

PHY294H

- Professor: Joey Huston
- email: huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
 - ◆ **Help-room hours: 12:40-2:40 Monday (note change);
3:00-4:00 PM Friday**
 - ◆ **hand-in problem for next Wed: 31.79**
- Quizzes by iclicker (sometimes hand-written)
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
 - ◆ lectures will be posted frequently, mostly every day if I can remember to do so

Ampere's law

- We could use Coulomb's law to calculate the electric field for any charge configuration, but we found that using Gauss' law was much easier for situations where there was a great deal of symmetry
- Similarly we can calculate the magnetic field from any configuration of currents using the Biot-Savart law, but we can calculate the magnetic field for symmetric situations much easier with Ampere's law
- Gauss' law involves a surface integral; Ampere's law involves a line integral



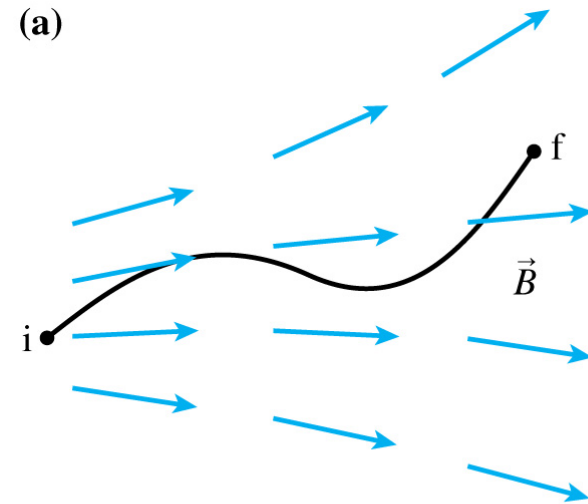
Andre Ampere

Line integral

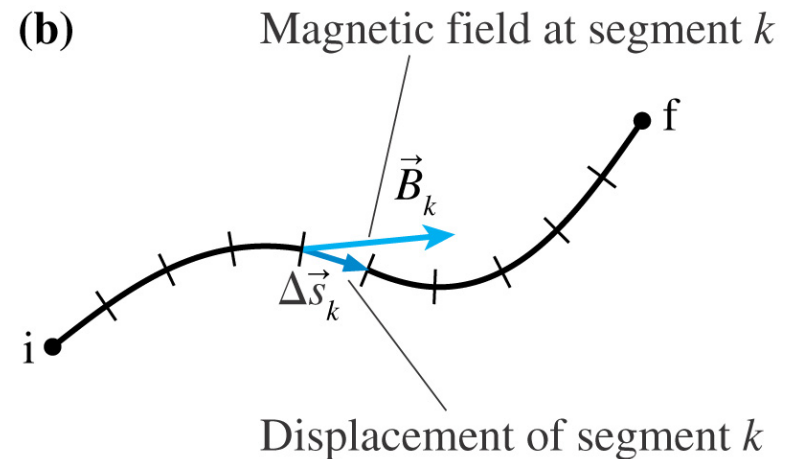
- Suppose I have a magnetic field in a region of space and I want to integrate the dot product of \vec{B} and \vec{s} along the path from i to f

- Then I get

$$\sum_k \vec{B}_k \cdot \Delta \vec{s}_k \rightarrow \int_i^f \vec{B} \cdot d\vec{s}$$

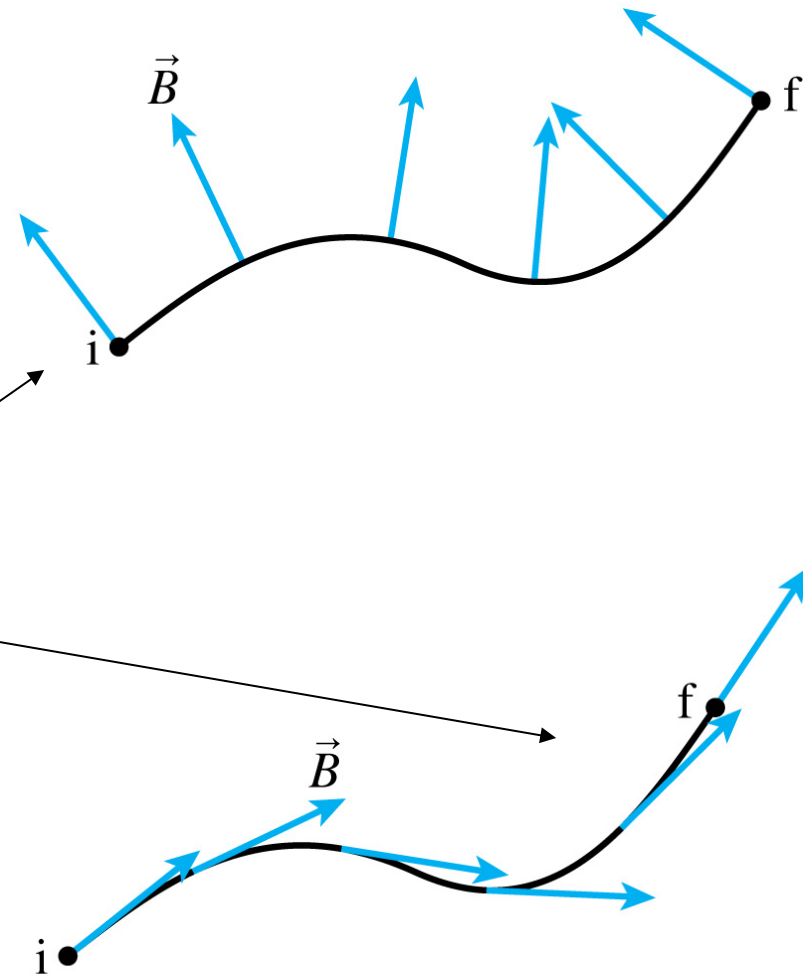
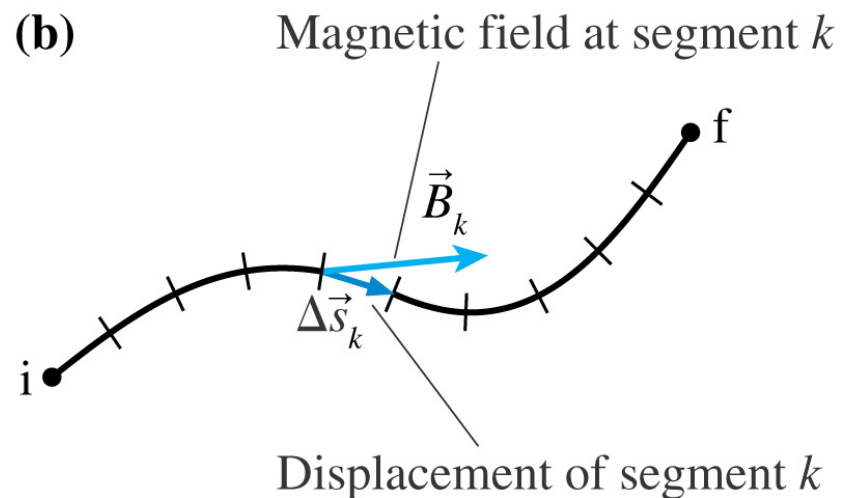


The line passes through a magnetic field.



Line integrals

- In general this line integral can be quite complex
- However, like for Gauss' law, we only use it only simple situations, especially when $\vec{B} \cdot d\vec{s} = 0$ and $\vec{B} \cdot d\vec{s} = B ds$



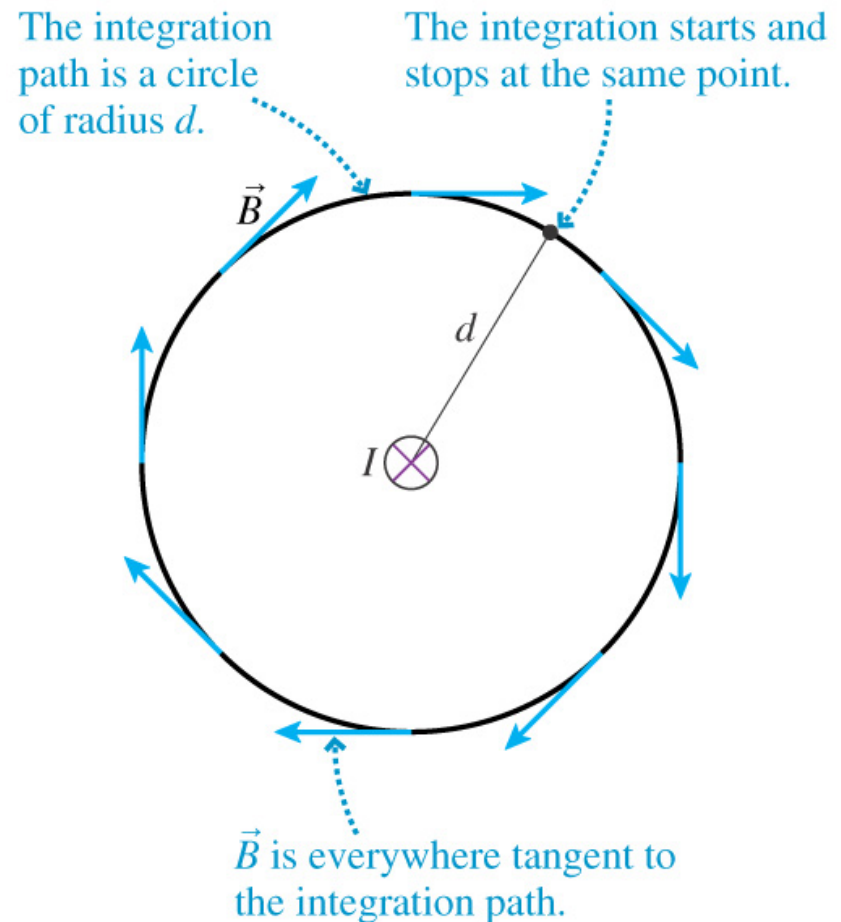
Ampere's law

- So Ampere noted that the electric field from a long straight current was given by

