PHY294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Help-room hours: 12:40-2:40 Monday (note change); 3:00-4:00 PM Friday**
  - **hand-in problem for next Wed: 31.79**
- Quizzes by iclicker (sometimes hand-written)
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

Biot-Savart law in terms of currents

- We have written the B-S law in terms of a charge ΔQ moving with a velocity v

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{\Delta Q\vec{v}x\hat{r}}{r^2}$$

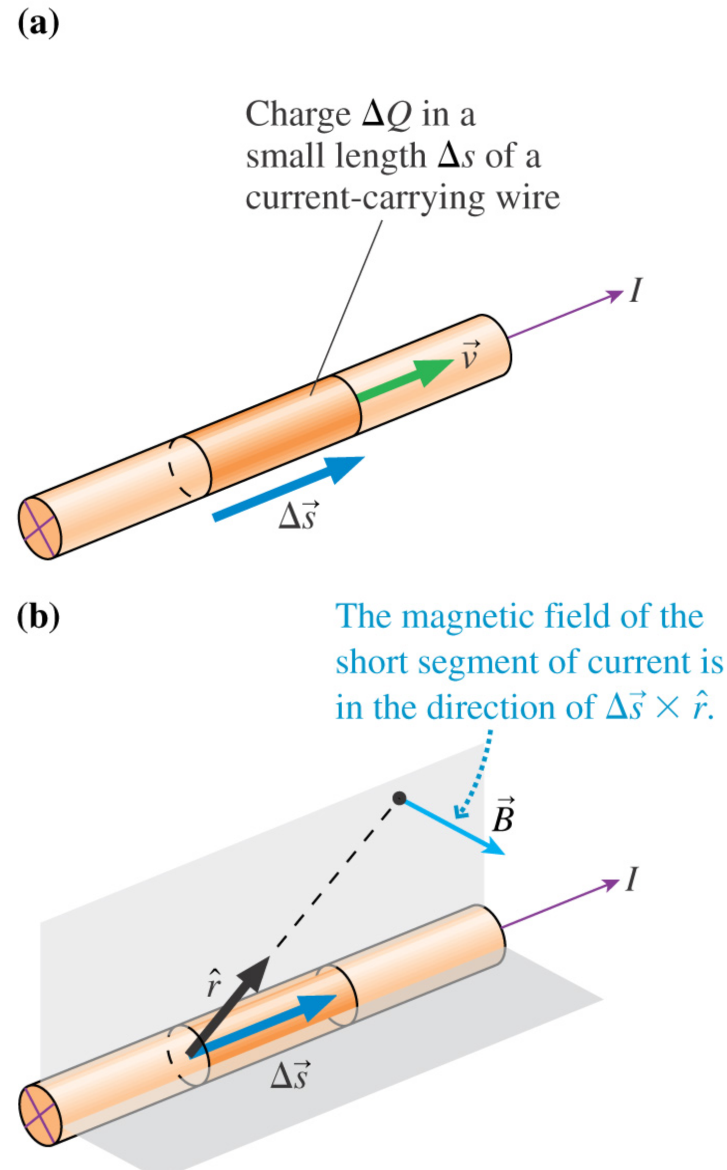
- But it's usually more convenient to define in terms of a current I

$$\Delta Qv = \Delta Q\frac{\Delta s}{\Delta t} = I\Delta s$$

- so

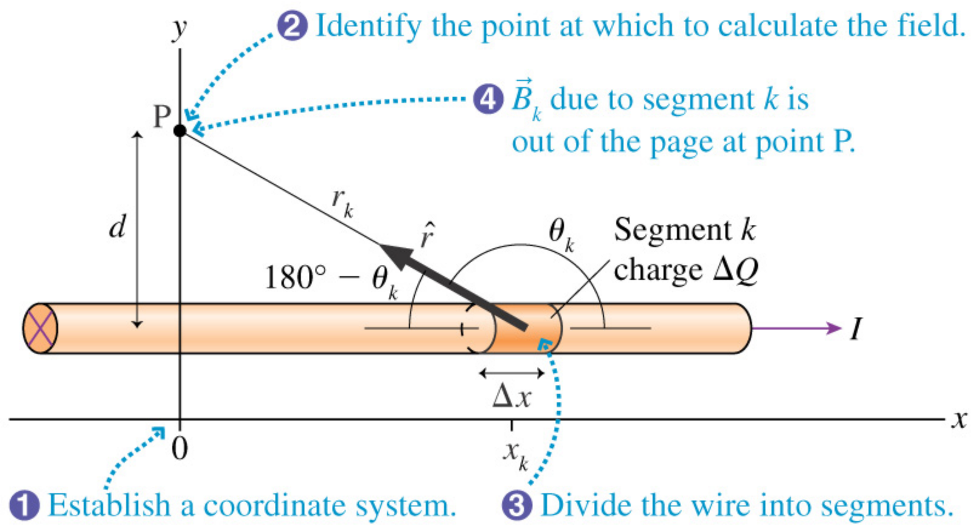
$$\vec{B} = \frac{\mu_o}{4\pi}\frac{I\vec{\Delta s}x\hat{r}}{r^2}$$

…or $$\vec{dB} = \frac{\mu_o}{4\pi}\frac{I\vec{ds}x\hat{r}}{r^2}$$



Magnetic field from a current in a straight wire

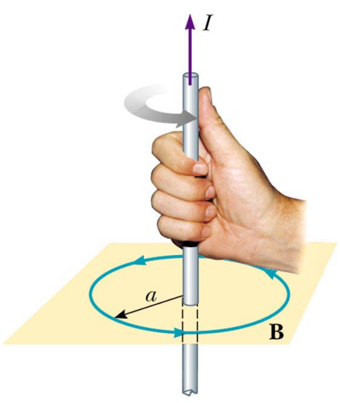
- We want to calculate the magnetic field at point P, which we have chosen to be on the y axis, from a current I
- We will apply the Biot-Savart law to a section of the wire Δx and then integrate over the entire length of the wire
- Note that from the right-hand rule that the resulting magnetic field (at point P) is out of the plane of the page



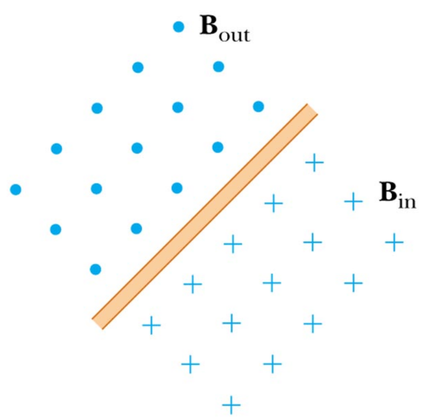
$$\vec{B}_k = \frac{\mu_o}{4\pi} \frac{I \vec{\Delta x} x \hat{r}}{r^2}$$

