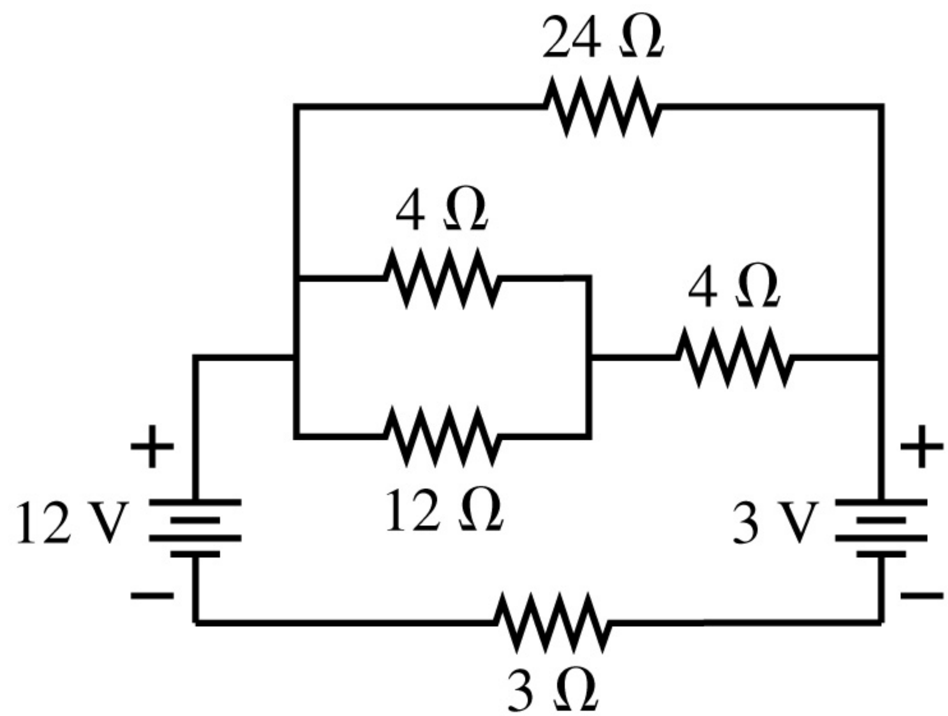
Example

- What is the equivalent resistance between a and b?
- Re-draw (if needed)
- Decide what resistors are in series and what resistors are in parallel, and then simplify



Example

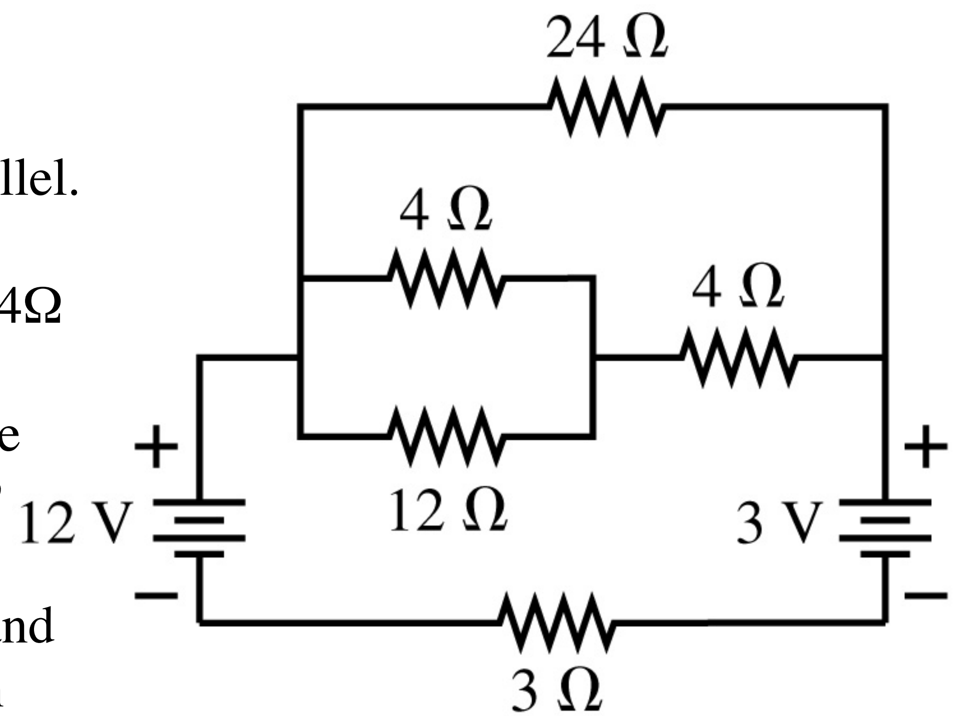
- What's the current through each of the resistors?



