PHY294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **2nd MP assignment due Wed Jan. 27; second hand-written problem (27.51) as well; I added it on to the MP assignment for convenience, but it still needs to be turned in with a complete solution**
  - **Help-room hours: 12:40-2:40 Tues; 3:00-4:00 PM Friday**
- Quizzes by iclicker (sometimes hand-written)
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

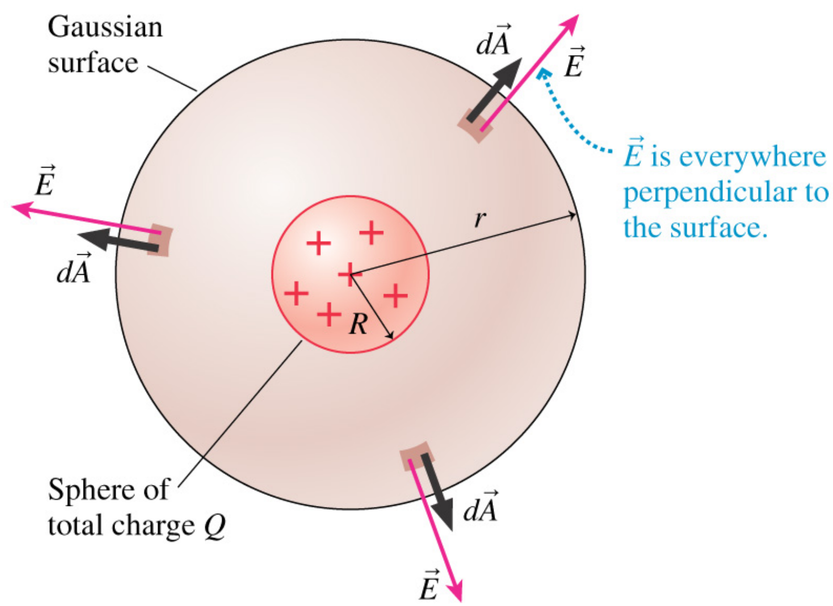
TABLE 25–1

ELECTRIC FIELDS AND POTENTIALS FOR VARIOUS CHARGE CONFIGURATIONS

Charge Configuration	*Magnitude of Electric Field*
Point charge	$\dfrac{q}{4\pi\epsilon_0 r^2}$
Infinite line of uniform charge density λ	$\dfrac{\lambda}{2\pi\epsilon_0 r}$
Parallel, oppositely charged plates of uniform charge density σ, separation d	$\dfrac{\sigma}{\epsilon_0}$
Charged disk of radius R, along axis at distance x	$\dfrac{Q}{2\pi\varepsilon_0 R^2}\left(\dfrac{\sqrt{R^2+x^2}-x}{\sqrt{R^2+x^2}}\right)$
Charged spherical shell of radius R	$r \geq R$: $\dfrac{Q}{4\pi\epsilon_0 r^2}$ $r < R$: 0
Electric dipole	Along bisecting axis only, far away: $\dfrac{p}{4\pi\epsilon_0 r^3}$
Charged ring of radius R, along axis	$\dfrac{Qx}{4\pi\epsilon_0(R^2+x^2)^{3/2}}$
Uniformly charged nonconducting solid sphere of radius R	$r \geq R$: $\dfrac{Q}{4\pi\epsilon_0 r^2}$ $r < R$: $\dfrac{Qr}{4\pi\epsilon_0 R^3}$

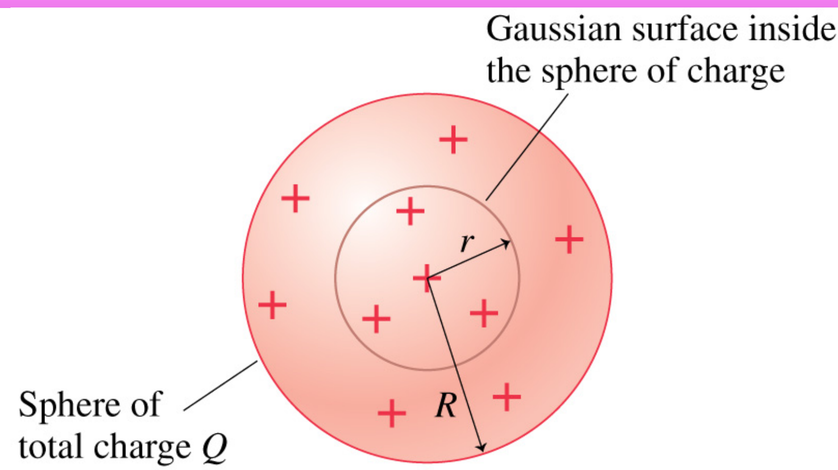
Gauss' law

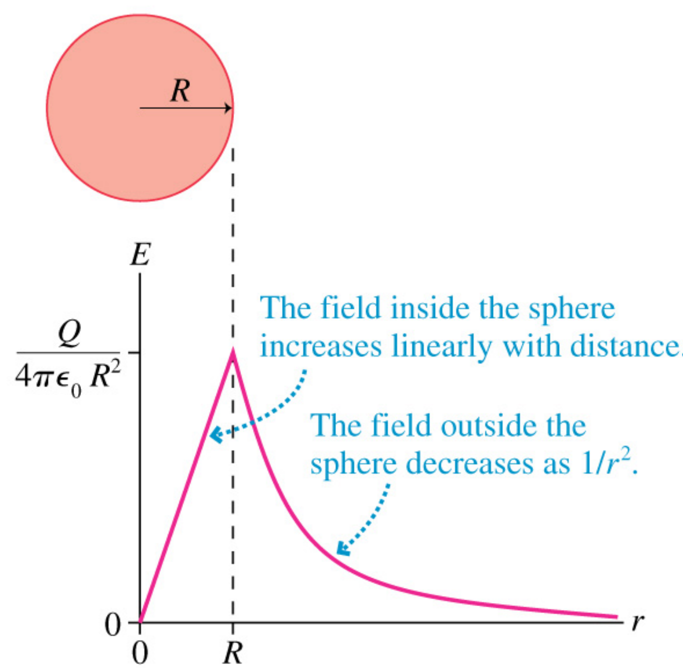
- Caveats
  - Gauss' law applies only to a closed surface
  - Gaussian surface is not a physical surface, just a mathematical construct
  - can't find the electric field from Gauss' law alone; need to apply it in situations where we can already guess at the shape of the field
- Let's start with the field outside of a spherical charge distribution