Example

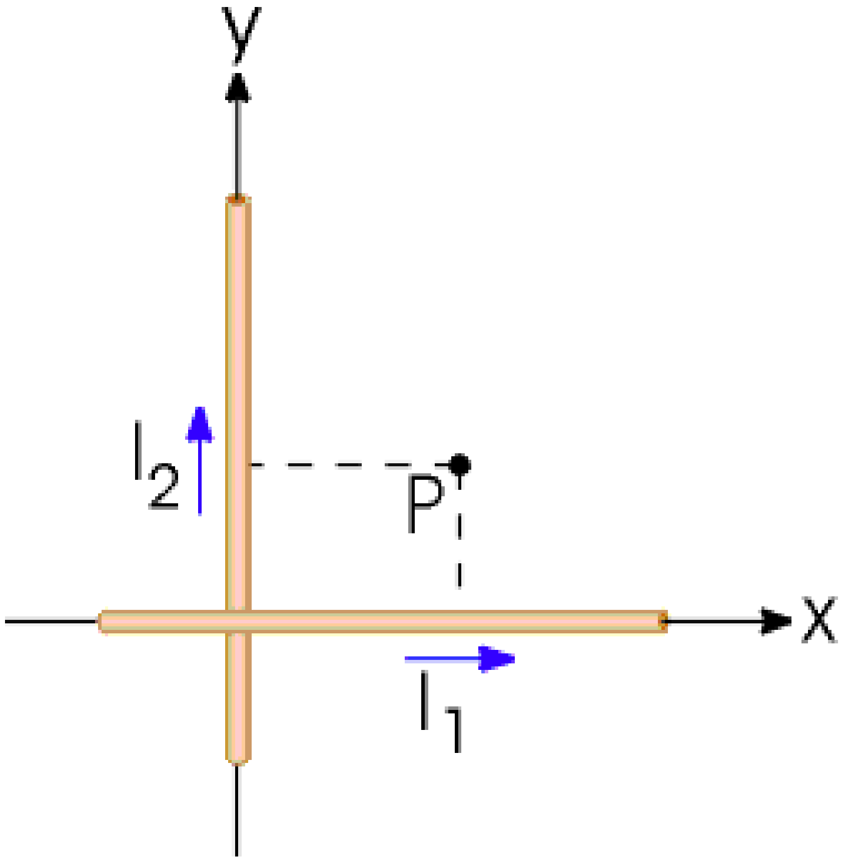
- Suppose I_1=6.69 A and I_2=5.40 A
- What is the magnetic field at point P (3.79 m, 2.14 m)?



(a)



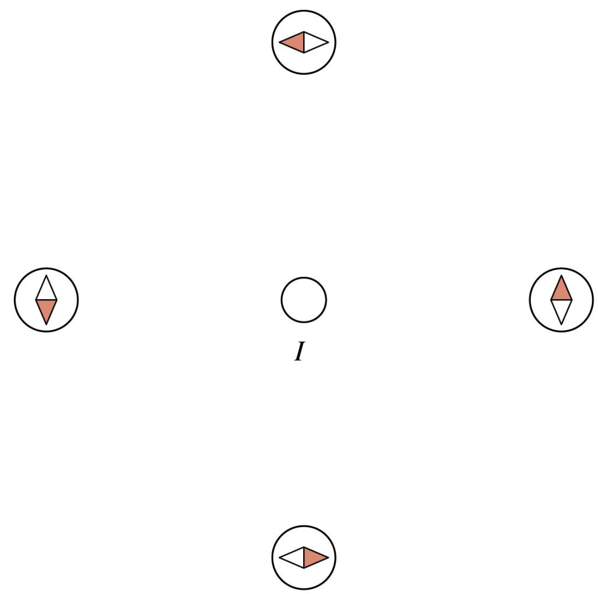
(c)



iclicker question

A long, straight wire extends into and out of the screen. The current in the wire is

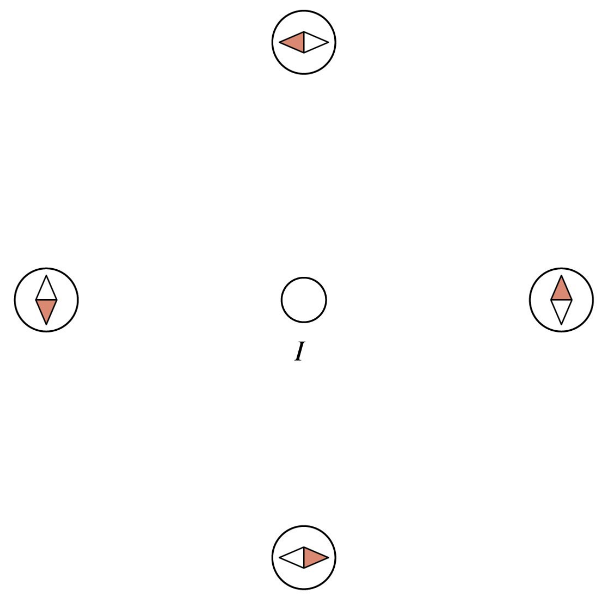
A. Into the screen.
B. Out of the screen.
C. There is no current in the wire.
D. Not enough info to tell the direction.



iclicker question

A long, straight wire extends into and out of the screen. The current in the wire is

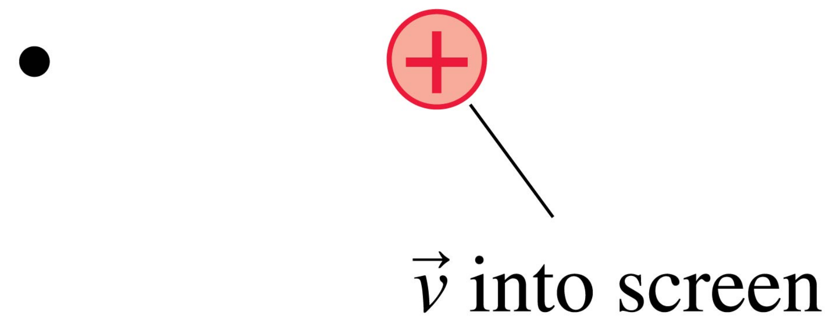
A. Into the screen.
B. Out of the screen.
C. There is no current in the wire.
D. Not enough info to tell the direction.



Iclicker question

What is the direction of the magnetic field at the position of the dot?

A. Into the screen.
B. Out of the screen.
C. Up.
D. Down.
E. Left.



Iclicker question

What is the direction of the magnetic field at the position of the dot?

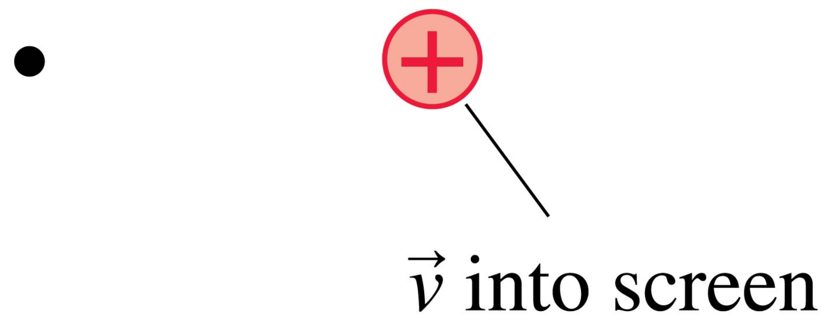
A. Into the screen.

B. Out of the screen.

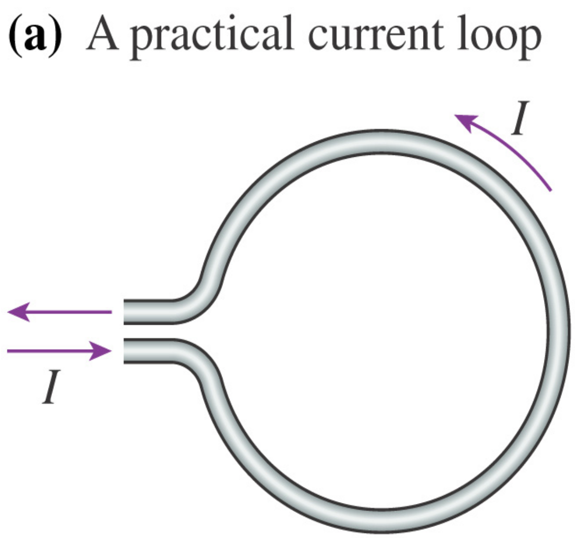
C. Up.

