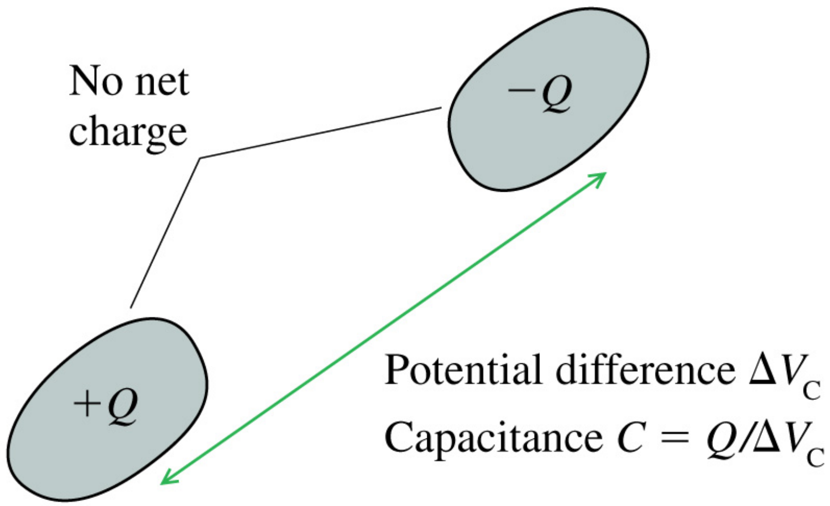
PHY294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Problem 29.78 (already assigned) will be the hand-in problem for 4th MP assignment (due Wed Feb. 10)**
  - **Help-room hours: 12:40-2:40 Tues; 3:00-4:00 PM Friday**
- Quizzes by iclicker (sometimes hand-written)
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

Capacitance

$$Q = \Delta V_c C$$

- So the amount of charge stored on a capacitor depends on the voltage across the plates of the capacitor and the value of the capacitance
- The capacitance depends on geometric factors
- Simplest capacitor is a parallel plate capacitor but can have any configuration

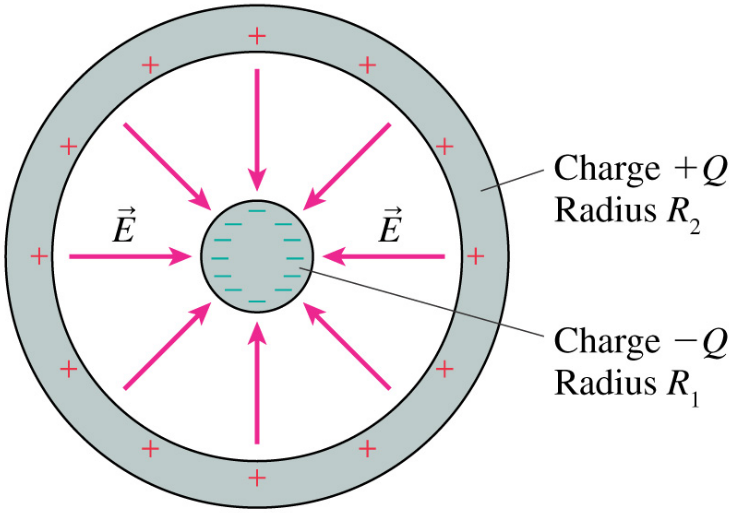


Spherical capacitor

- Here I've stored a charge -Q on a conducting sphere of radius R_1 placed inside a conducting shell of radius R_2, which has a charge of +Q
- It's a capacitor; what's the capacitance?
- Use the definition of capacitance

$$C = \frac{Q}{\Delta V_c}$$

- The numerator is easy; it's the denominator that's harder
  - we have to calculate ΔV_c, but that's something we know how to do; the electric field between the two conductors is that of a point charge of value -Q