$$B = \frac{\mu_o I}{2\pi d}$$

- ...and that the field was everywhere tangent to a circle with radius d
- In this case, it's easy to evaluate an integral around the entire circle

$$\oint \vec{B} \cdot d\vec{s} = B(2\pi d) = \mu_o I$$



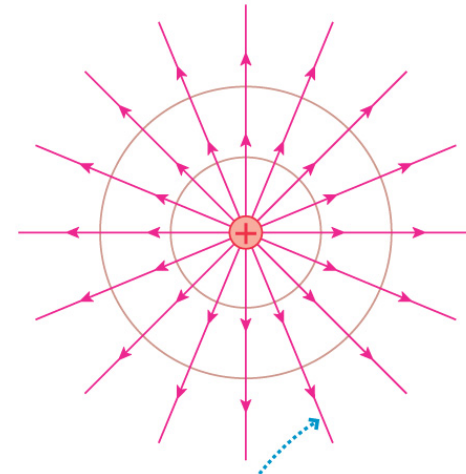
Remember Gauss' law

- The integral over the closed surface did not depend on the shape/volume of the surface, only the charge enclosed

$$\int_A \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_o}$$

- A similar result holds for Ampere' s law

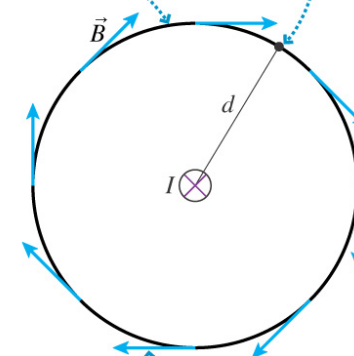
$$\oint \vec{B} \cdot d\vec{s} = \mu_o I_{enclosed}$$



Every field line passing through the smaller sphere also passes through the larger sphere. Hence the flux through the two spheres is the same.

The integration path is a circle of radius d .

The integration starts and stops at the same point.



\vec{B} is everywhere tangent to the integration path.

Consider another closed path

- I integrate along the two circular paths C_1 and C_2 (and the radial segments ab and cd)

$$\begin{aligned}\oint_C \vec{B} \cdot d\vec{s} &= \oint_{C_1} \vec{B} \cdot d\vec{s} + \oint_{C_2} \vec{B} \cdot d\vec{s} \\ &= -B_1(2\pi r_1) + B_2(2\pi r_2)\end{aligned}$$

- ◆ along ab and cd , the integrals are zero because B is perpendicular to ds

- No current is enclosed by the full curve; the integral is zero

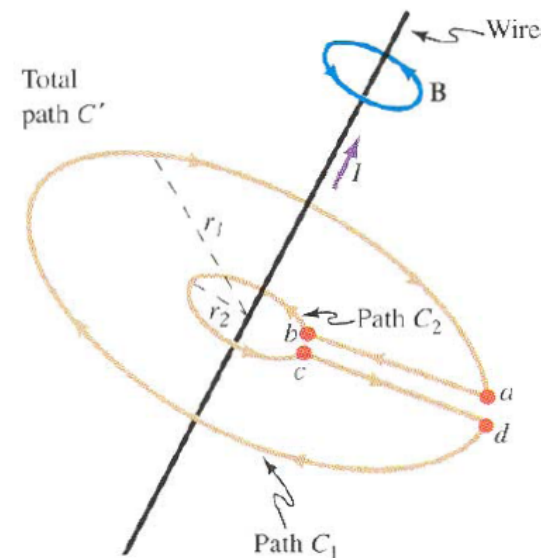


FIGURE 30-8 Path C' consists of a clockwise circle C_1 of radius r_1 , a leg a that moves in to a distance r_2 from the wire, a counterclockwise circle C_2 of radius r_2 , and a leg b that moves out to r_1 . The magnetic field follows a counterclockwise circle.

Magnetic field inside a wire

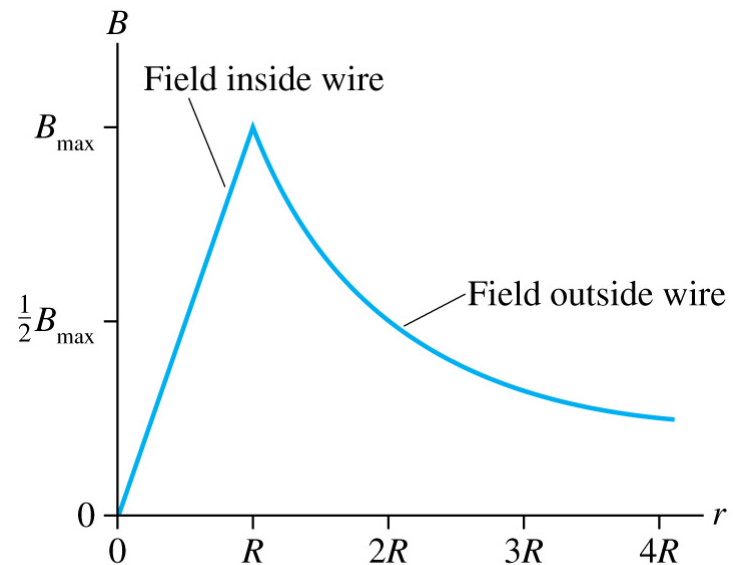
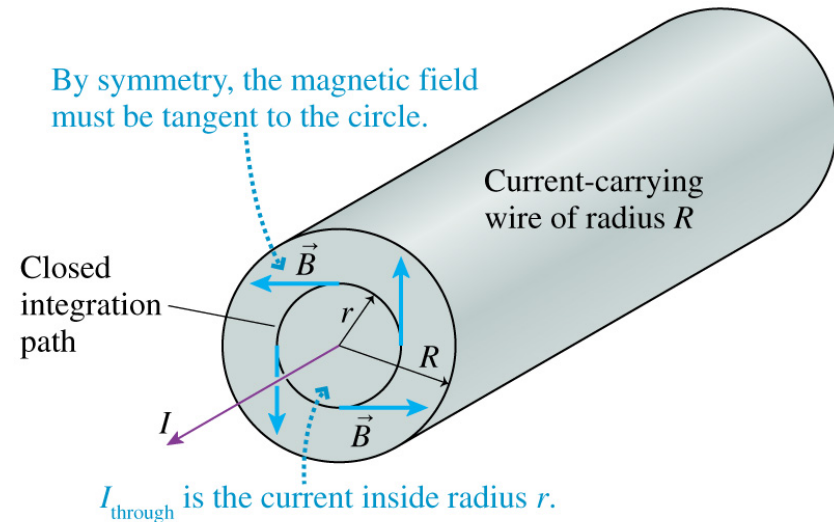
- We know what the magnetic field looks like outside a wire carrying a current I
- What about inside?
- We can use Ampere's law