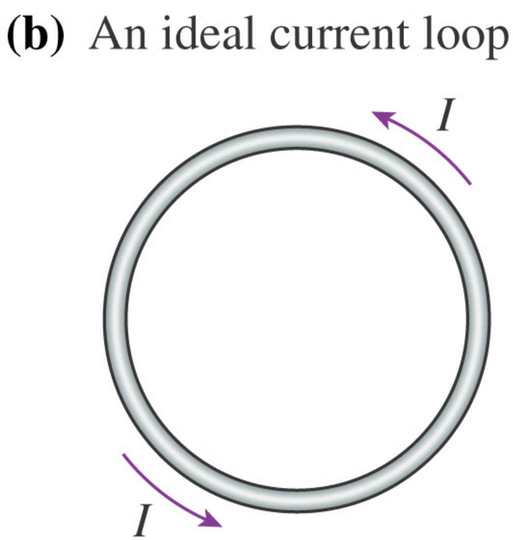
D. Down.

E. Left.

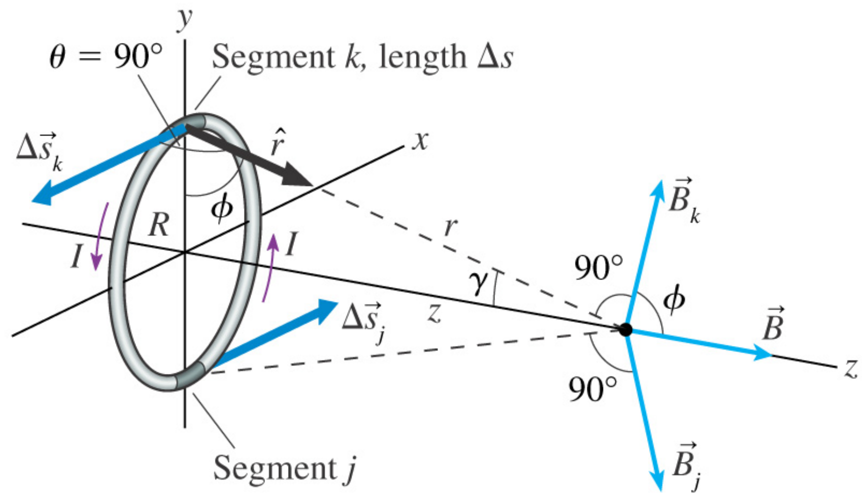


B from a current loop





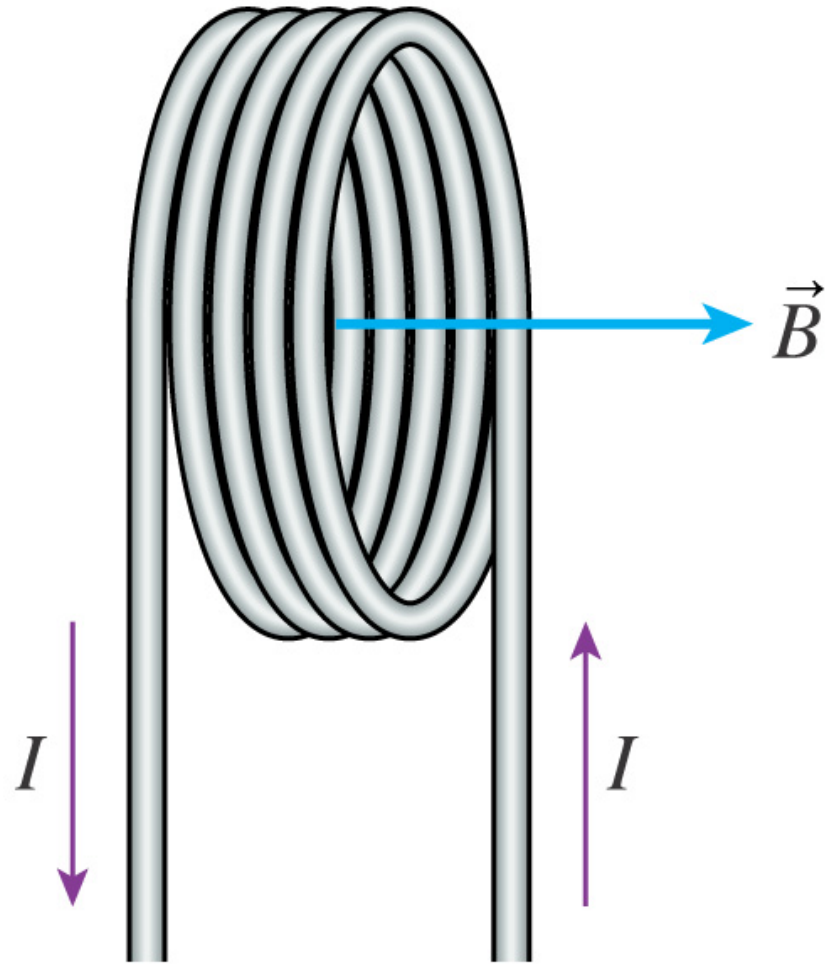
- Put the loop in the xy plane and the axis of the loop along the z axis



- Consider segment k; direction of $\vec{B}_k$ is given by the cross product $\Delta\vec{s}_k \times \hat{r}$; $\vec{B}_k$ is perpendicular to both $\Delta\vec{s}_k$ and to $\hat{r}$

B for a coil

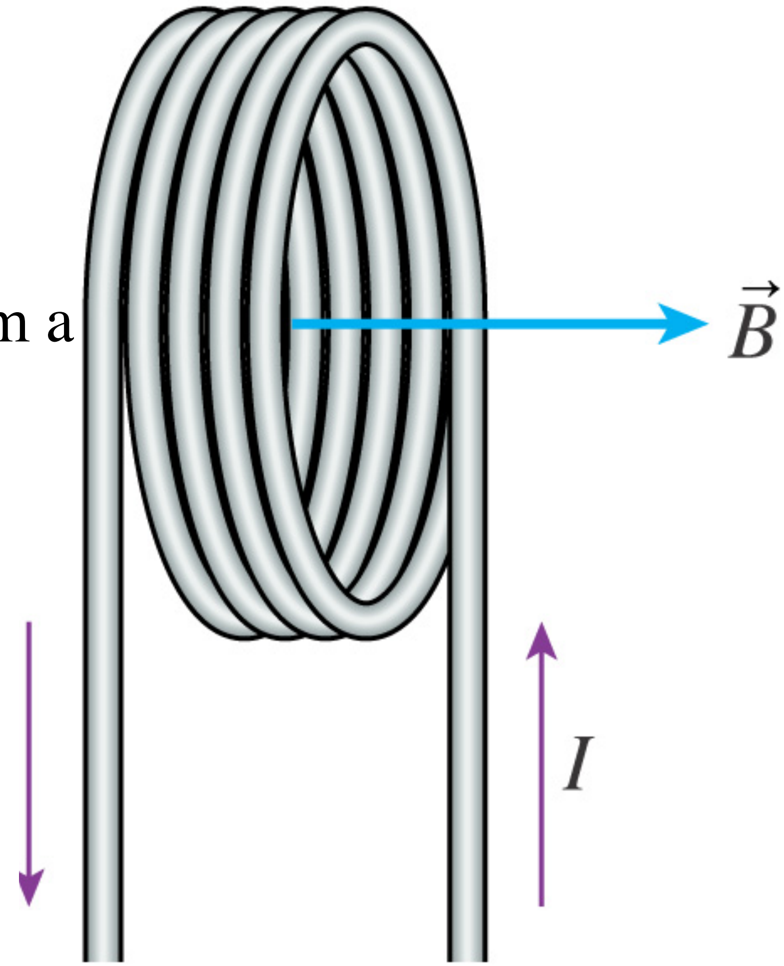
$$B_{coil,center} = \frac{\mu_o}{2}\frac{NI}{R_{coil}}$$



B for a coil

$$B_{coil,center} = \frac{\mu_o}{2} \frac{NI}{R_{coil}}$$

It's very easy to confuse this formula for the one for the B field from a long straight wire. No π here.