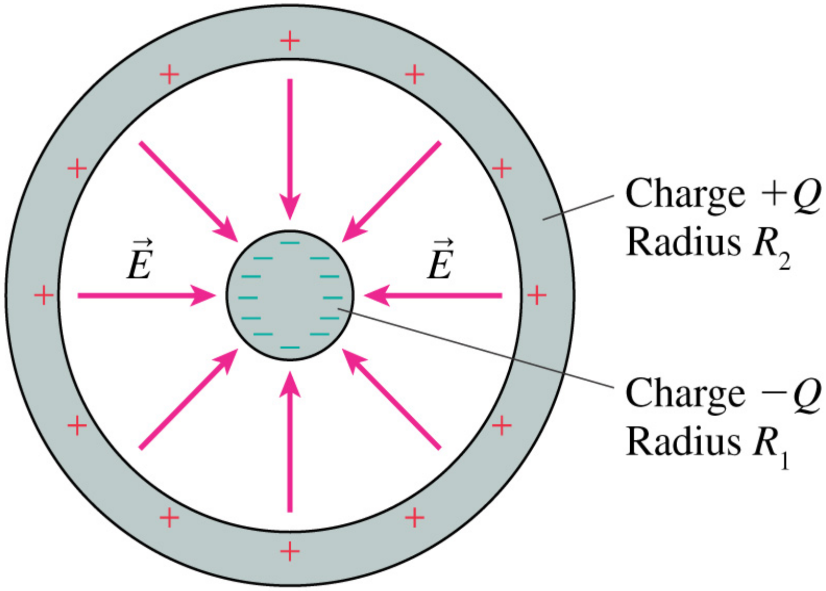
$$\Delta V_c = -\int_{si}^{sf} E_s ds = -\int_{R_1}^{R_2} \frac{-Q}{4\pi\varepsilon_o r^2} dr$$

$$\Delta V_c = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

Spherical capacitor

- The capacitance is the charge stored per voltage

$$C = \frac{Q}{\Delta V_c} = \frac{Q}{\dfrac{Q}{4\pi\varepsilon_o}\left[\dfrac{1}{R_1} - \dfrac{1}{R_2}\right]} = 4\pi\varepsilon_o \frac{R_1 R_2}{R_2 - R_1}$$