Gauss' law applications

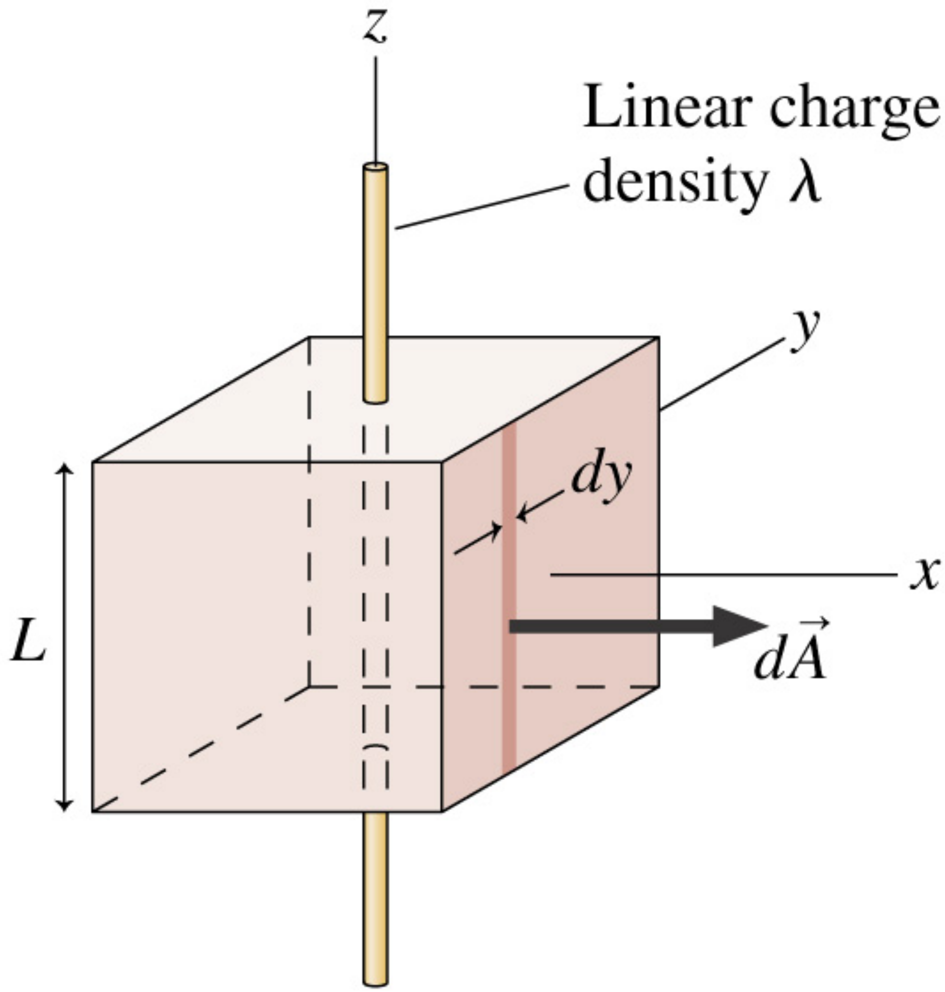
- Now consider the electric field inside a sphere of charge
- Again, choose a spherical Gaussian surface, or radius $r < R$
- Electric field increases linearly inside the sphere and then drops off as $1/r^2$ outside