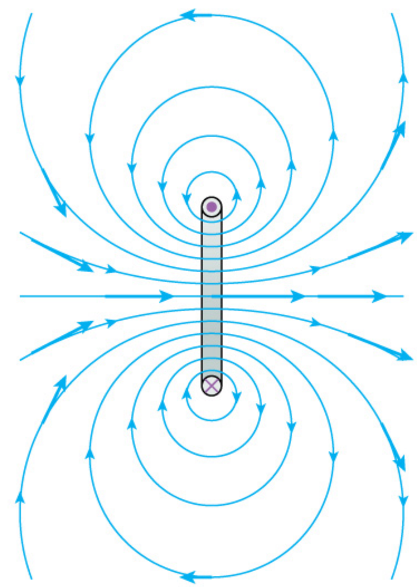
Magnetic field from a loop of current-3D

- To find the direction of the field, you can stick your thumb (right hand) in the direction of the current, and your fingers will curl in the direction of the magnetic field lines
    - can be painful but works in general
- …or you can curl your hand in the direction of the current and your thumb will point in the direction of the field
    - less painful but works only in the center of the loop
- Field along the z-axis is given by

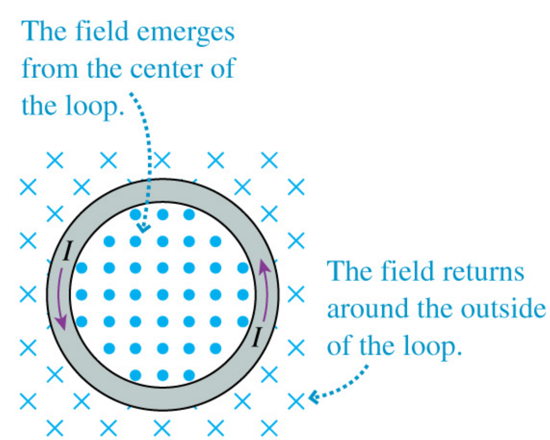
$$B_{z-axis} = \frac{\mu_o}{2} \frac{IR_{coil}^2}{(z^2 + R_{coil}^2)^{3/2}}$$

- In general case, calculation is pretty difficult

(a) Cross section through the current loop



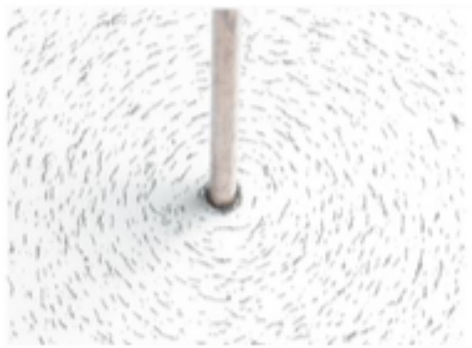
(b) The current loop seen from the right



demo

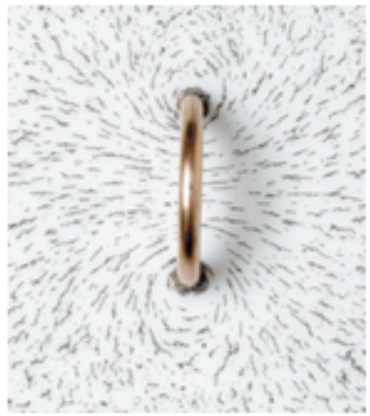
Electric Currents Create Magnetic Fields

A long, straight wire



$$\vec{B}_{wire} = \frac{\mu_0}{2\pi}\frac{I}{d}$$

A current loop

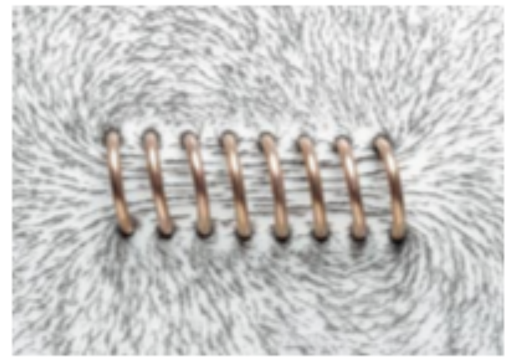


$$\vec{B}_{loop} = \frac{\mu_0}{2}\frac{IR^2}{\left(z^2 + R^2\right)^{3/2}}$$

B field at the center (z = 0) of a coil is:

$$\vec{B}_{\text{loop center}} = \frac{\mu_0}{2}\frac{I}{R}$$

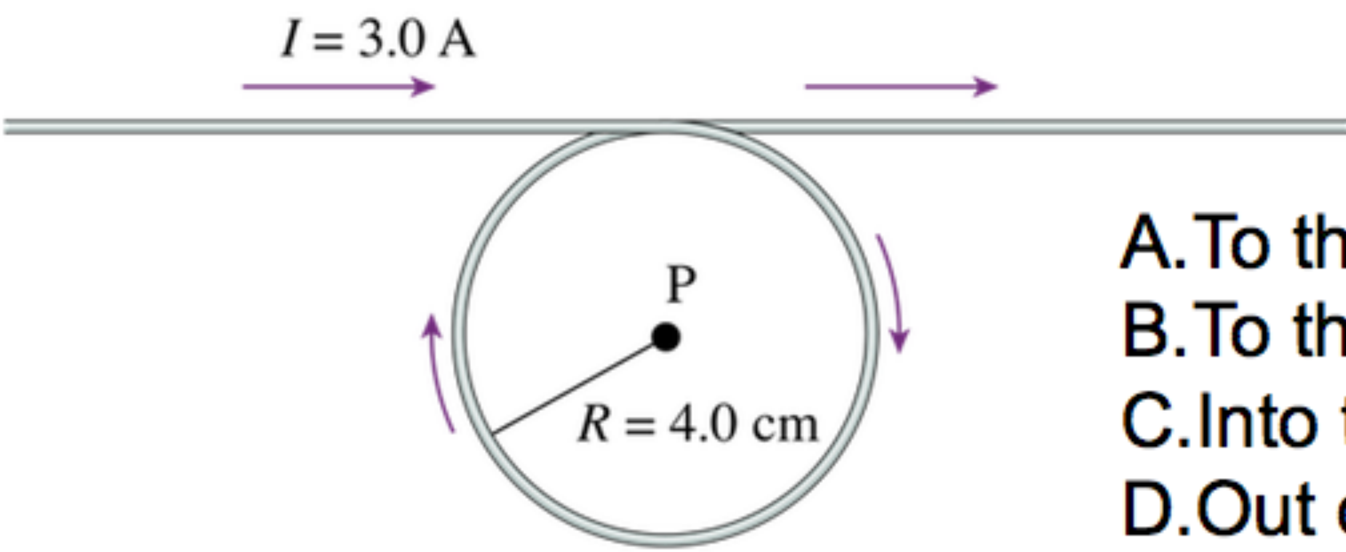
A N-turn Coil



$$\vec{B}_{\text{coil center}} = \frac{\mu_0}{2}\frac{NI}{R}$$

iclicker question

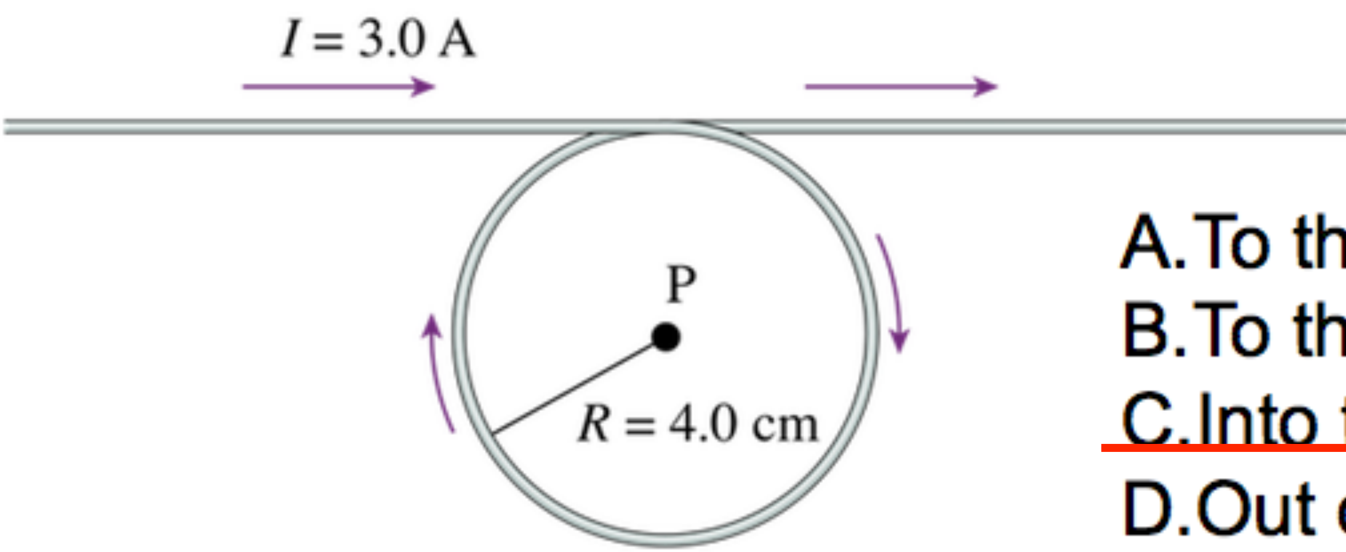
In what direction does the B field point at the center of the loop (point P)?



A. To the right
B. To the left
C. Into the page
D. Out of the page

iclicker question

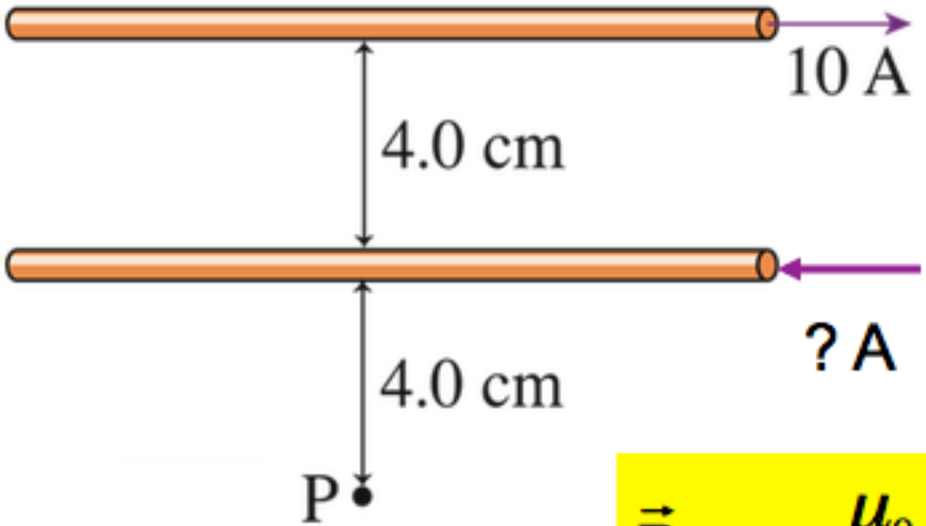
In what direction does the B field point at the center of the loop (point P)?



A. To the right
B. To the left
C. Into the page
D. Out of the page

Iclicker question

The magnetic field at point P is zero. What is the magnitude of the current in the lower wire?

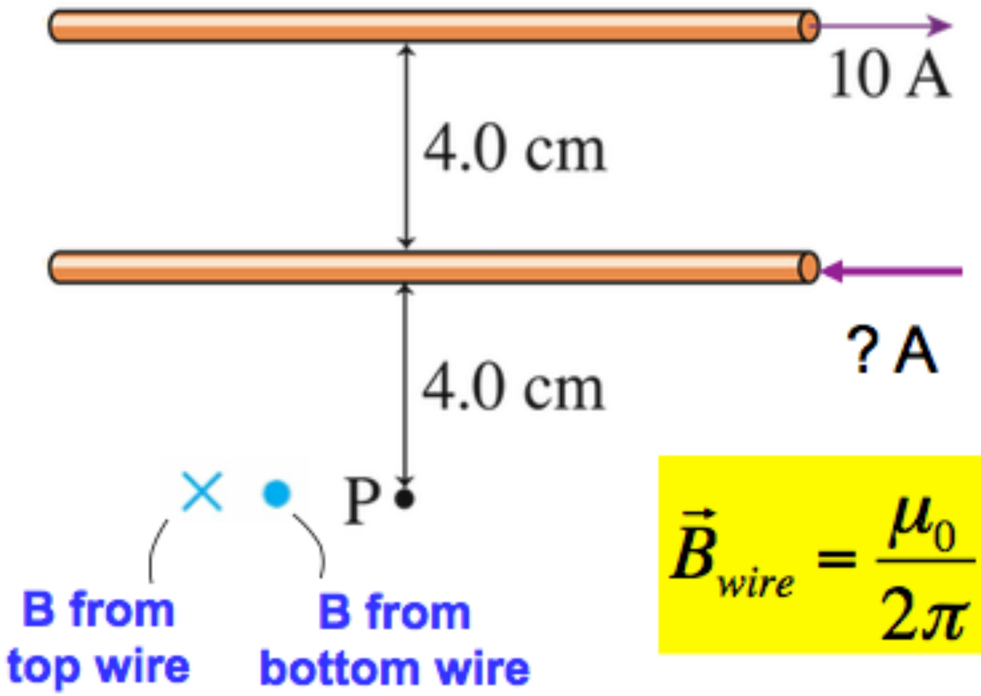


$$\vec{B}_{wire} = \frac{\mu_0}{2\pi} \frac{I}{d}$$

A. 100 A
B. 50 A
C. 25 A
D. 10 A
E. 5 A

Iclicker question

The magnetic field at point P is zero. What is the magnitude of the current in the lower wire?