Example

- What's the current through each of the resistors?

Below, I denote all equivalent resistances in *italics*.

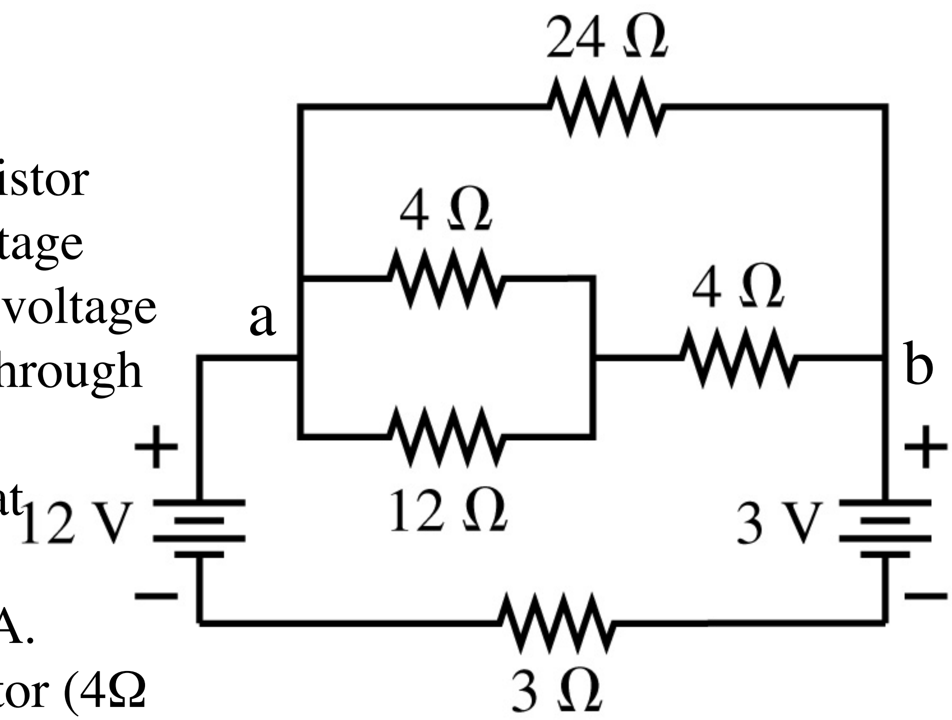


1) The 4Ω and 12Ω resistors are in parallel. Replace them by a *3Ω* resistor.
2) That *3Ω* resistor is in series with the 4Ω resistor. Replace them by a *7Ω* resistor.
3) That *7Ω* resistor is in parallel with the 24Ω resistor. Replace them with a *5.4Ω* resistor.
4) Now I have a circuit with one loop, and two batteries and two resistors: *5.4Ω* in series with 3Ω. These two resistors are in series so the total resistance is *8.4Ω*. The sum of the two batteries is 9V. (They oppose each other.)

5) The current through the 3Ω resistor and the *5.4Ω* resistor is thus 1.07A.

Example

- What's the current through each of the resistors?