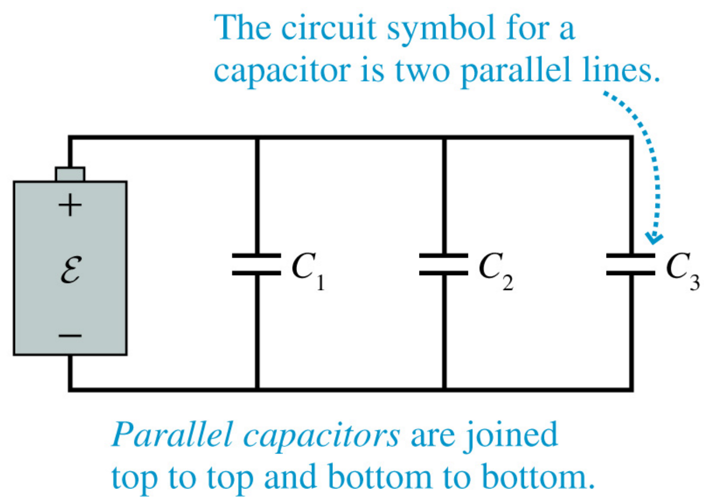


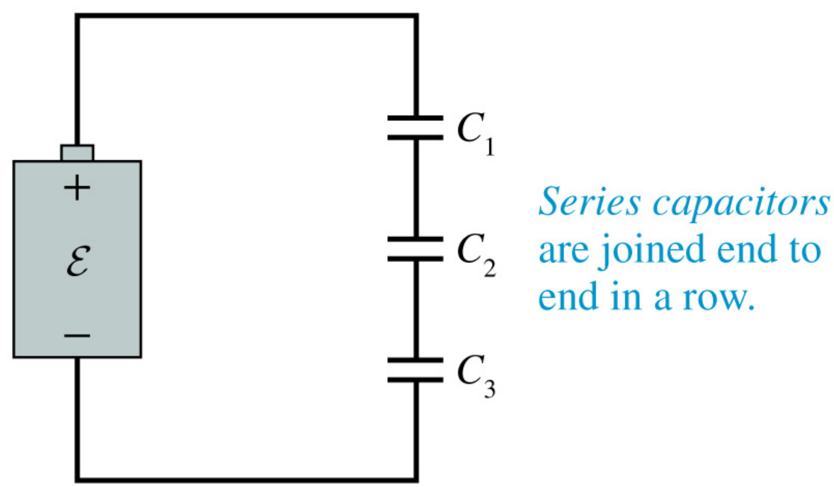
- Note that the capacitance depends on ε_o and on the geometry of the conductors
- What if R_2 goes to infinity?

$$C = 4\pi\varepsilon_o R_1$$

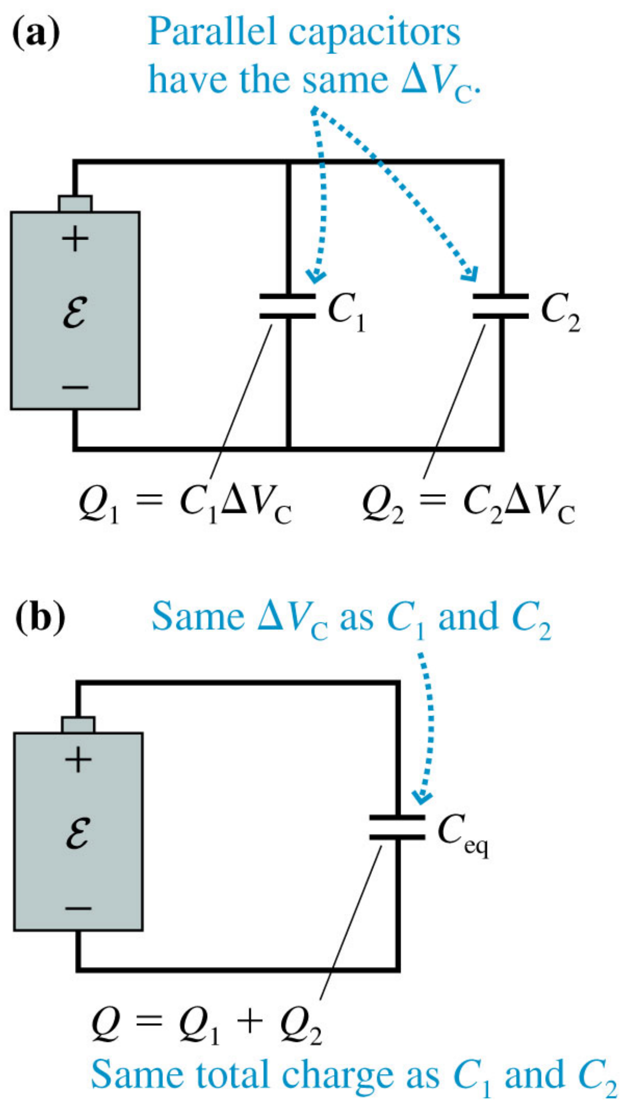
Capacitors in combination

- Often, we encounter more than one capacitor in a circuit, either joined in series or in parallel
- We would like to answer the question as to what the equivalent capacitance of these combinations is