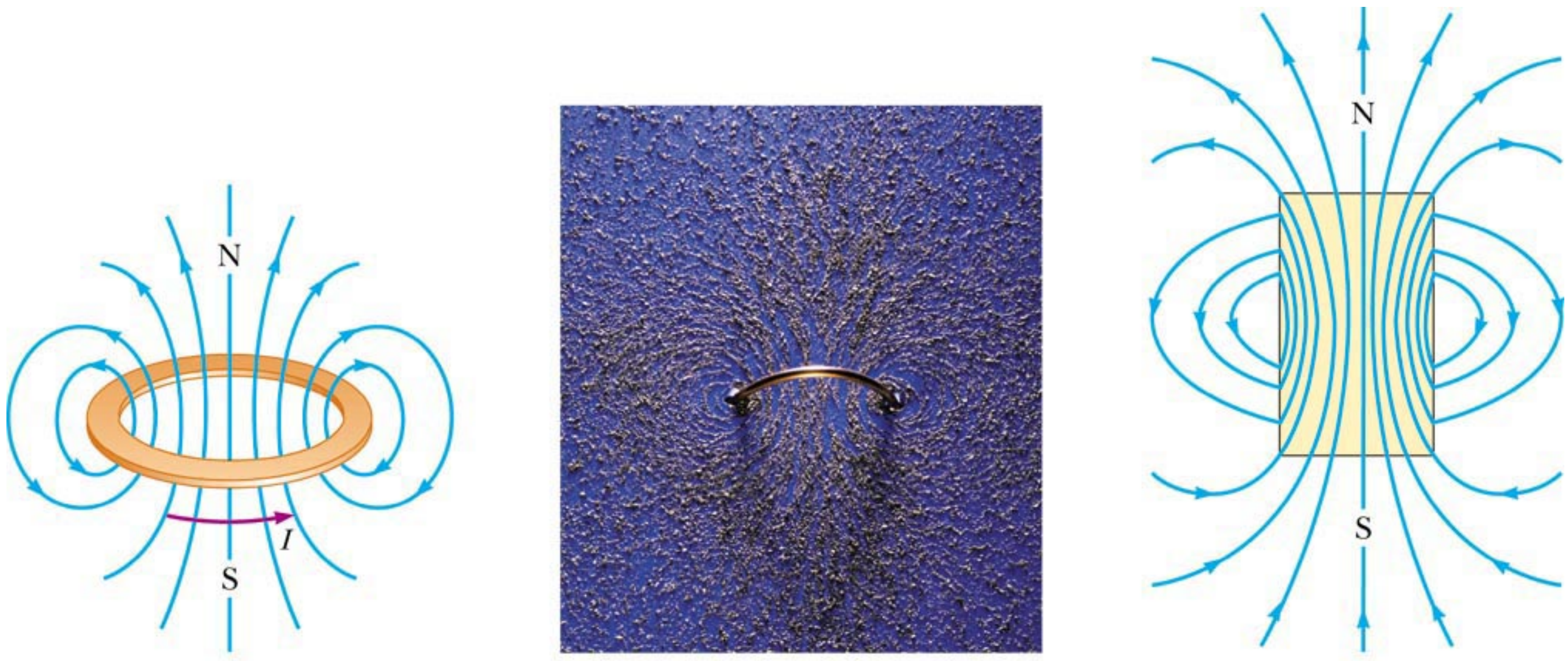


$$\vec{B}_{wire} = \frac{\mu_0}{2\pi}\frac{I}{d}$$

A. 100 A
B. 50 A
C. 25 A
D. 10 A
E. 5 A

Compare the magnetic field lines from coil and magnet

Both are magnetic dipoles



(a) (b) (c)

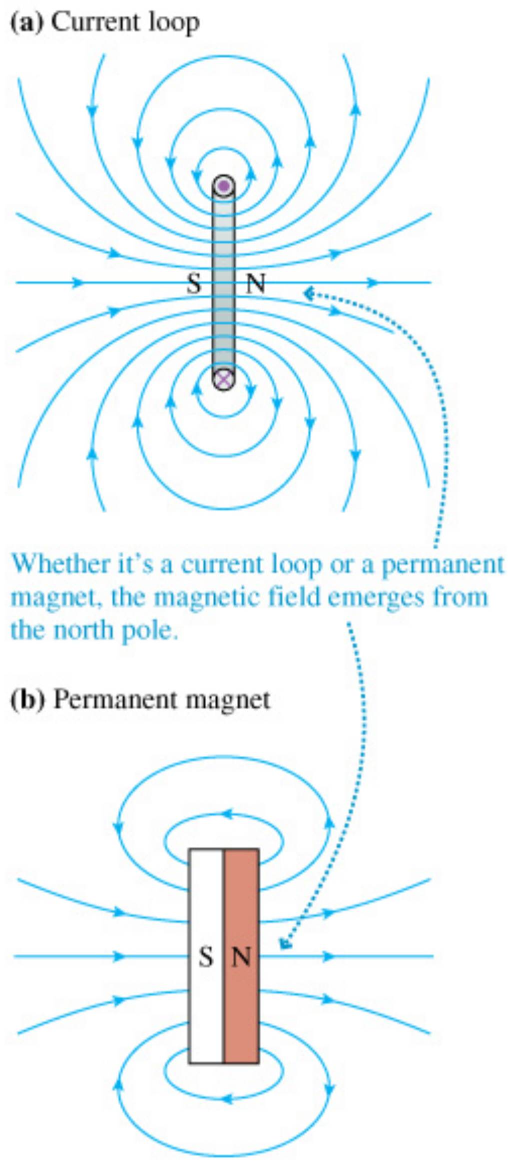
Dipoles

- A (flat) permanent magnet and a current loop generate the same magnetic field
- For current loop

$$B_{z-axis} = \frac{\mu_o}{2} \frac{IR_{coil}^2}{(z^2 + R_{coil}^2)^{3/2}}$$

- Let z>>R, then we can write

$$B_{z-axis} = \frac{\mu_o}{2} \frac{IR_{coil}^2}{z^3} = \frac{\mu_o}{2\pi} \frac{I\pi R_{coil}^2}{z^3} = \frac{\mu_o}{4\pi} \frac{2IA}{z^3}$$



Dipoles

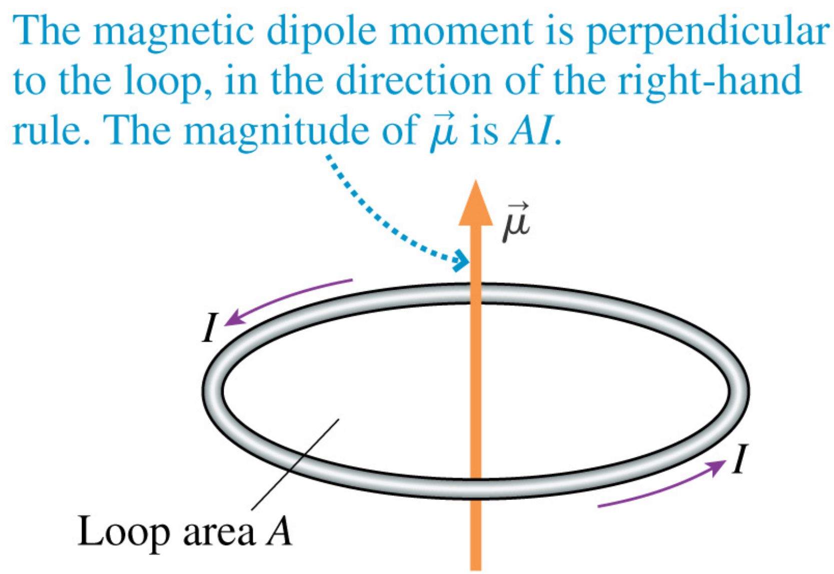
- Define magnetic dipole moment of magnitude μ=IA, then I can write the magnetic field for a current loop along the axis of the loop as

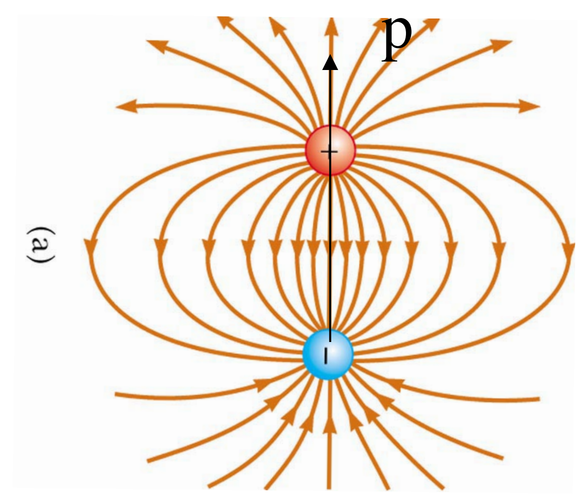
$$\vec{B}_{z-axis} = \frac{\mu_o}{4\pi}\frac{2\vec{\mu}}{z^3}$$