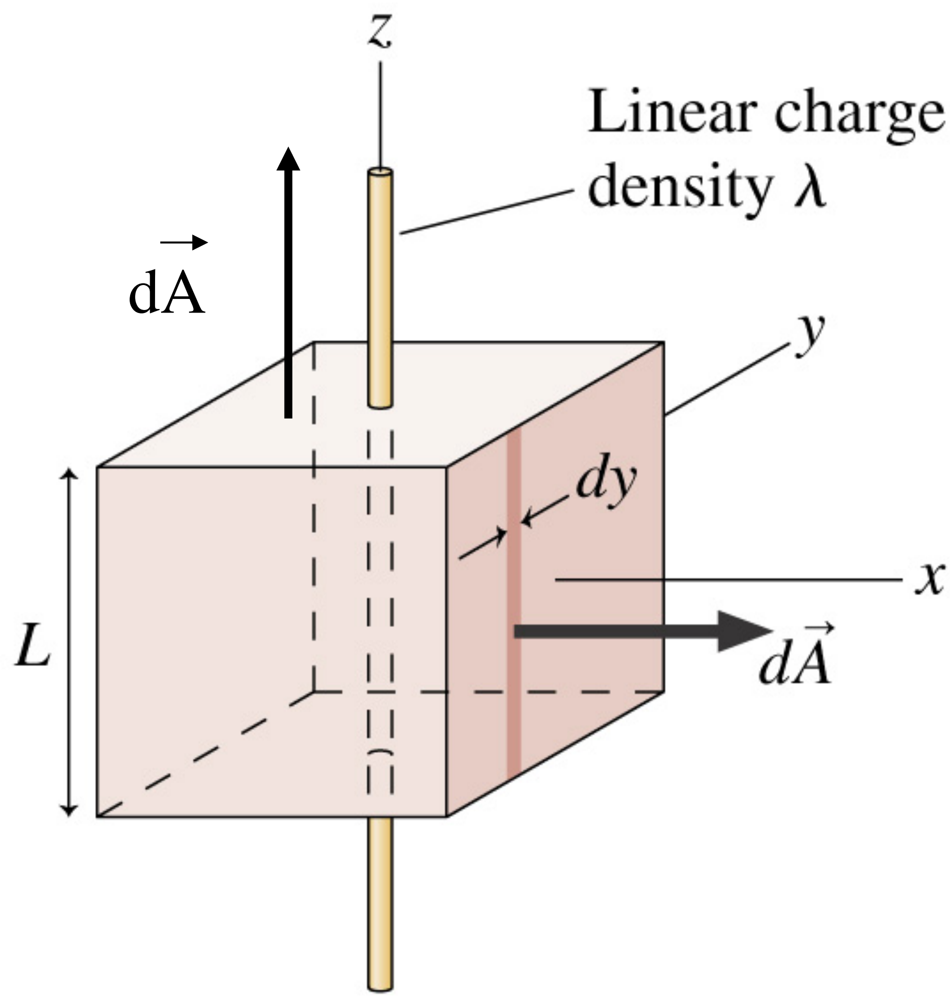
A harder integral

- Suppose I have a wire with a uniform charge density λ passing through the center of a Gaussian surface (a cube) of side L
- What is the electric flux through the cube?
- I'll do this the easy way and the hard way
- Easy way:
  - $Q_{in}/\varepsilon_o = \lambda L/\varepsilon_o$
  - $Q_{in}=\lambda L$
- Hard way:
  - evaluate the integral



A harder integral

- Let's make life a bit easier for ourselves
- We know the electric field from a long straight line of charge points perpendicularly away from the line
  - so there is no component of the electric field along the z-axis
  - so $\vec{E} \cdot d\vec{A}$ will be zero for both the top and bottom of the cube
- So I just have to integrate over the 4 sides of the cube
  - but from the symmetry each side should give the same result, so I just have to do one side
  - but neither r nor cosθ is constant so this is not a trivial integral



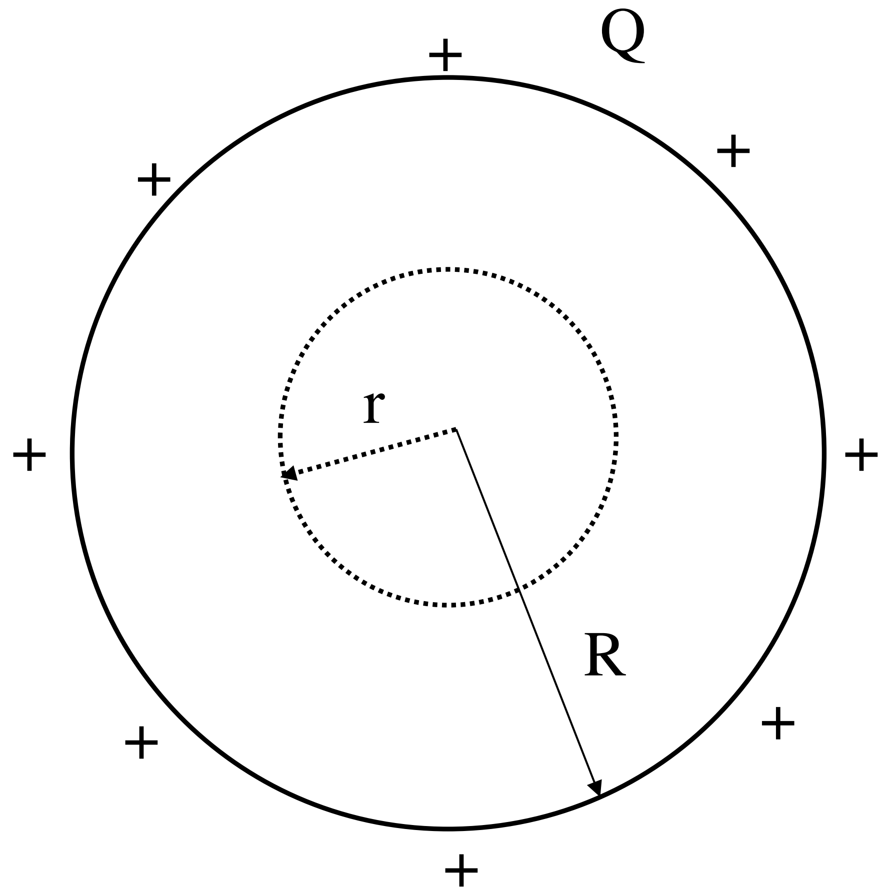
…to the whiteboard

Spherical shell

- Let's consider a Gaussian surface inside a uniformly charged hollow spherical shell

$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

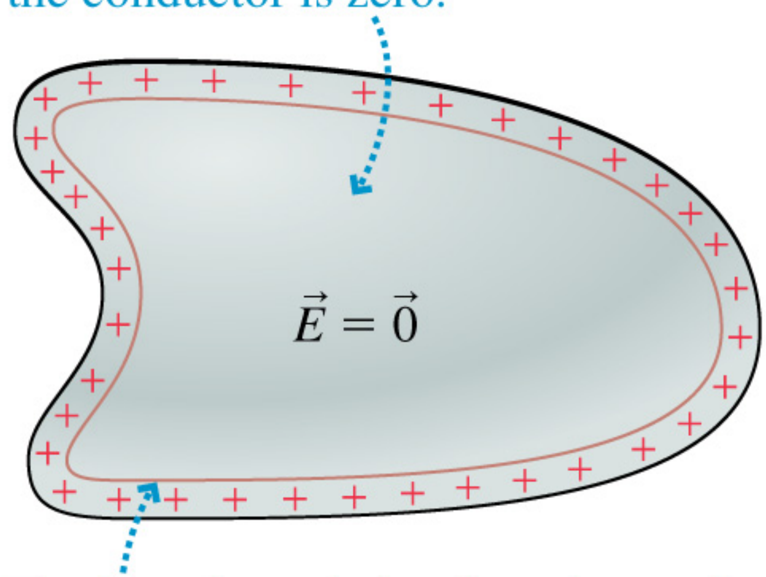
- By symmetry, E should be uniform on spherical surface of radius r
  - $E4\pi r^2 = Q_{in}/\varepsilon_o$
  - $Q_{in} = 0$
  - $E=0$