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{\text{enclosed}}$$

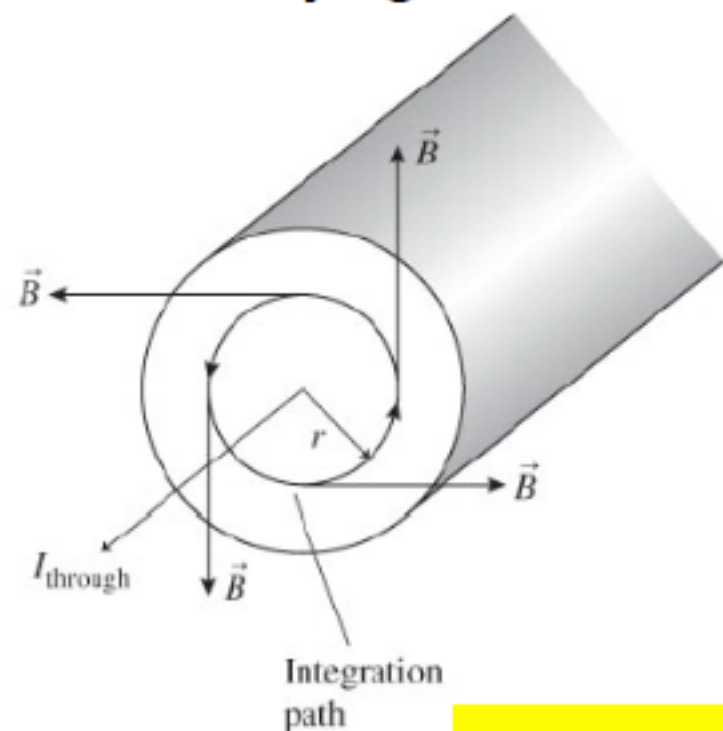
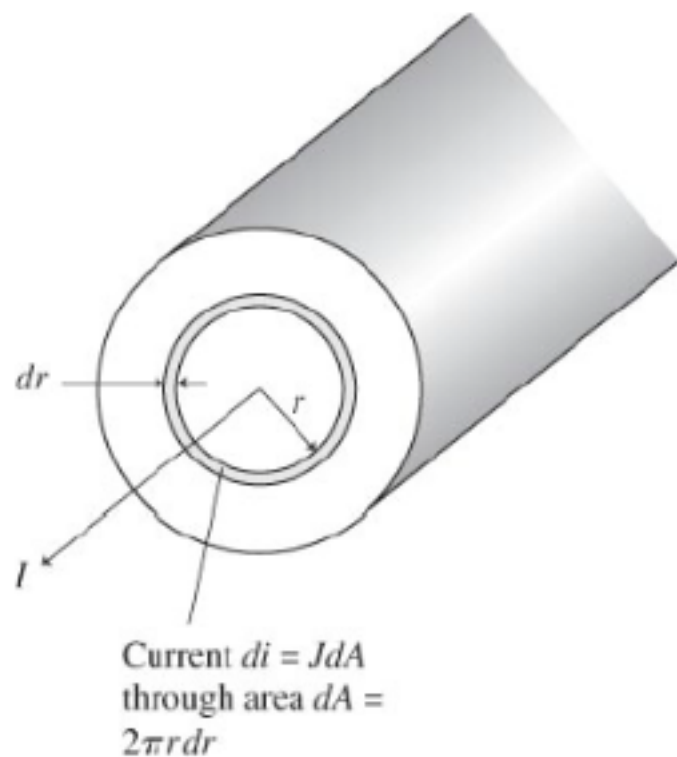
$$I_{\text{enclosed}} = \frac{\pi r^2}{\pi R^2} I = \frac{r^2}{R^2} I$$

$$B(2\pi r) = \mu_o \frac{r^2}{R^2} I$$

$$B = \frac{\mu_o I r}{2\pi R^2}$$



Find an expression for the B field strength inside a wire (at radius r) that has a **non-uniform current density**, $J = J_0 r/R$, where J_0 is a constant. At $r = R$, does this result agree with the B field outside a straight current-carrying wire?

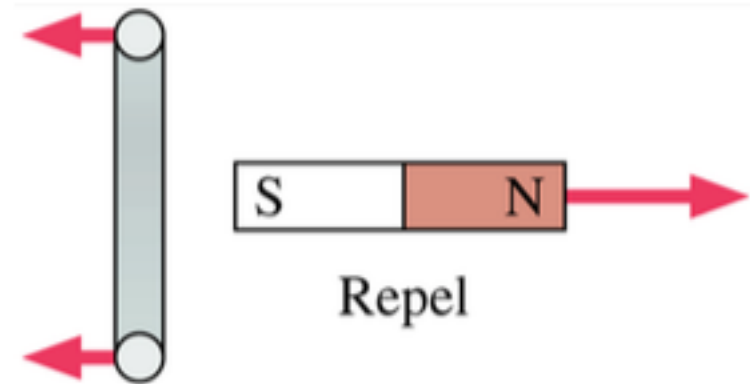


$$\vec{B}_{\text{wire}} = \frac{\mu_0}{2\pi} \frac{I}{R}$$

iclicker question

What is the current direction in the loop?

- A. Out of the page at the top of the loop, into the page at the bottom.
- B. Out of the page at the bottom of the loop, into the page at the top.

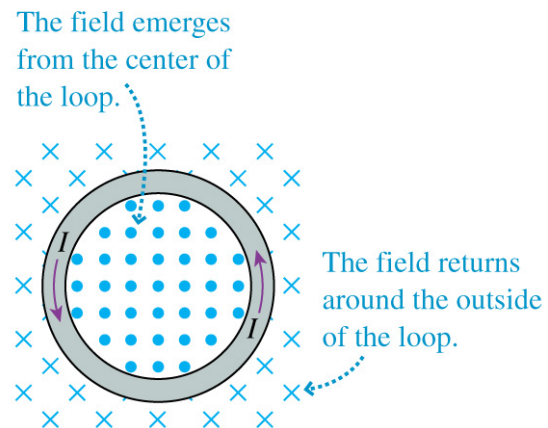


Magnetic field from a coil

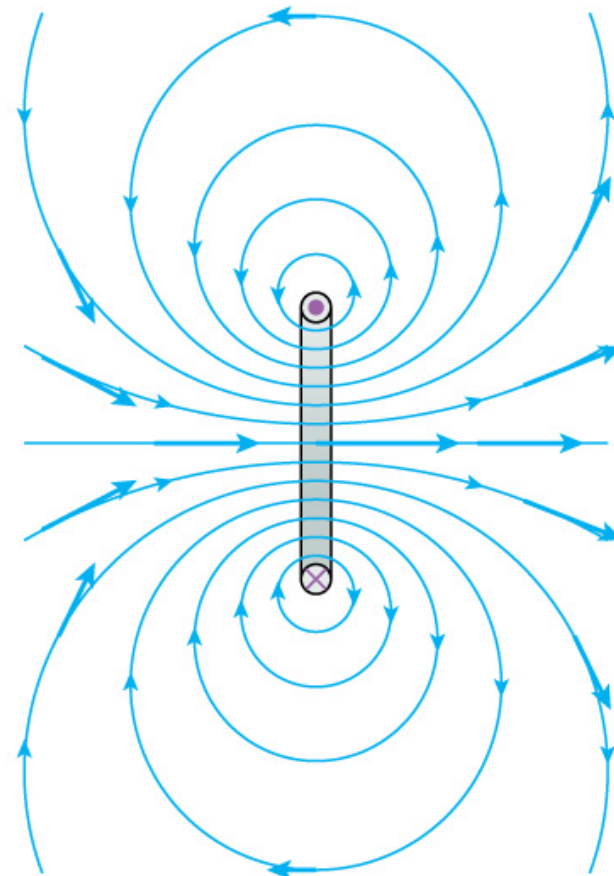
- Field along the z-axis is given by

$$B_{z-axis} = \frac{\mu_o}{2} \frac{IR_{coil}^2}{(z^2 + R_{coil}^2)^{3/2}}$$

(b) The current loop seen from the right



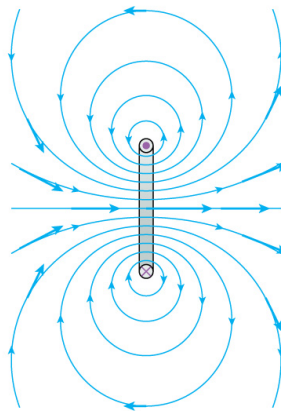
(a) Cross section through the current loop



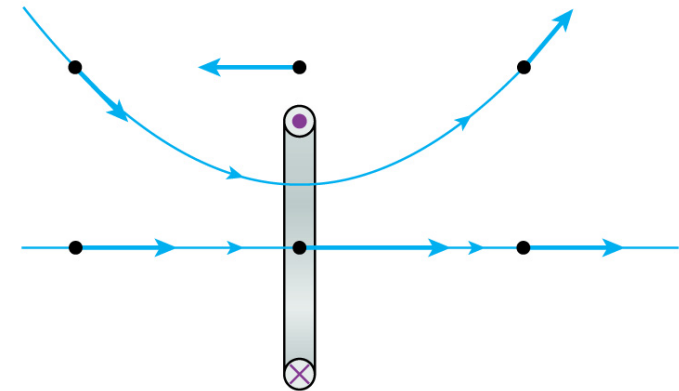
Stacking loops together

- The total magnetic field is the sum of the magnetic fields from the individual coils
- Note that the fields reinforce each other inside the coil volumes but fight against each other outside

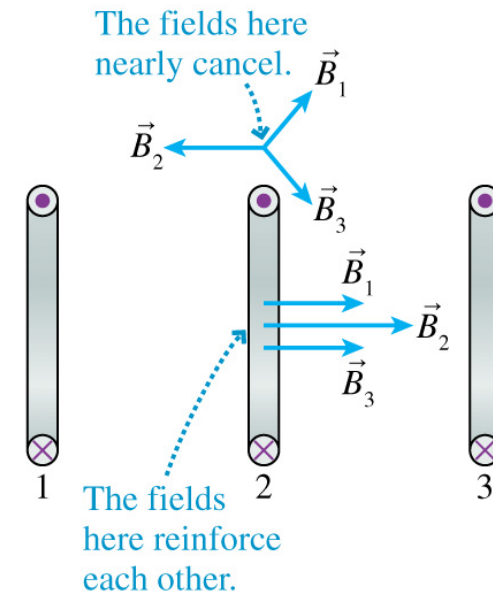
(a) Cross section through the current loop