  - where the direction of $\vec{\mu}$ is the direction of $\vec{B}$ (from south pole to north pole)

- Remember that for the electric dipole, the field along the axis is given by

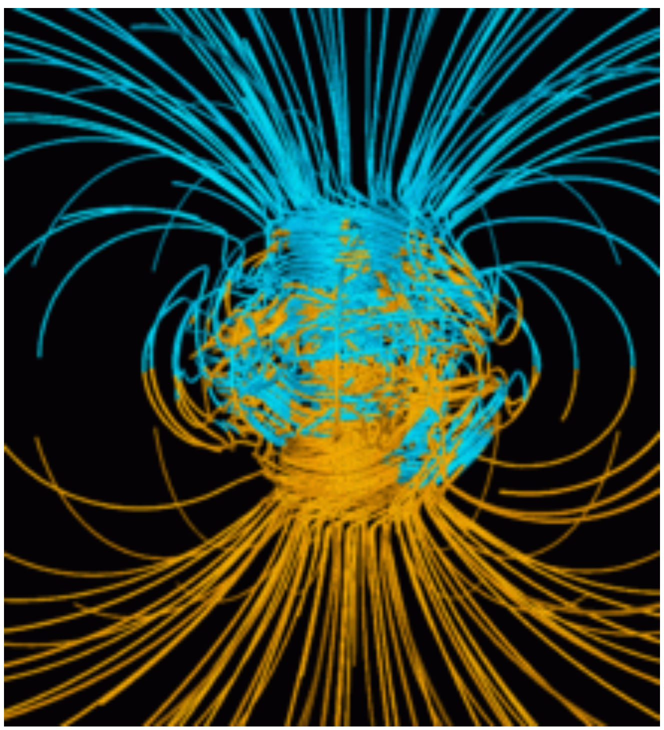
$$\vec{E}_{z-axis} = \frac{1}{4\pi\varepsilon_o}\frac{2\vec{p}}{z^3}$$

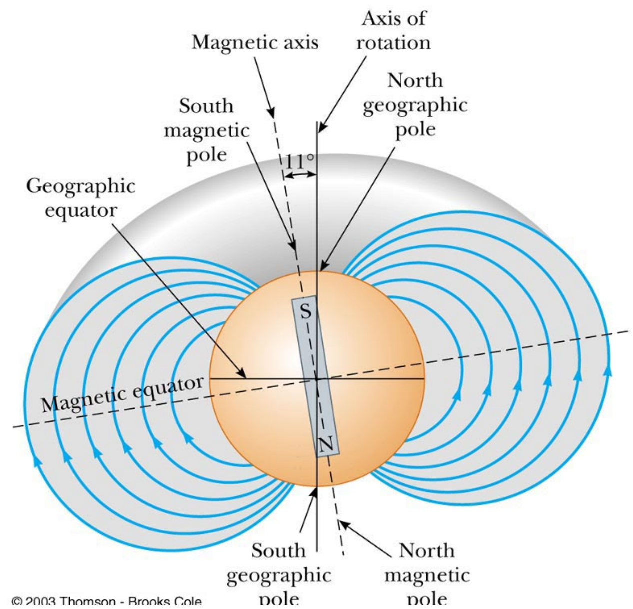


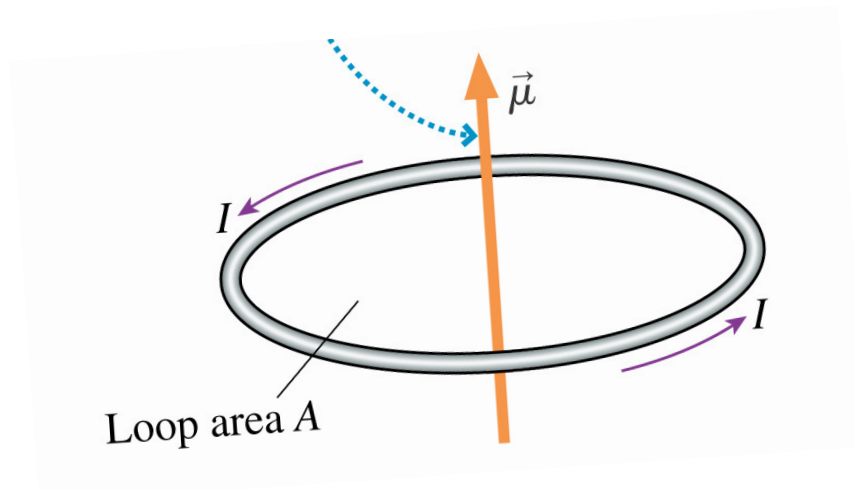


Example

- The earth's magnetic dipole moment is 8.0×10^{22} A·m^2
  - Suppose the current is in a ring of radius 2×10^6 m (in the molten interior); what is the current?







Ampere's law

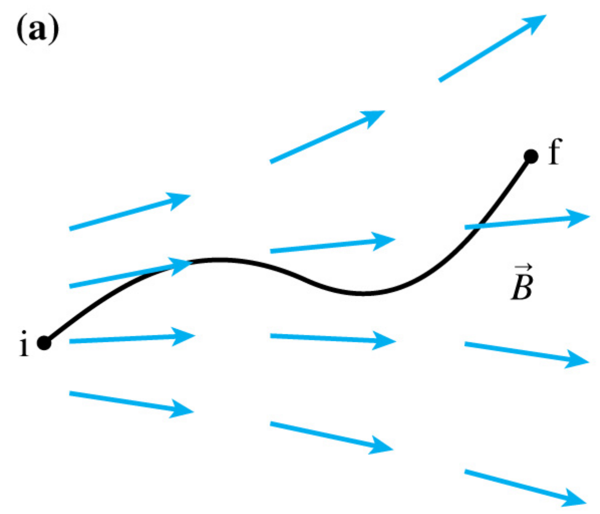
- We could use Coulomb's law to calculate the electric field for any charge configuration, but we found that using Gauss' law was much easier for situations where there was a great deal of symmetry
- Similarly we can calculate the magnetic field from any configuration of currents using the Biot-Savart law, but we can calculate the magnetic field for symmetric situations much easier with Ampere's law
- Gauss' law involves a surface integral; Ampere's law involves a line integral

Andre Ampere

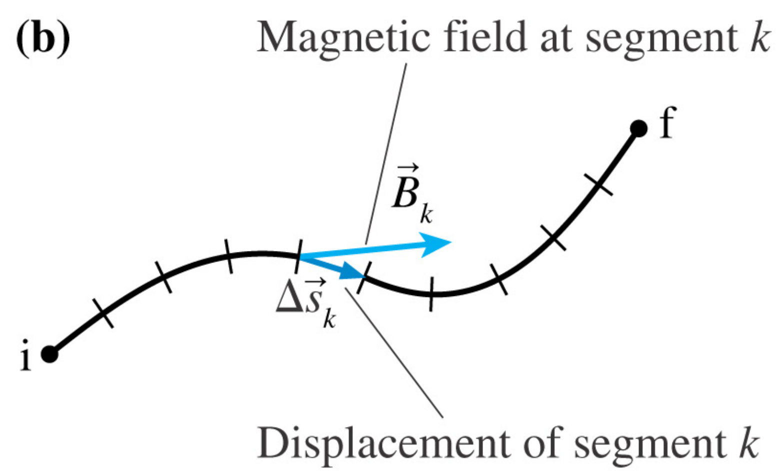
Line integral

- Suppose I have a magnetic field in a region of space and I want to integrate the dot product of B and s along the path from i to f
- Then I get

$$\sum_k \vec{B}_k \cdot \vec{\Delta s}_k \rightarrow \int_i^f \vec{B} \cdot \vec{ds}$$