Capacitors in parallel



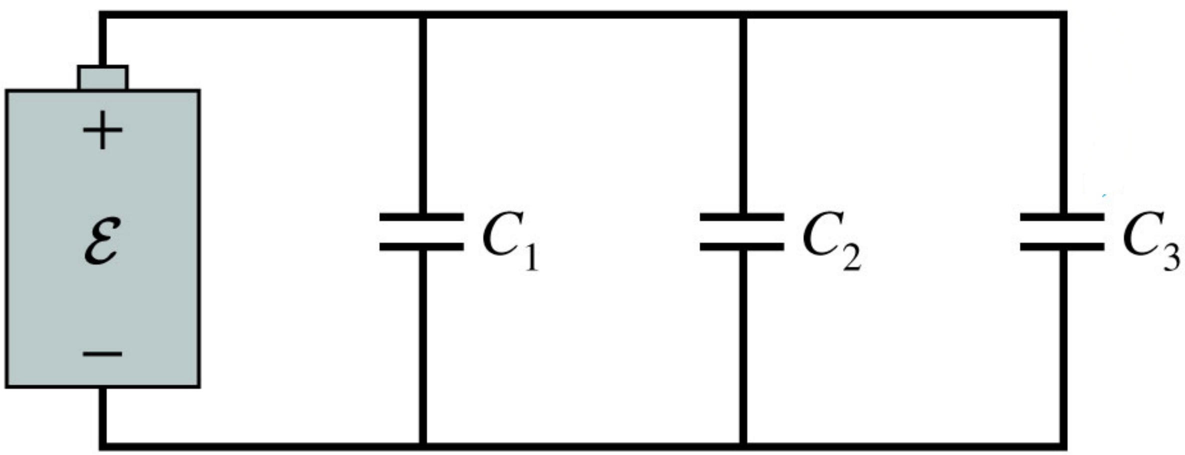
- Consider two capacitors C_1 and C_2 connected in parallel.
- The total charge drawn from the battery is $Q = Q_1 + Q_2$.
- In figure (b) we have replaced the capacitors with a single "equivalent" capacitor:

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{Q_1 + Q_2}{\Delta V_C} = \frac{Q_1}{\Delta V_C} + \frac{Q_2}{\Delta V_C}$$

$$C_{eq} = C_1 + C_2$$

If capacitors C_1, C_2, C_3, … are in parallel, their equivalent capacitance is:

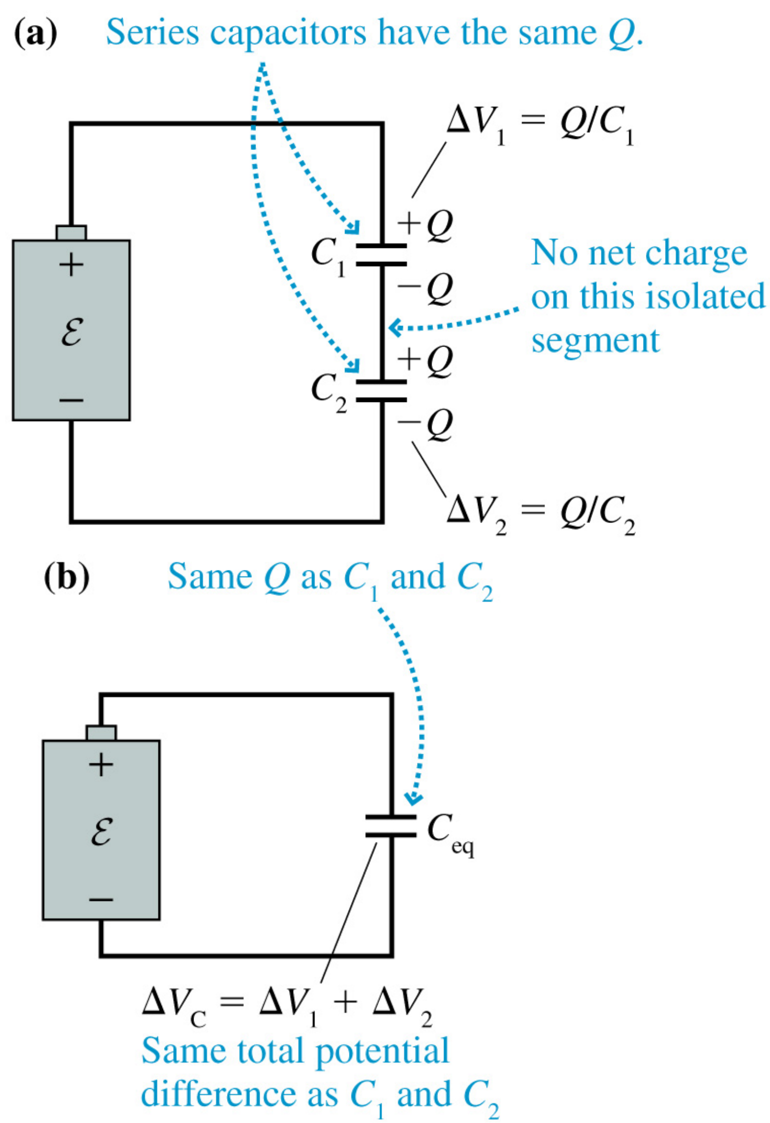
$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel capacitors)}$$