(a) A single loop

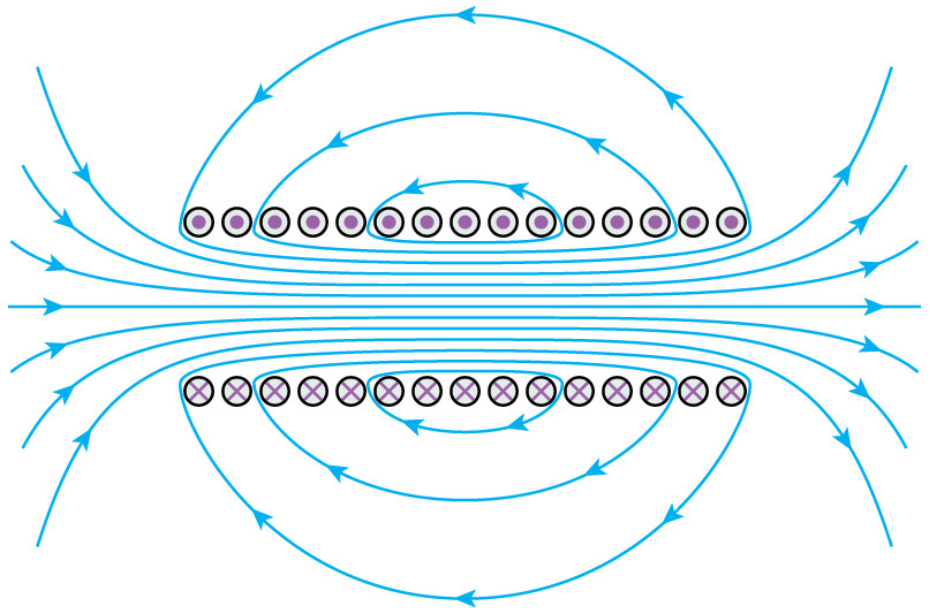


(b) A stack of three loops



Solenoid

- If I stack many coils close together, I have a solenoid
- The magnetic field is very uniform inside the solenoid and close to zero outside
- In an ideal solenoid, the field is perfectly uniform inside the solenoid and zero outside



Magnetic field inside a solenoid

- Can use Ampere's law to calculate the field inside a solenoid

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_o I_{\text{enclosed}}$$

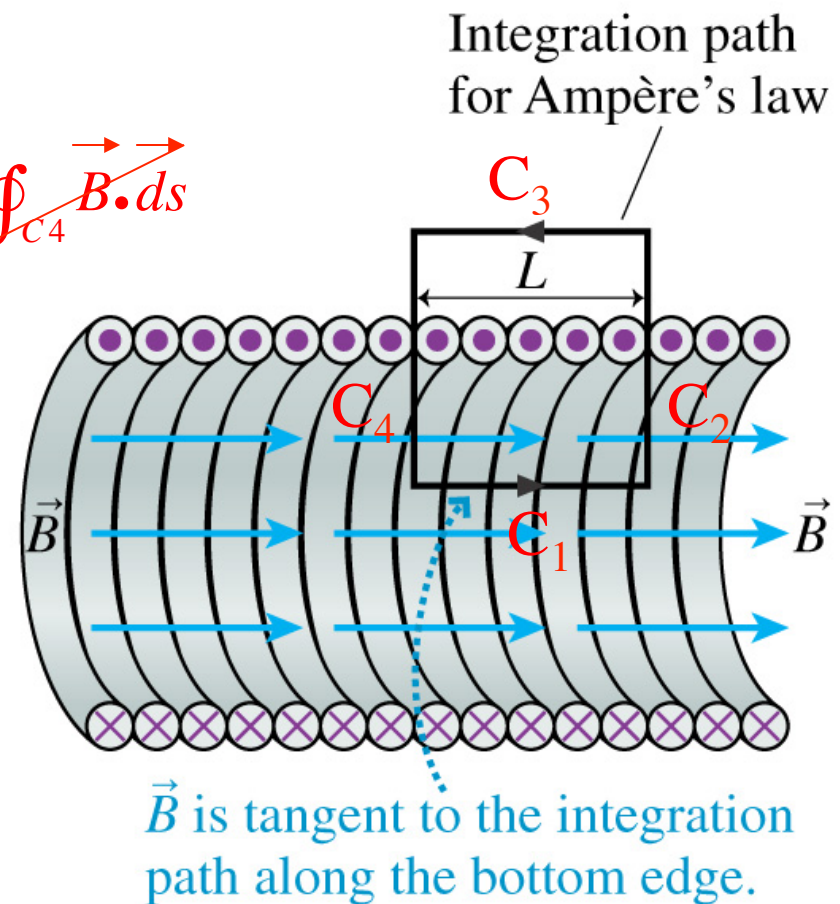
$$= \oint_{C_1} \vec{B} \cdot d\vec{s} + \cancel{\oint_{C_2} \vec{B} \cdot d\vec{s}} + \cancel{\oint_{C_3} \vec{B} \cdot d\vec{s}} + \cancel{\oint_{C_4} \vec{B} \cdot d\vec{s}}$$