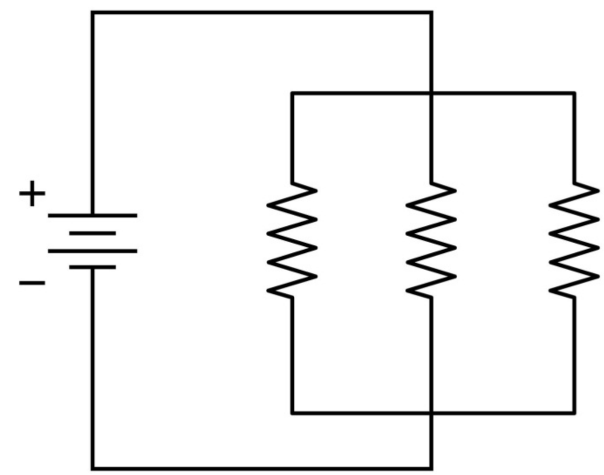
6) The voltage drop across the *5.4Ω* resistor is (1.07A)(*5.4Ω*)=5.78V. This is the voltage between points a and b. This is also the voltage across the 24Ω resistor. So the current through that resistor is 0.24A.

7) But the current into a is 1.07A, so that means that the current through the *7Ω* resistor (the branch from a to b) is 0.83A.

8) The voltage drop across the *3Ω* resistor (4Ω and 12Ω in parallel) is 5.78V-(0.83A)(4Ω)=2.46V.

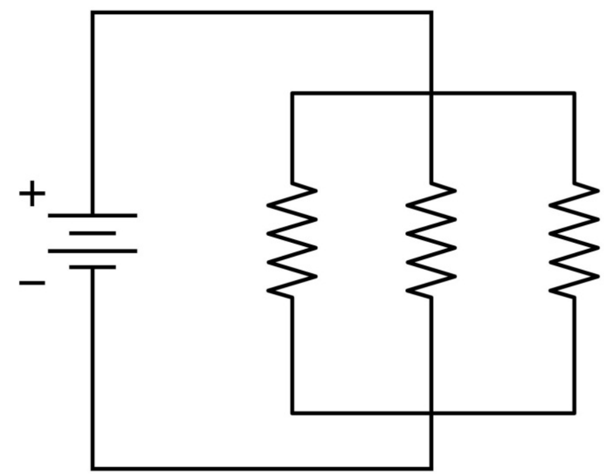
9) Thus, the current through the 4Ω resistor (in parallel with the 12Ω) is 0.615A, leaving 0.215A to go through the 12Ω resistor.

What things about the resistors in this circuit are the same for all three?



A. Current *I*.
B. Potential difference Δ*V*.
C. Resistance *R*.
D. A and B.
E. B and C.

What things about the resistors in this circuit are the same for all three?



A. Current I.

B. Potential difference ΔV.

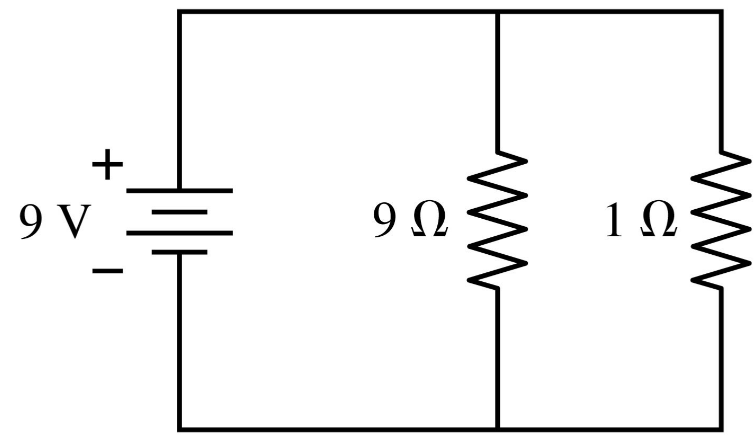
C. Resistance R.

D. A and B.

E. B and C.

Which resistor dissipates more power?