Capacitors in series

- Same idea; replace two or more capacitors in series with one capacitor able to store the same charge (given a voltage)
- In this case, we know that the two capacitors store the same amount of charge, but that the voltage is not necessarily the same
- But we also know that the sum of the voltages across C_1 and C_2 must equal the total voltage provided by the battery

$$\Delta V = \varepsilon = \Delta V_1 + \Delta V_2$$



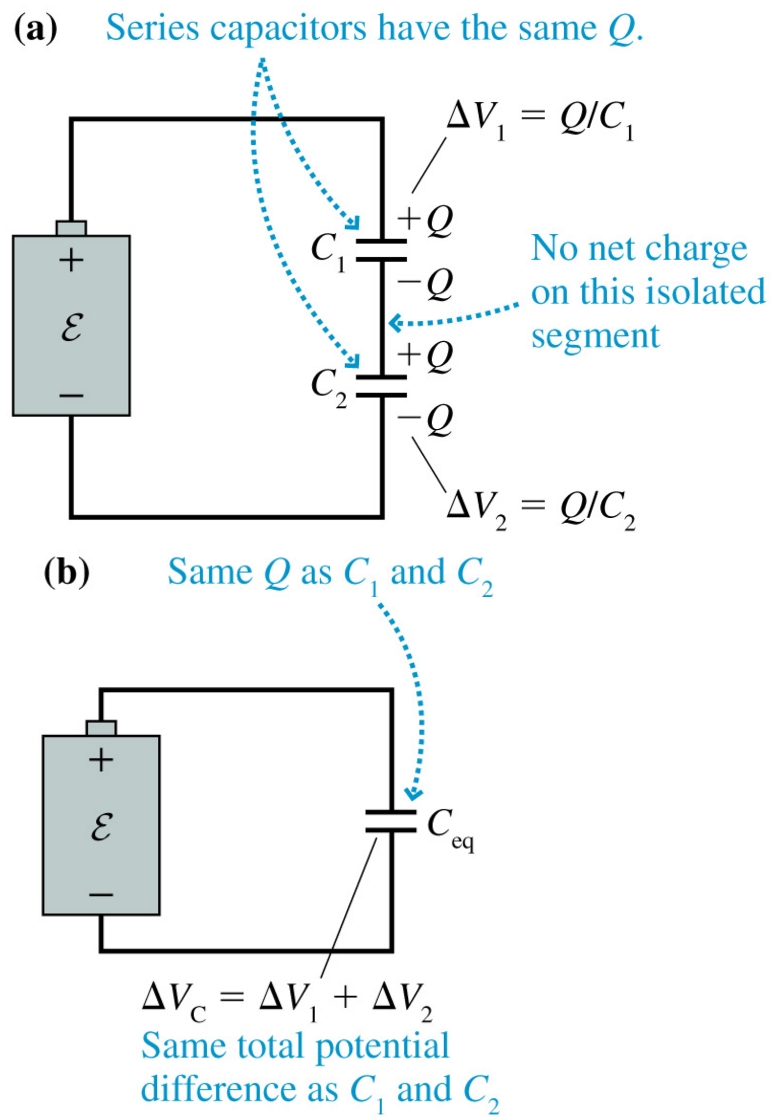
Capacitors in series

- For capacitors in series, you add the reciprocals of the individual capacitances to find the reciprocal of the equivalent capacitance