$$= BL = \mu_o NI$$

- where N is the number of coils in a length L of the solenoid

$$B = \frac{\mu_o NI}{L} = \mu_o nI$$

- where n is the number of coils per unit length

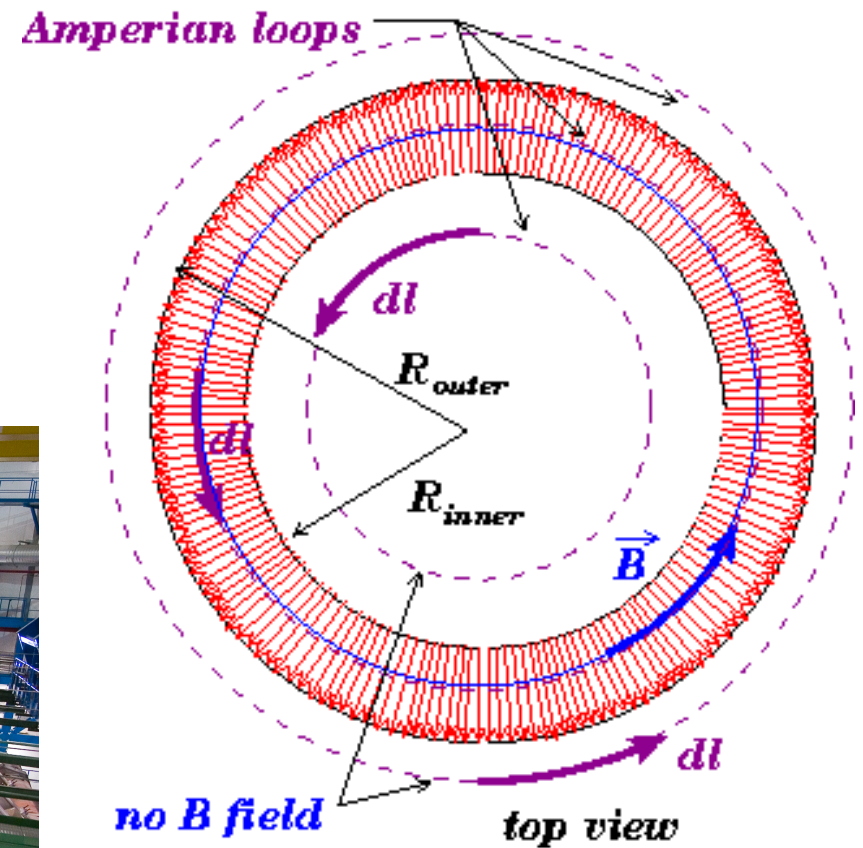


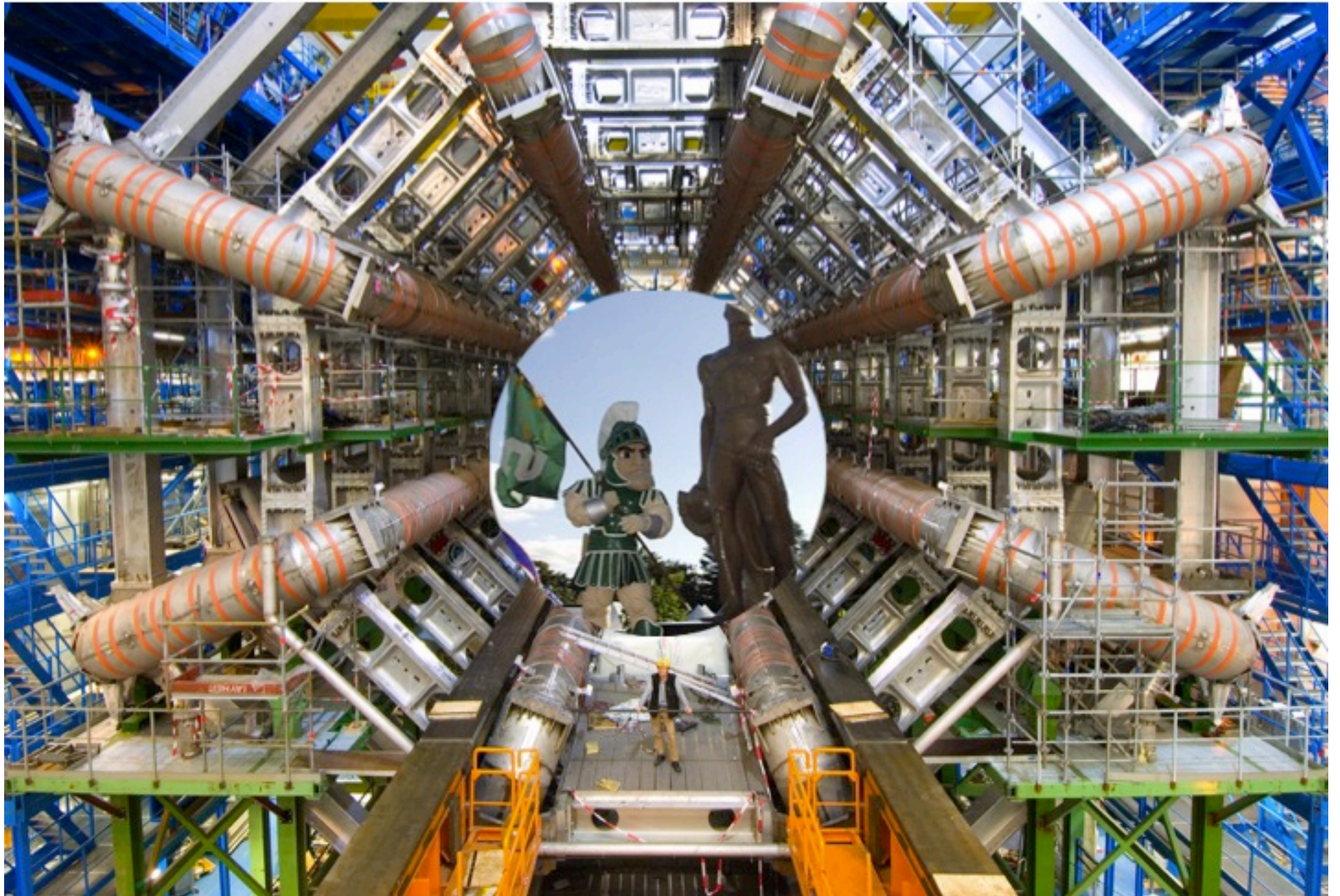
Toroids

$$\oint \vec{B} \cdot d\vec{l} = \mu_o \sum I_{\text{enclosed}}$$

$$B(2\pi r) = \mu_o NI$$

$$B = \frac{\mu_o NI}{2\pi r}$$

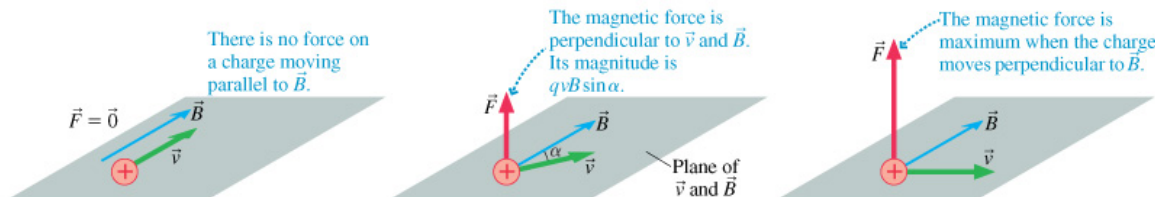




Magnetic forces on electric charges

- So we believe in the existence of magnetic fields
 - ◆ and magnets exert forces on each other
- What kind of force is exerted on an electric charge?
- Suppose I do a series of experiments involving electric charges and magnetic fields

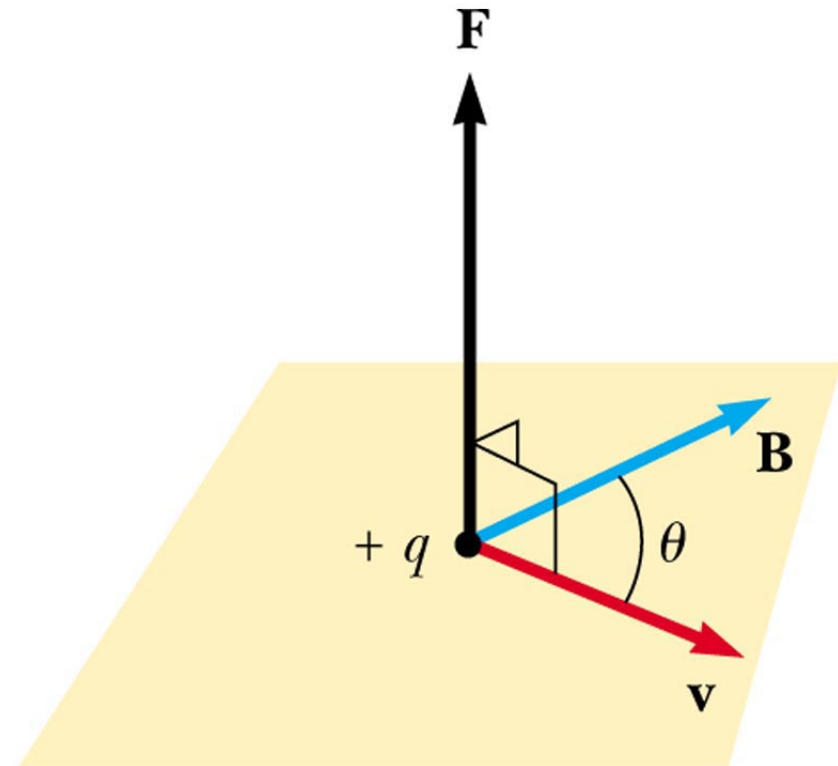
a charge q in a region of magnetic field



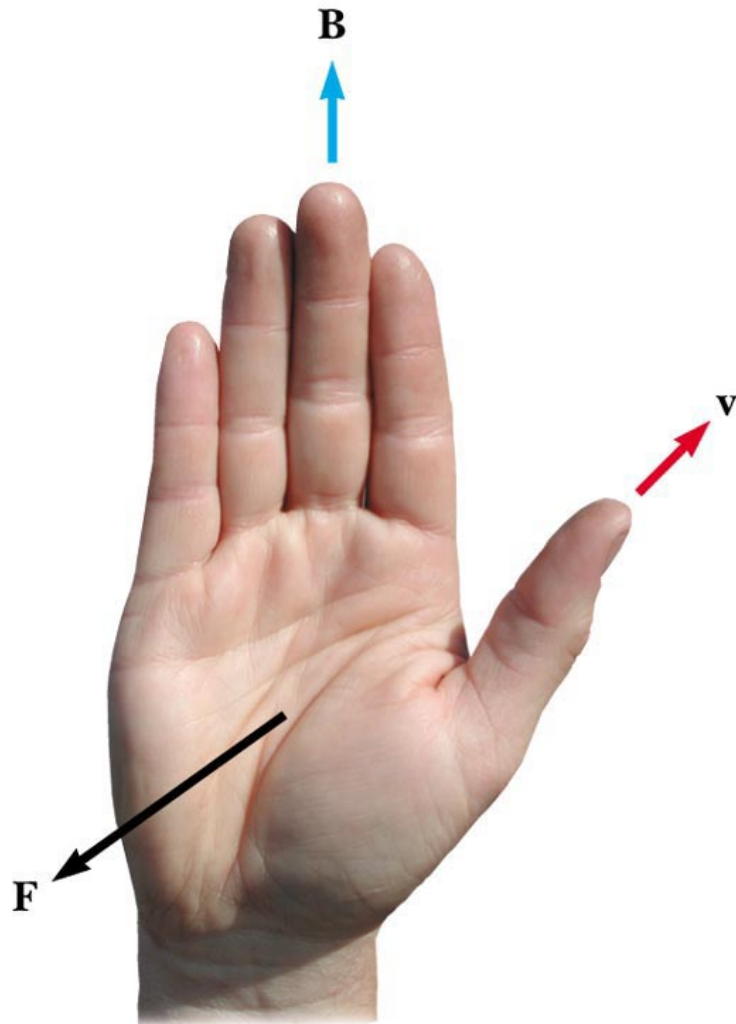
1. The force on q is proportional to the magnitude of q
 2. The force on q is proportional to the magnitude of B
 3. The force on a negative charge is opposite the force on a positive charge
 4. The force is proportional to the velocity v
 5. The magnitude and direction of F depends on the angle between v and B
1. If v is parallel or anti-parallel to B , F is zero
 2. If v is not parallel to B , force is perpendicular to B and to v
 3. If v makes an angle of θ to B , then F is proportional to $\sin \theta$