Charged conductors

- Thus the electric field inside a conductor (due to static charges) must be zero
- Since the electric field is zero, the electric flux must be zero through any surface in the interior of a conductor
- If the flux is 0, then the charge enclosed by any Gaussian surface must be 0
- All free charge then resides on the surface of a conductor

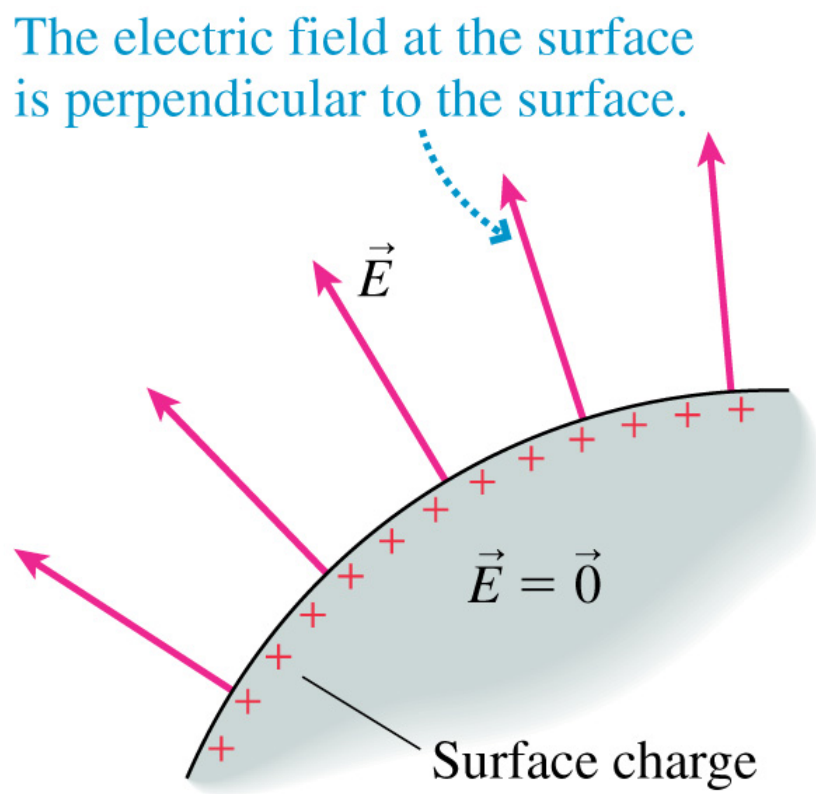
The electric field inside the conductor is zero.



The flux through the Gaussian surface is zero. There's no net charge inside the conductor. Hence all the excess charge is on the surface.