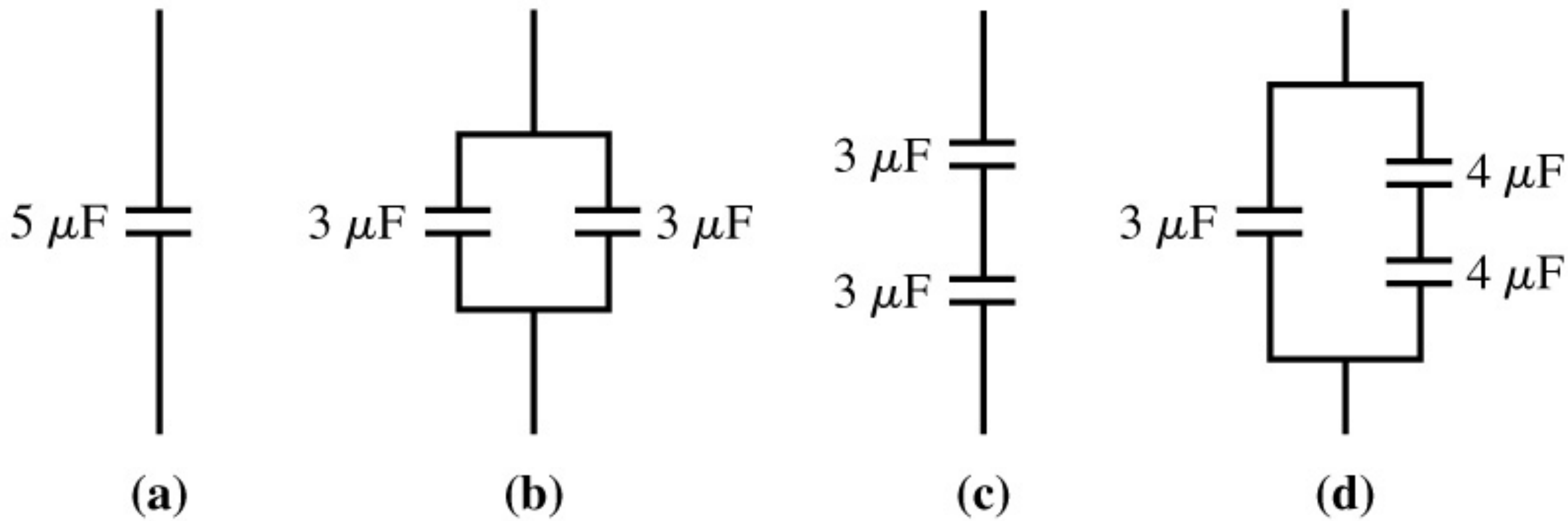
$$\Delta V = \varepsilon = \Delta V_1 + \Delta V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\Delta V = \frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}\left(+ \frac{1}{C_3} + \ldots \right)$$