Magnetic force

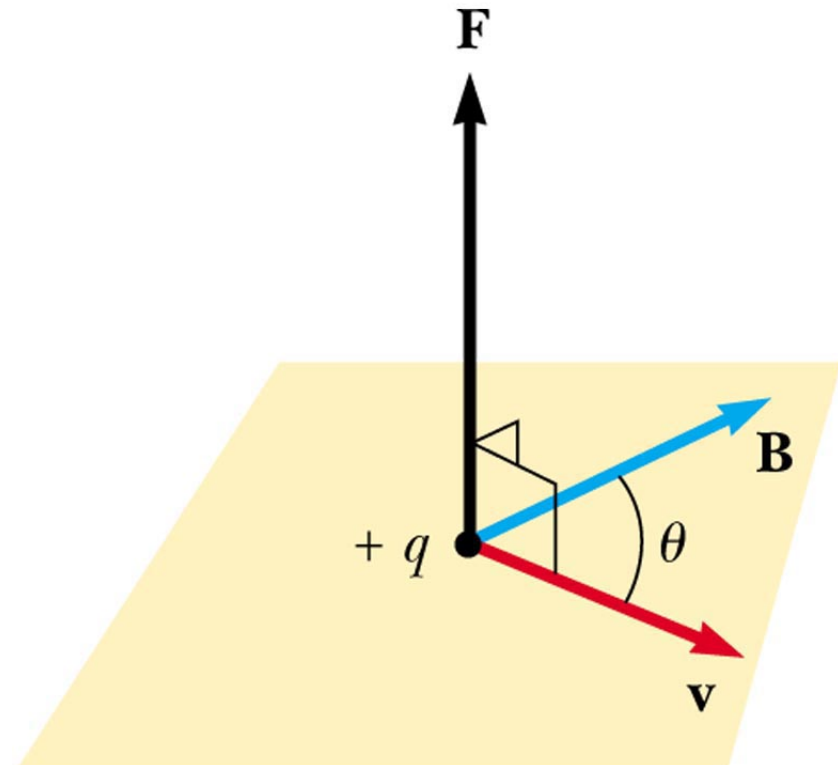
- $|F| = qvB \sin \theta$ gives the magnitude
- What about the direction?
- F is perpendicular to v and to B
- Which way?
 - ◆ right-hand rule



Talk to the hand



© 2003 Thomson - Brooks Cole



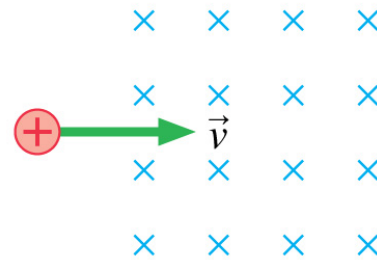
© 2003 Thomson - Brooks Cole

$$\vec{F} = q\vec{v} \times \vec{B}$$

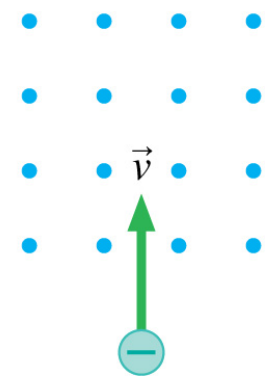
Examples

- What is the initial direction of deflection for the charged particles entering the magnetic fields shown to the right?

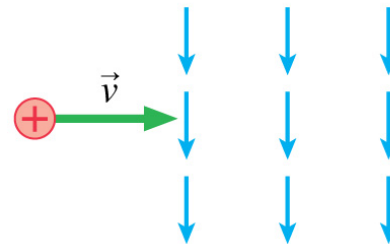
(a)



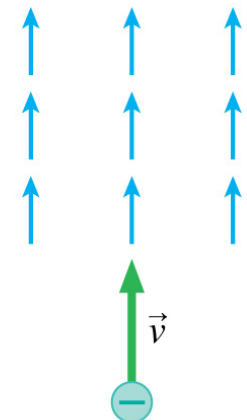
(b)



(a)



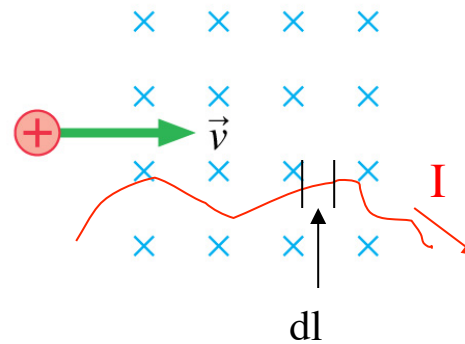
(b)



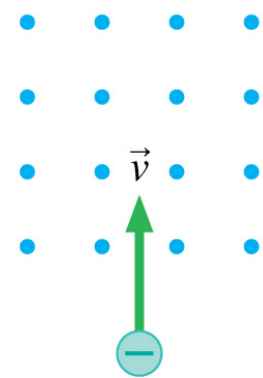
Forces on currents

- An electric current is just a bunch of moving electric charges
- So there should be a force acting on an electric current moving in a magnetic field
- What direction?
- The direction that the magnetic force would be on a moving positive charge
- Consider the force on an element $d\vec{l}$ of a current-carrying wire
 - ◆ $q = Idl$

(a)



(b)



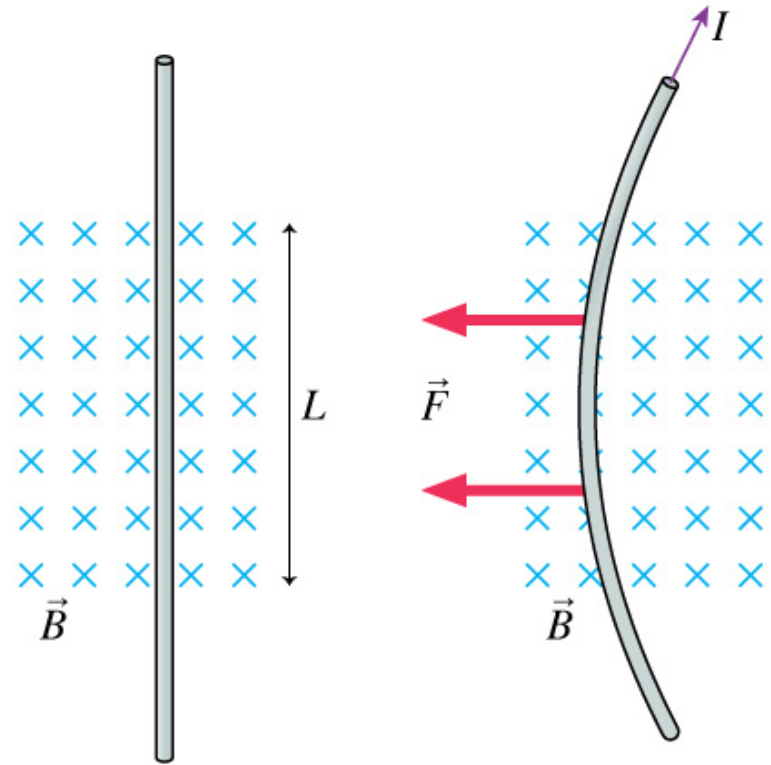
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\vec{F} = Id\vec{l} \times \vec{B}$$

...where $d\vec{l}$ has the direction of the current I

Force on a current

- Same direction as force on a positive charge moving in the direction of the current