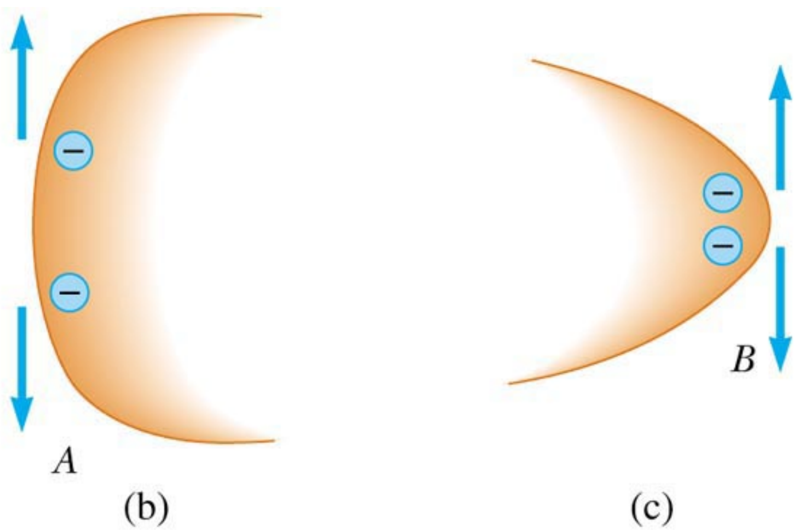
Electric field near a conductor

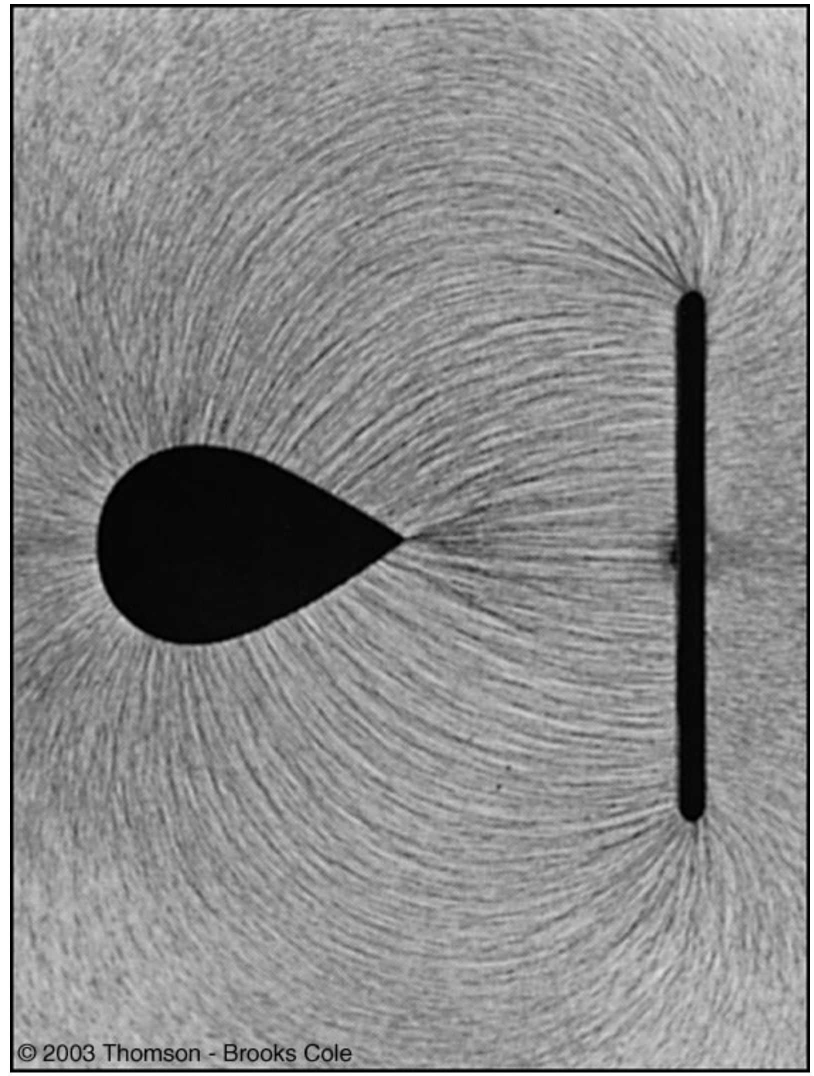
- In addition, we can state the the electric field near the surface of a conductor has to be perpendicular to that surface
- Suppose that the electric field had a component parallel to the surface
  - then this would cause a force on the conduction electrons that would cause them to move, until the electric field at the surface was perpendicular again



Sharp edges

- If I have a charged object with a pointy end, then the electric field lines are more concentrated there
  - because electrons are more concentrated there
- So lightning rods have pointy ends





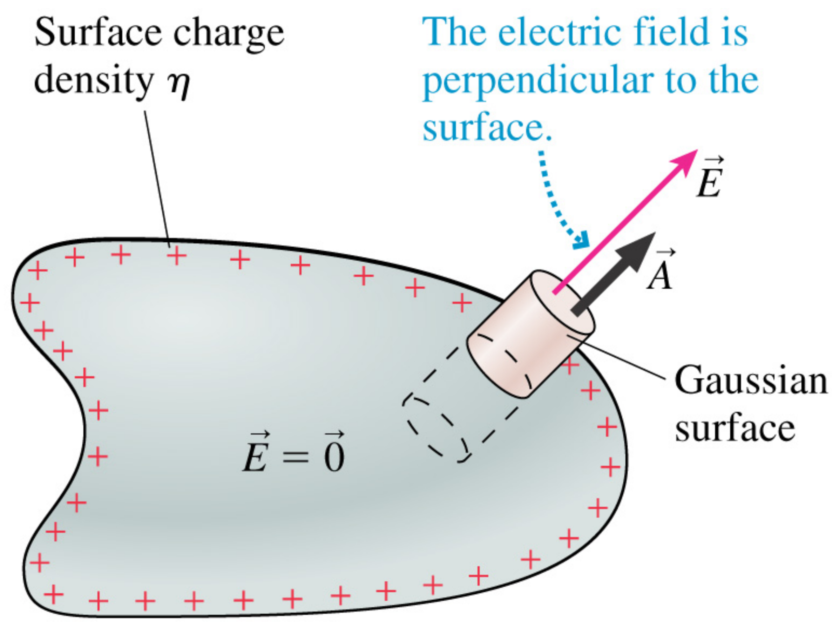
demo

Lightning rods

- In the United States, the pointed lightning rod conductor, also called a “lightning attractor” or “Franklin rod” was invented by Benjamin Franklin in 1749 as part of his groundbreaking explorations of electricity
- Franklin speculated that with an iron rod sharpened to a point at the end,*"The electrical fire would, I think, be drawn out of a cloud silently, before it could come near enough to strike [...]."*