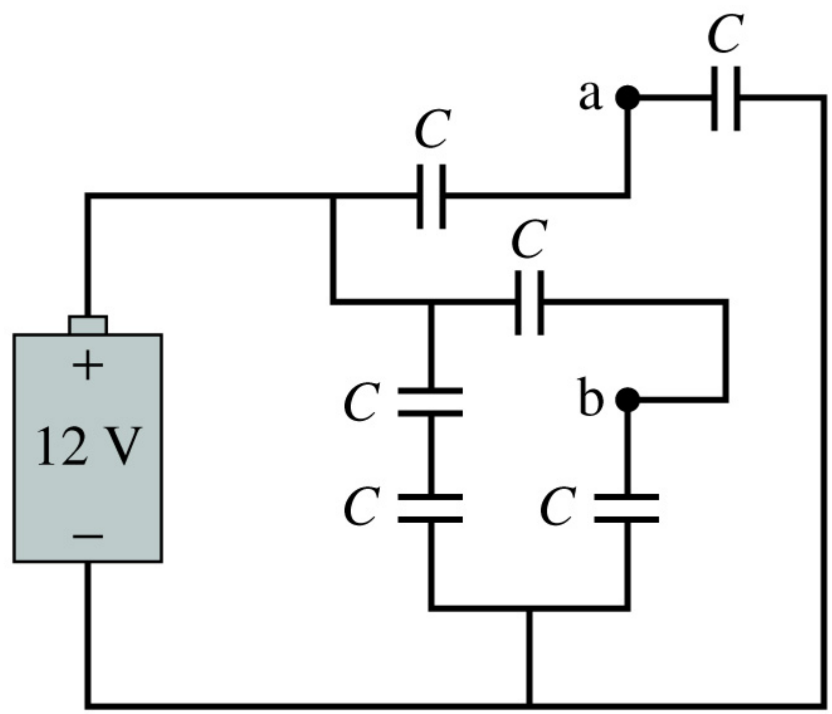


Which combination has the highest capacitance?



Example

- What is the equivalent capacitance of these six capacitors?
- What is the voltage between a and b?



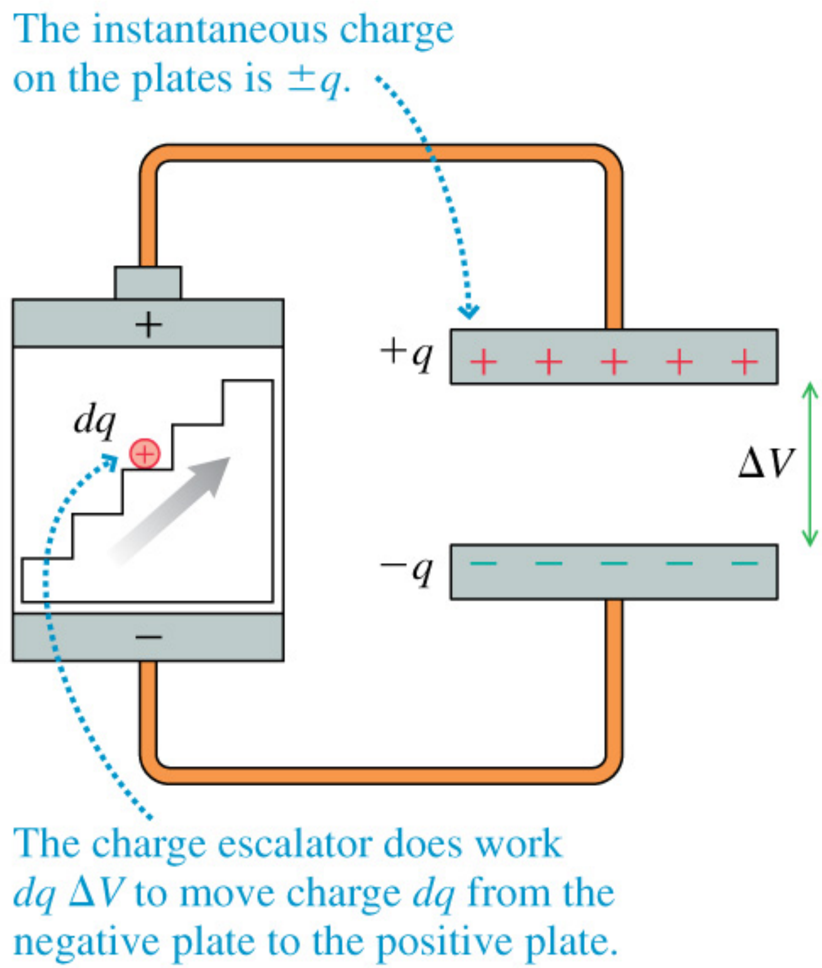
Energy stored in capacitor

- When the battery moves a charge dq from the bottom plate to the top plate, it represents an increase of potential energy stored in the capacitor

$$dU = dq\Delta V = \frac{qdq}{C}$$

- We can calculate the total potential energy stored in the capacitor when it becomes fully charged

$$U_C = \int dU = \frac{1}{C}\int_0^Q qdq = \frac{Q^2}{2C} = \frac{1}{2}C(\Delta V_C)^2$$