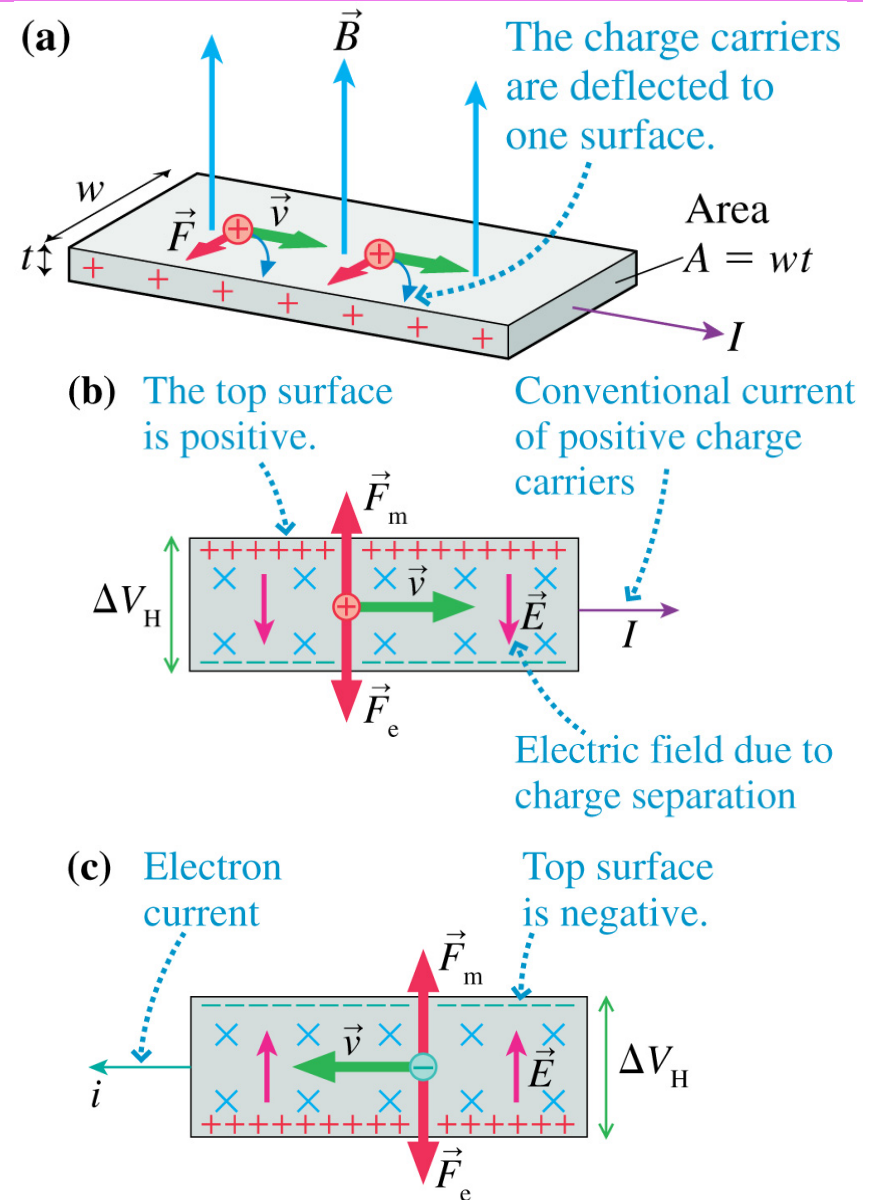


A wire is perpendicular to an externally created magnetic field.

A current through a wire that is fixed at the ends causes the wire to be bent sideways.

Hall effect

- Remember when I said it didn't matter that we assume that an electric current is the movement of positive charges (rather than the negatively charged electrons)
- Only a few ways you can tell the difference
- One way is the Hall effect
- Consider a current I in the direction indicated moving through a thin conductor
- Each charge in the current experiences a force equal to $e\vec{v}_d \times \vec{B}$, perpendicular to \vec{v} and \vec{B}
- This magnetic force causes a separation of charges so that one side of the conductor will be positive and one side negative
- Now it makes a difference whether the current is a flow of + charges to the right or - charges to the left



Hall effect

- The separation of charges creates an electric field that continues to build until the electric force cancels out the magnetic force ($F_e = F_m$)
- This creates a potential difference ΔV_H which can be measured (although it's small)
- When $F_e = F_m$

$$F_m = ev_d B = F_e = eE = e \frac{\Delta V}{w}$$

- The Hall voltage is

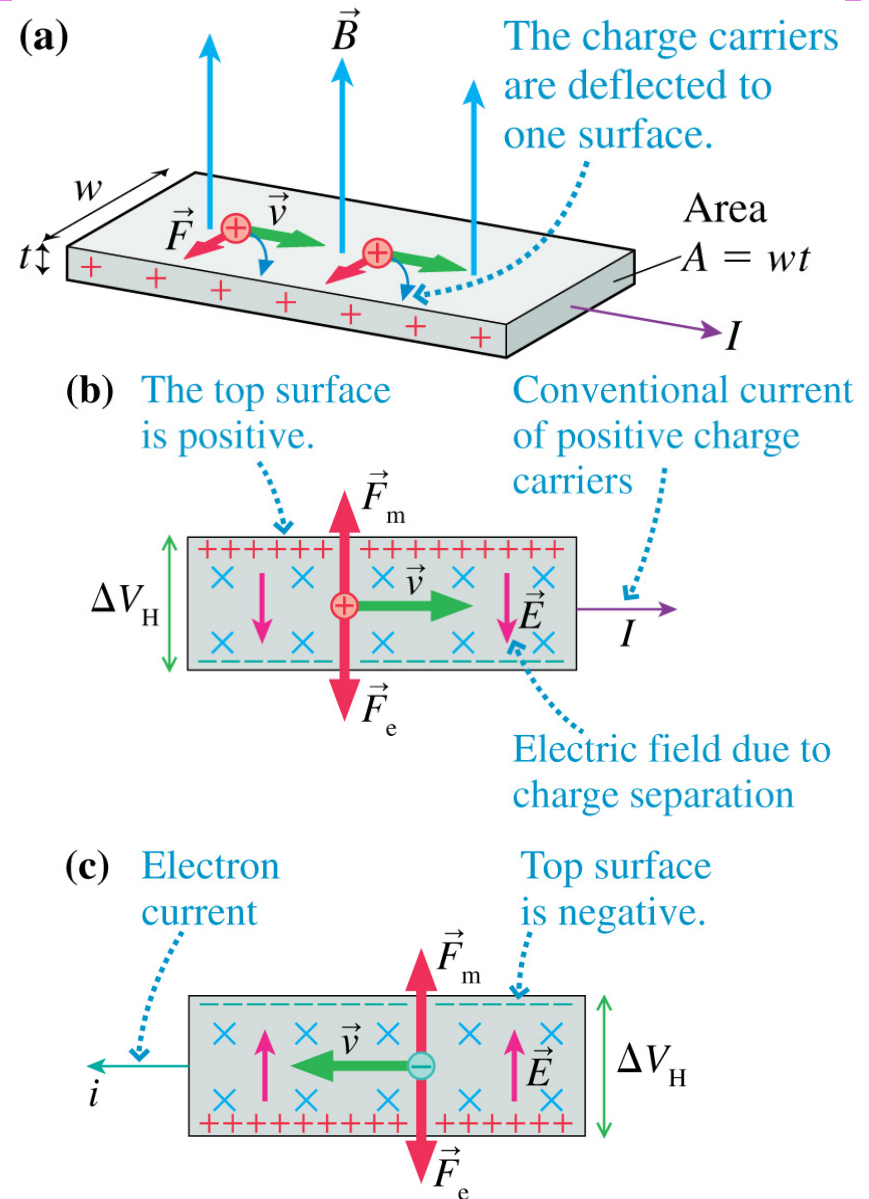
$$\Delta V_H = w v_d B$$

- Using the formula for v_d

$$v_d = \frac{I}{wtne}$$

- ΔV_H is

$$\Delta V_H = \frac{IB}{tne}$$



Forces between currents

- Now consider 2 current carrying wires separated by a distance r and carrying currents I_1 and I_2
- Current I_1 produces a magnetic field at the position of I_2 equal to

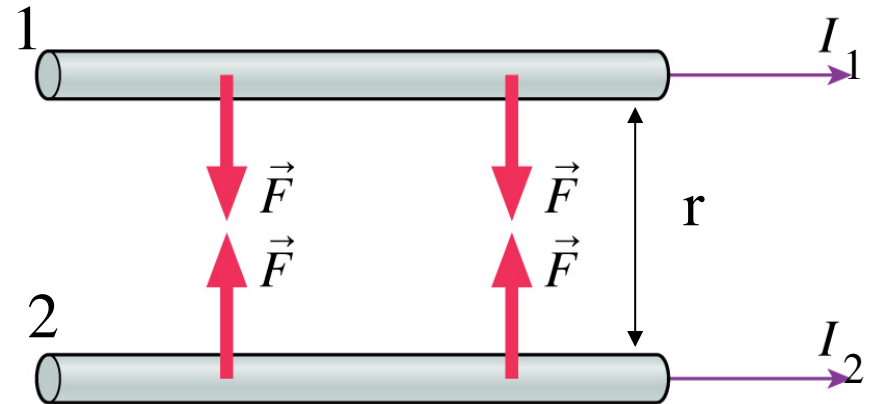
$$B_1 = \frac{\mu_o I_1}{2\pi r}$$

- A length l of current I_2 experiences a force given by

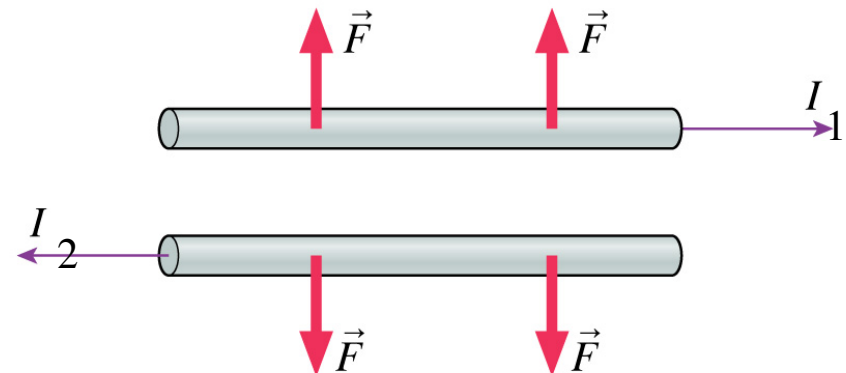
$$|\vec{F}_2| = I_2 l B_1 \sin \theta = I_2 l \frac{\mu_o I_1}{2\pi r} \sin \theta$$

$$|\vec{F}_2| = \frac{\mu_o}{2\pi} \frac{I_1 I_2 \sin \theta}{r}$$

- What's the direction of the force?
 - given by RHR



“Like” currents attract.