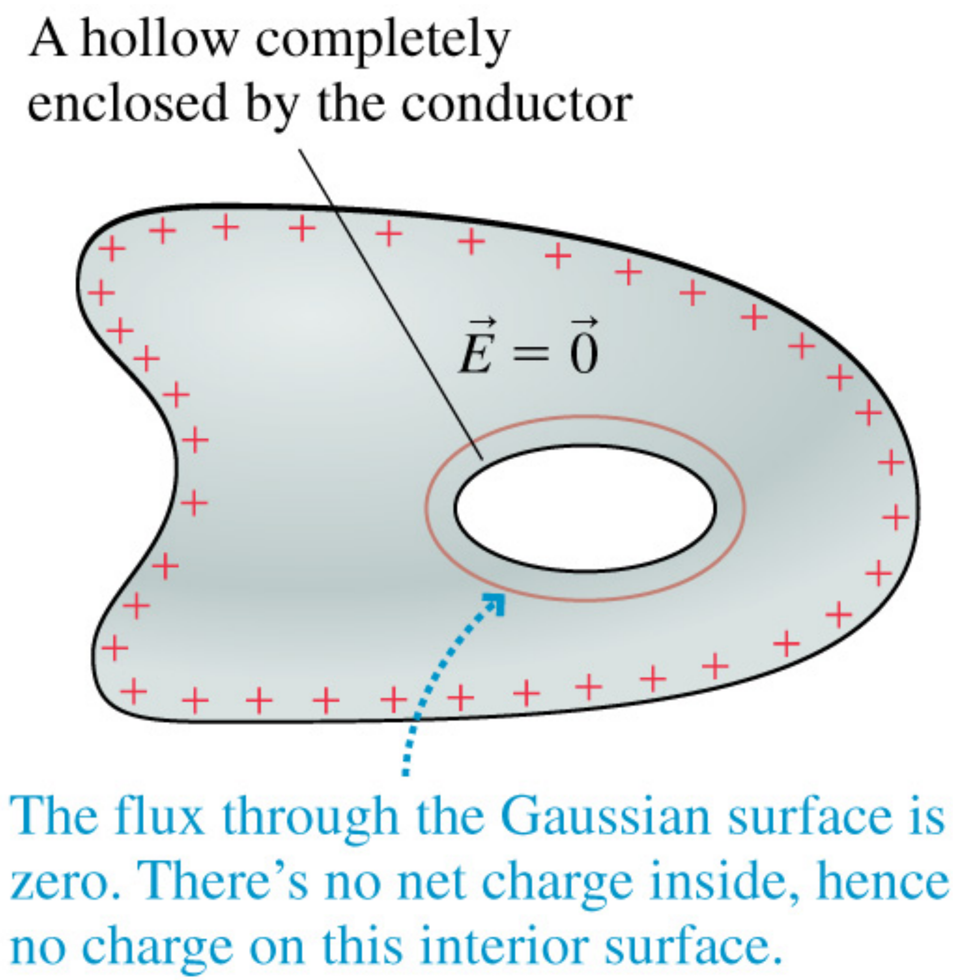
Electric field near a conductor

- Let's try a cylindrical Gaussian surface to calculate the value of E near the surface
- The only contribution is from the top of the cylinder
  - $EA = q_{in}/\varepsilon_o$
  - $q_{in} = \eta A$
  - $E = \eta/\varepsilon_o$



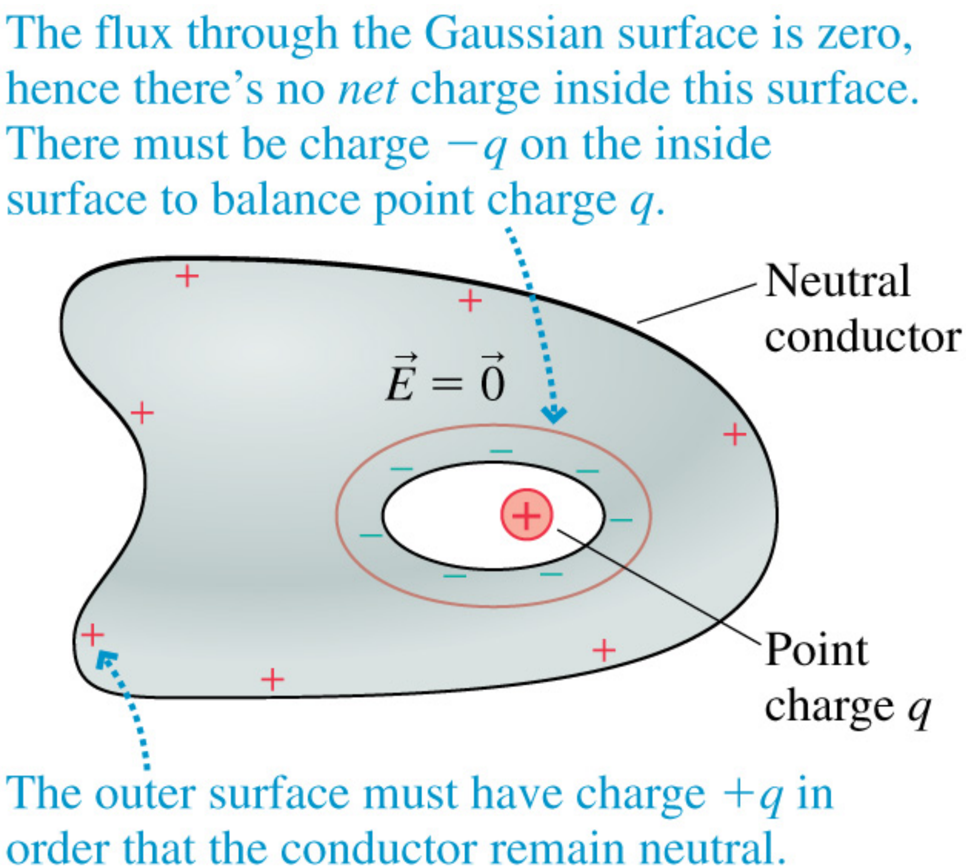
Suppose the interior of a conductor is hollow

- There is no electric field inside the conductor
- So there is no electric flux around the Gaussian surface shown
- So no net charge inside the hollow