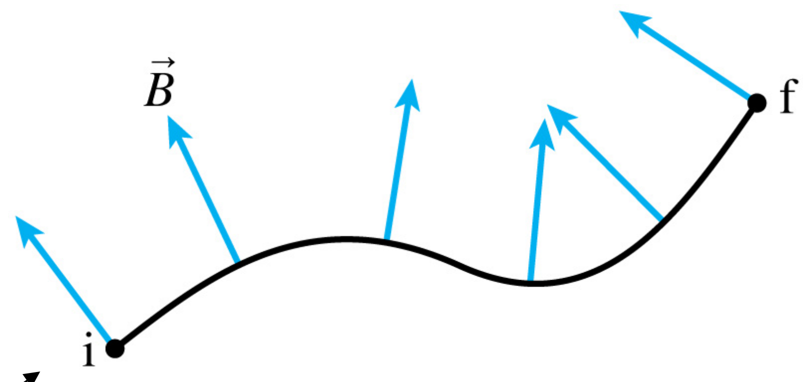


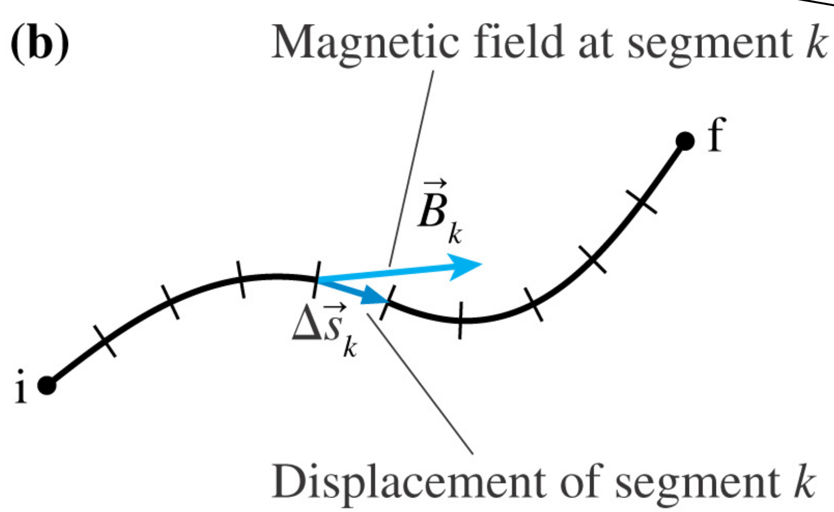
The line passes through a magnetic field.

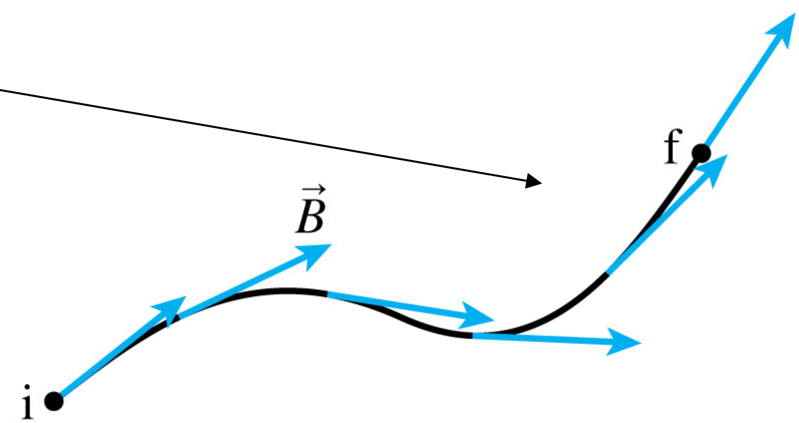


Line integrals

- In general this line integral can be quite complex
- However, like for Gauss' law, we only use it only simple situations, especially when $\vec{B}\cdot d\vec{s}=0$ and $\vec{B}\cdot d\vec{s}=Bds$







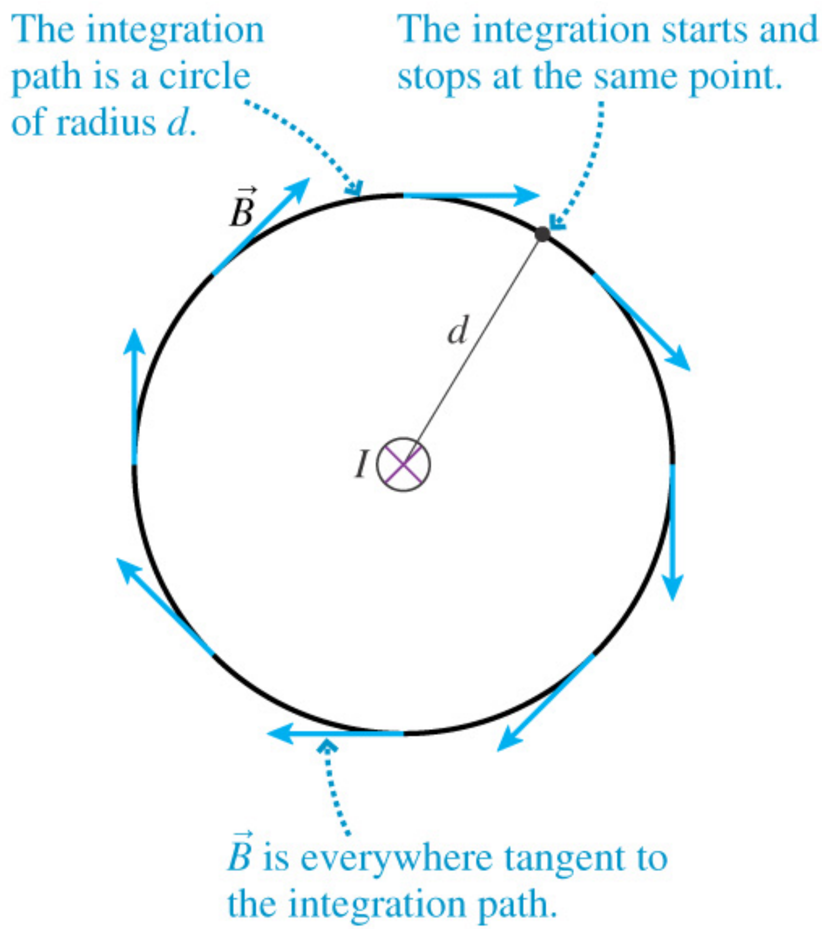
Ampere’s law

- So Ampere noted that the electric field from a long straight current was given by

$$B = \frac{\mu_o I}{2\pi d}$$

- …and that the field was everywhere tangent to a circle with radius d
- In this case, it’s easy to evaluate an integral around the entire circle

$$\oint \vec{B} \bullet \vec{ds} = B(2\pi d) = \mu_o I$$



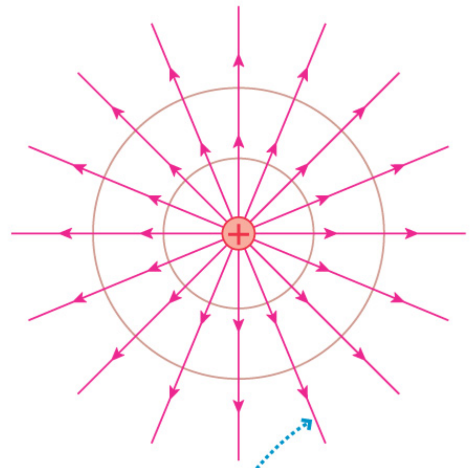
Remember Gauss' law

- The integral over the closed surface did not depend on the shape/volume of the surface, only the charge enclosed

$$\int_A \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

- A similar result holds for Ampere's law

$$\oint \vec{B} \cdot d\vec{s} = \mu_o I_{enclosed}$$



Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.