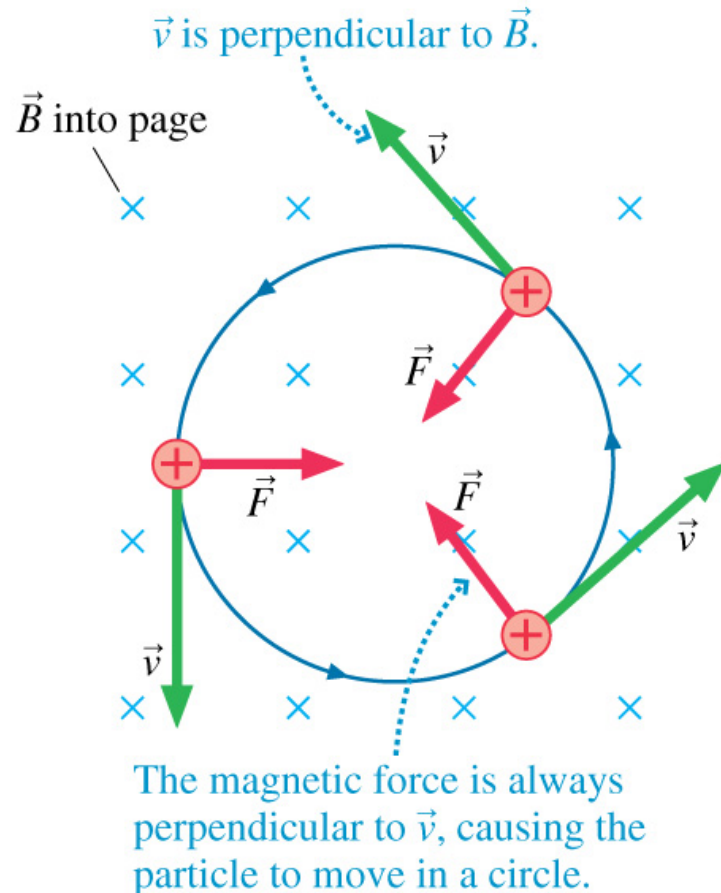
“Opposite” currents repel.

Force on a moving charge

- We've seen that the magnetic force acting on a moving electric charge is always perpendicular to the direction of motion
- A force that is always perpendicular to the motion changes the direction of motion but not the speed
- If I have a particle moving perpendicular to a uniform magnetic field, that particle will undergo uniform circular motion

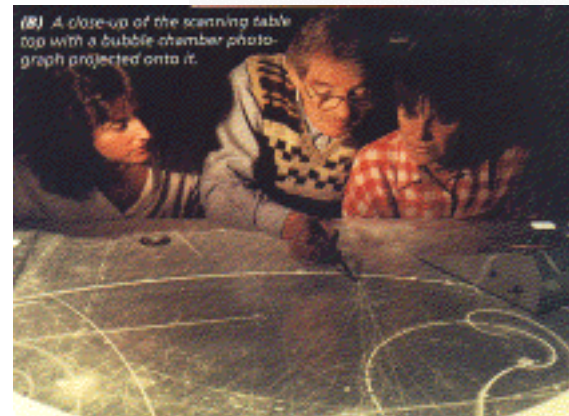
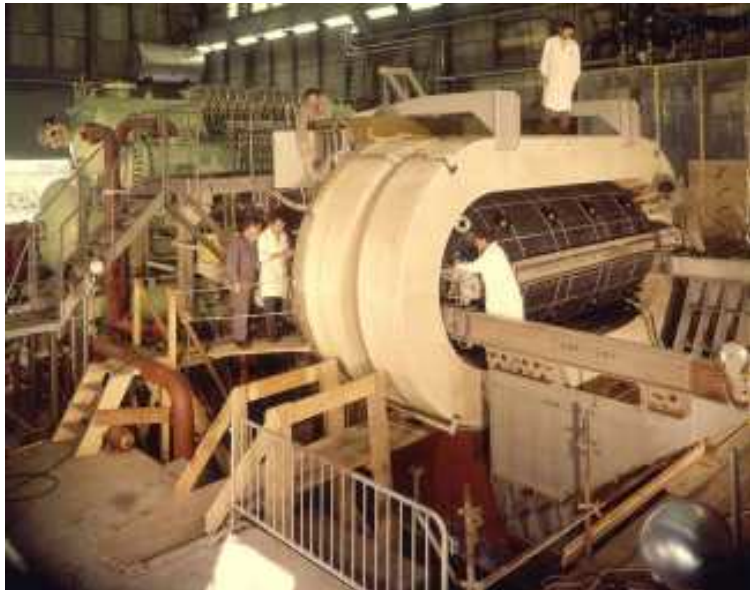
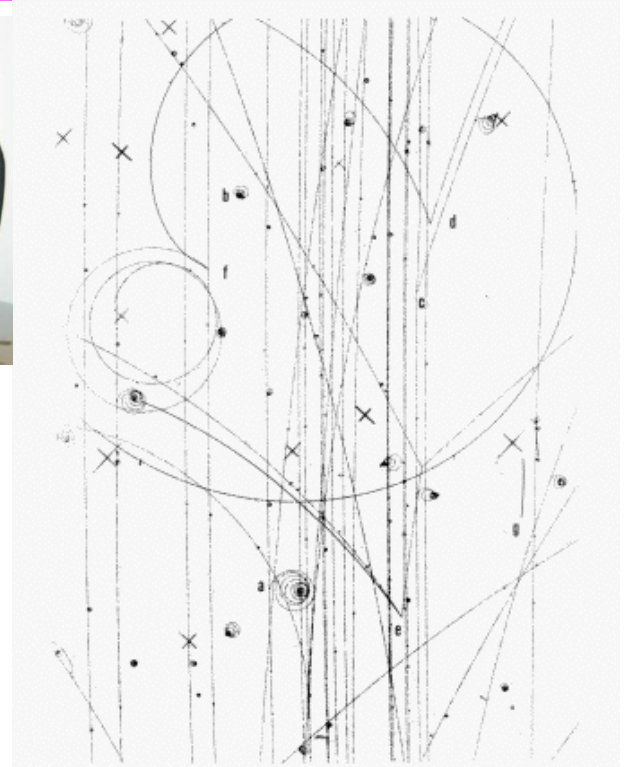
$$F_c = qvB = ma_r = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB}$$



Bubble chambers

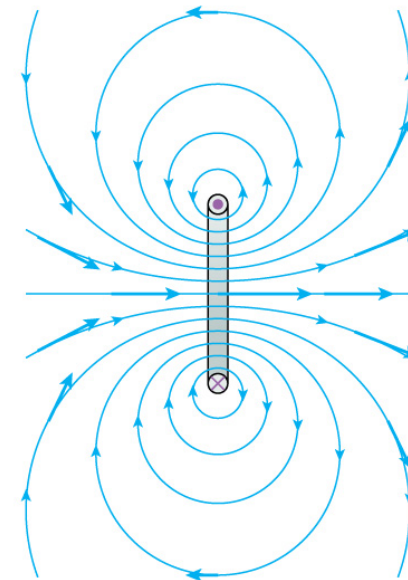
- Device formerly used in high energy physics experiments
 - ◆ invented by Donald Glaser who got the idea by staring at the bubbles in his glass of beer
- Charge particles interact in a boiling liquid (usually H_2) and leave tracks of bubbles
- Curvature in a magnetic field allows the determination of the particle momenta



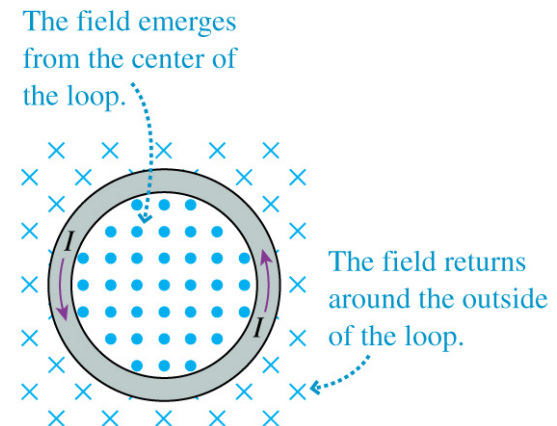
Magnetic field from a loop of current-3D

- To find the direction of the field, you can stick your thumb (right hand) in the direction of the current, and your fingers will curl in the direction of the magnetic field lines
 - ◆ can be painful but works in general
- ...or you can curl your hand in the direction of the current and your thumb will point in the direction of the field
 - ◆ less painful but works only in the center of the loop
- Field along the z-axis is given by
$$B_{z-axis} = \frac{\mu_o}{2} \frac{IR_{coil}^2}{(z^2 + R_{coil}^2)^{3/2}}$$
- In general case, calculation is pretty difficult

(a) Cross section through the current loop



(b) The current loop seen from the right



demo