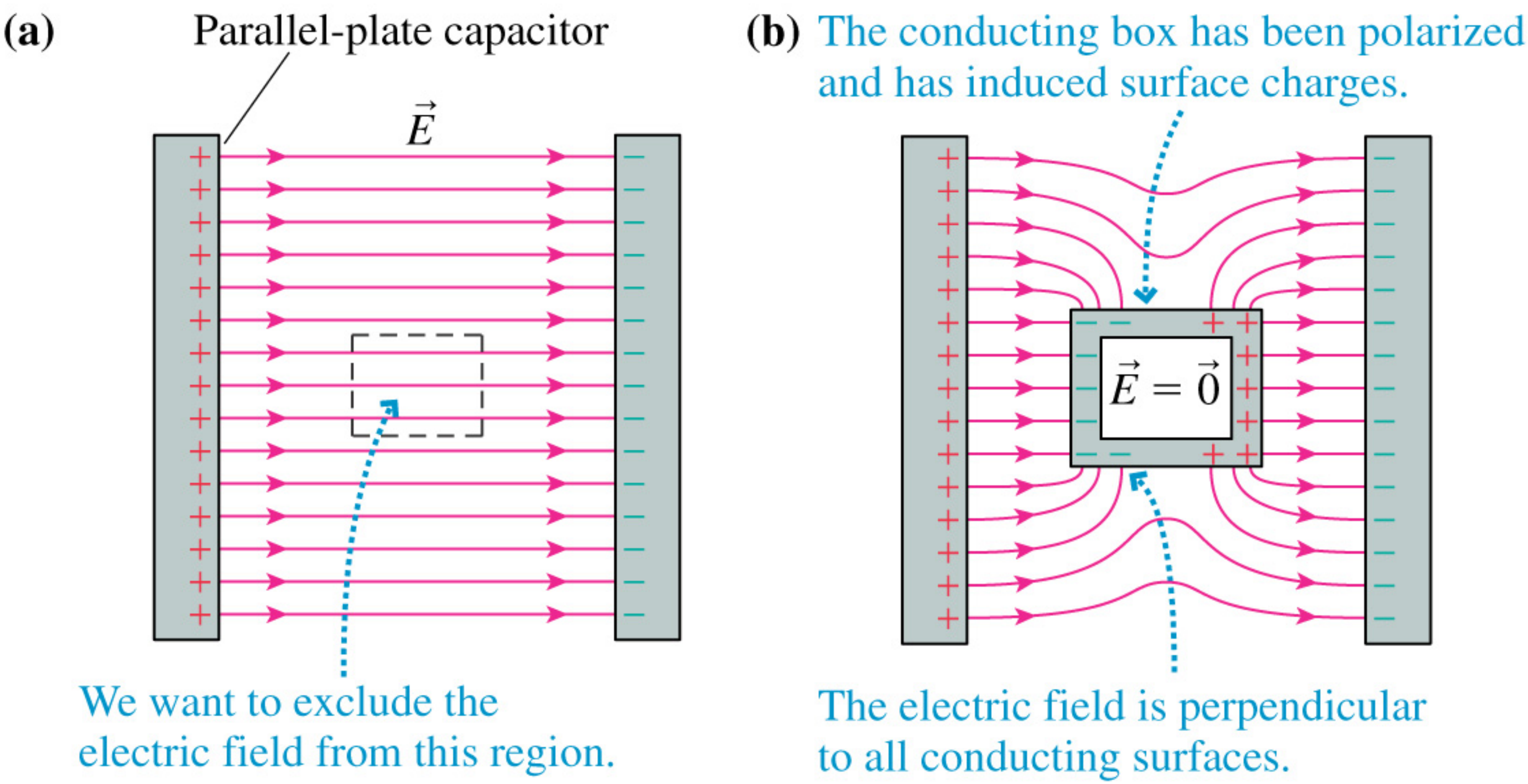


Conductors

- Let's put a positive charge inside the hollow
- If I put a Gaussian surface inside the hollow, there is a non-zero electric flux, so there's got to be a non-zero field
- But if the Gaussian surface is outside the hollow, then the E field has to be zero
    - thus the flux has to be zero
    - and the charge enclosed has to be zero
    - electrons must move to the surface of the hollow to cancel out the electric field due to the postive charge



Use conductors to screen electric fields

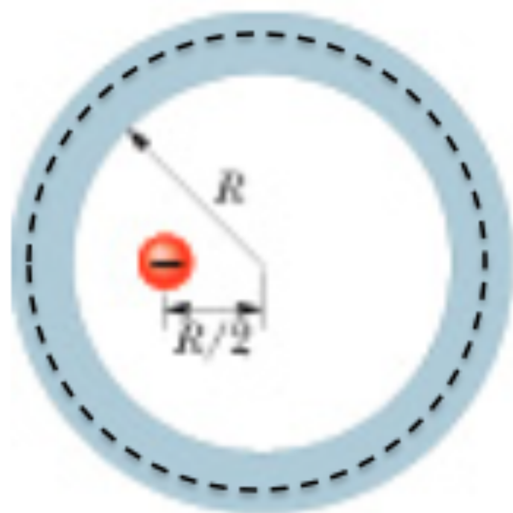


demo

iClicker question

A point charge of -5.0 C is off-center inside an electrically neutral spherical metal shell. What is the induced charge on the **inner surface** of the shell?