Uses for energy stored in capacitor

- Defibrillator

- Flashlamp

Uses for energy stored in capacitor

A capacitor charged to 1.5 V stores 2.0 mJ of energy. If the capacitor is charged to 3.0 V, it will store

A. 1.0 mJ.
B. 2.0 mJ.
C. 4.0 mJ.
D. 6.0 mJ.
E. 8.0 mJ.

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D. 6.0 mJ.

✔ **E. 8.0 mJ.** $U_C \propto (\Delta V)^2$

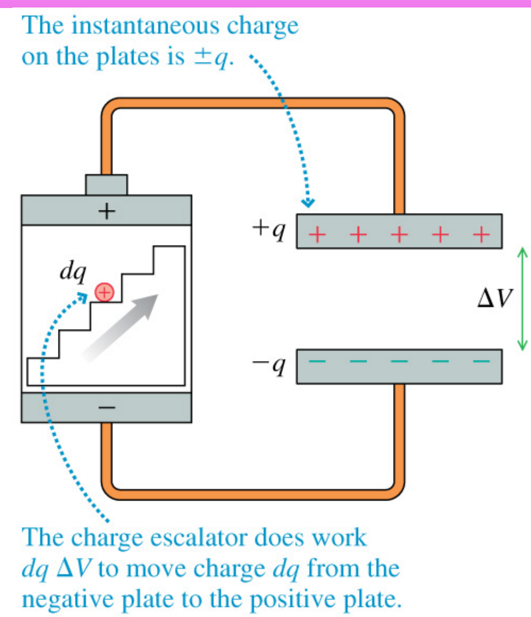
Energy in electric field

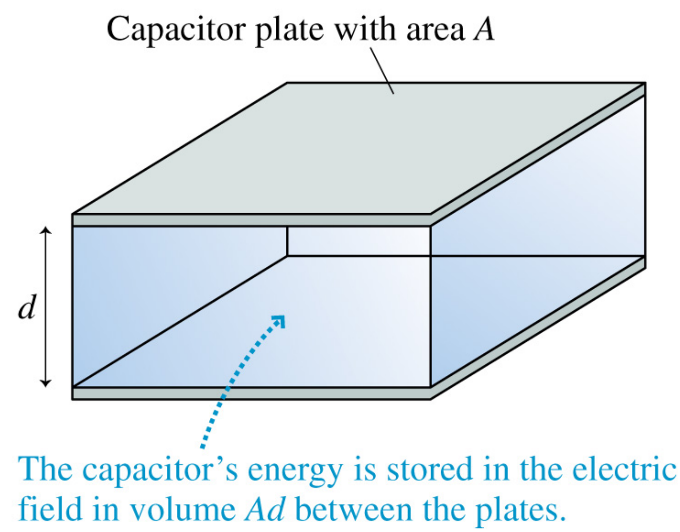
- You can think of the energy stored in the capacitor as being stored in the electric field created between the two plates of the charged capacitor
- Take a parallel plate capacitor for convenience

$$U_C = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}\frac{\varepsilon_o A}{d}(Ed)^2$$

$$U_C = (\frac{1}{2}\varepsilon_o E^2)(Ad)$$

$$u_E = U_C / Vol = \frac{1}{2}\varepsilon_o E^2$$ (energy density)





Example

- What is the energy density in the electric field at the surface of a 1.0 cm diameter sphere charged to a potential of 1000 V?
- What is the energy stored in the capacitor?

Farad capacitor at Fermilab