A. The 9 Ω resistor.
B. The 1 Ω resistor.
C. They dissipate the same power.

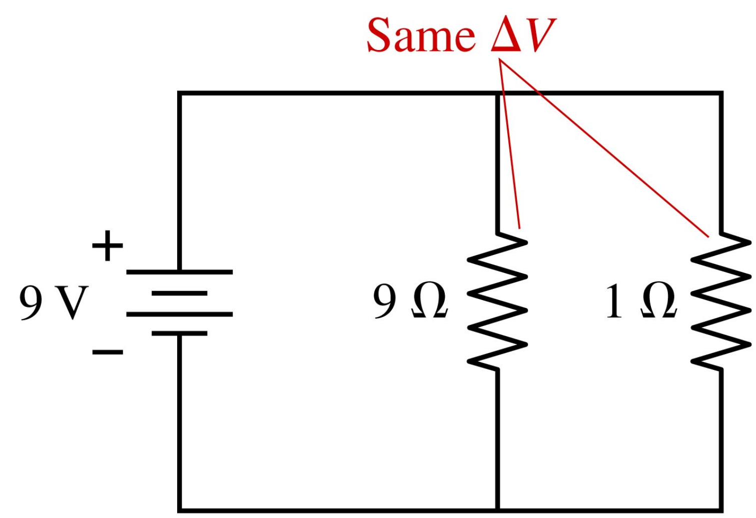


Which resistor dissipates more power?

A. The 9 Ω resistor.

✔ **B. The 1 Ω resistor.**

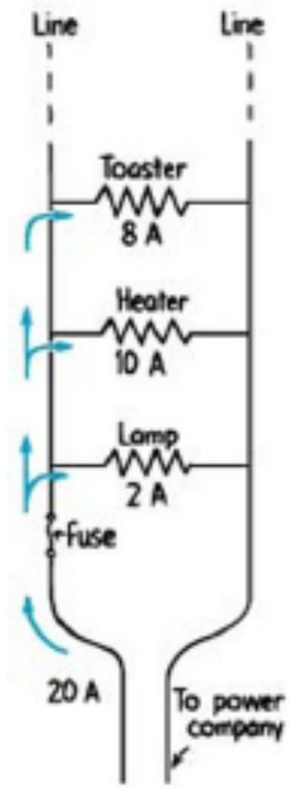
C. They dissipate the same power.



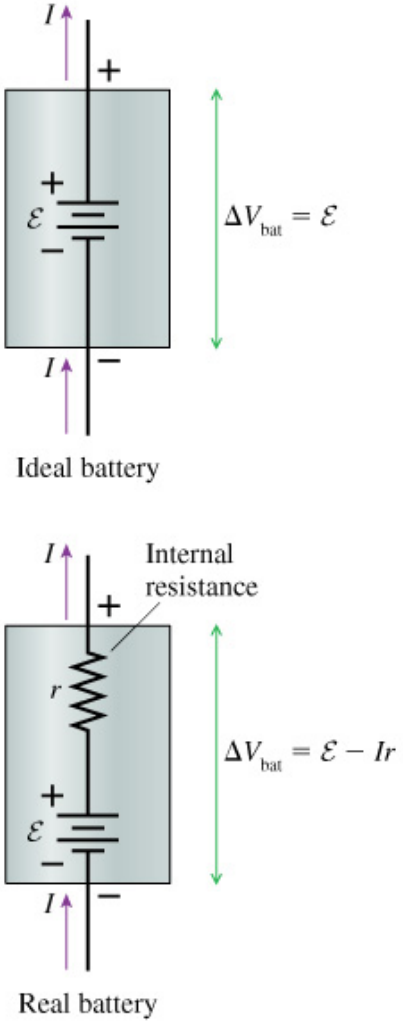
$$P = \frac{(\Delta V)^2}{R}$$

Household circuits

- Are wired in parallel, so that multiple appliances can be used at the same time
- The more appliances connected, the more current is drawn
- In order to prevent too much current being drawn, a fuse (circuit breaker) is part of the circuit



Real batteries



What does this have to do with starting your car in the winter?

Equivalent resistance

- So in a simple circuit like the one on the right, the total resistance is the sum of the external resistor R and the internal resistance r
- Typically r is small (<<1 Ω) for a car battery, so if you short a car battery, watch out