A. 0 C
B. +5 C
C. -5 C
D. +10 C
E. -10 C



iClicker question

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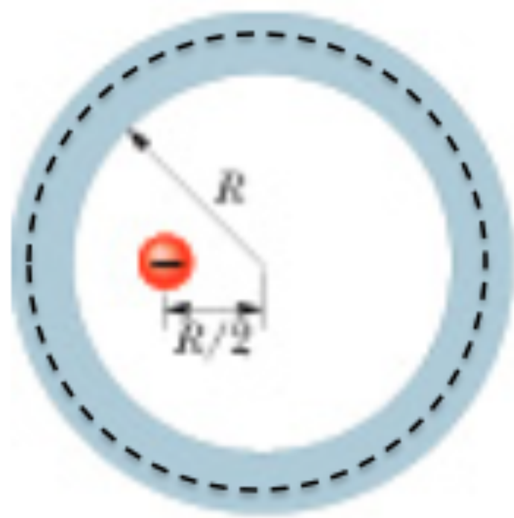
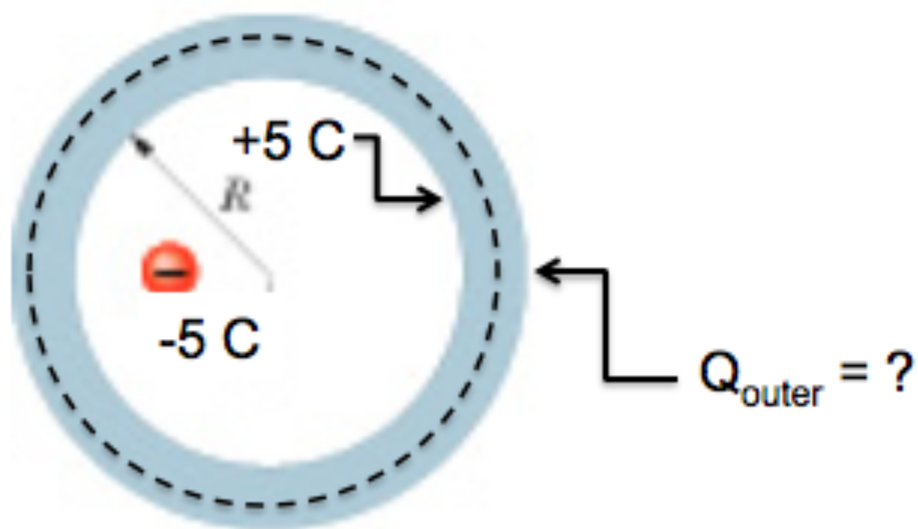


The electric field inside of the conductor is zero. So the Gaussian surface integral inside the conductor is zero. So the charge enclosed must be zero.

iClicker question

A point charge of -5.0 C is off-center inside an electrically **neutral** spherical metal shell. What is the induced charge on the **outer surface** of the shell?

A. 0 C
B. +5 C
C. -5 C
D. +10 C
E. -10 C



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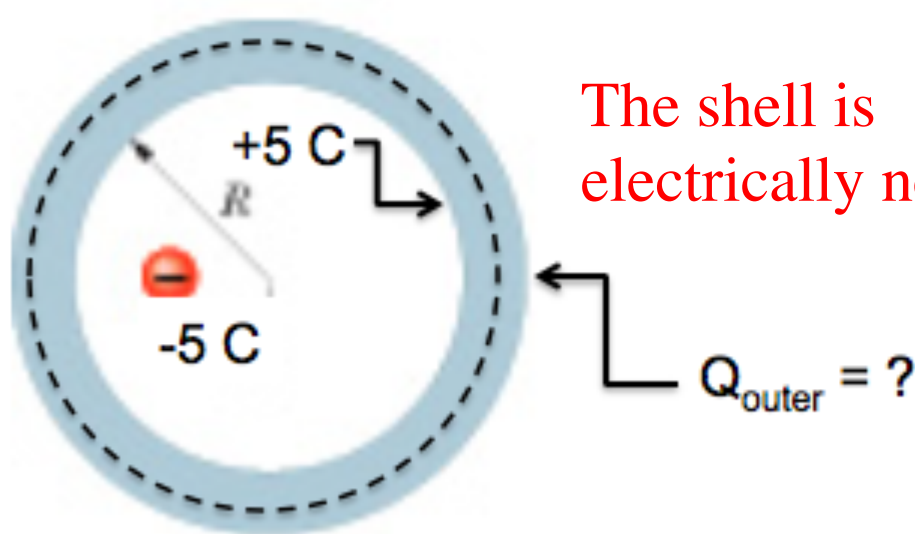
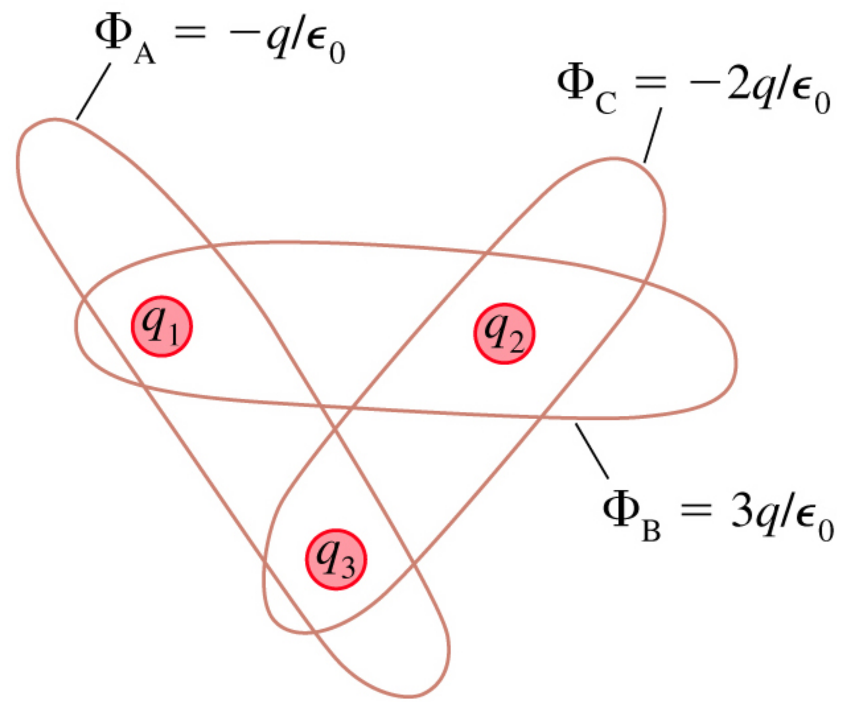


The shell is electrically neutral.

Problem

- Given the fluxes through the Gaussian surfaces, what are the values of the charges q_1, q_2 and q_3?



Step back: how well do we know Gauss' and Coulomb's law

- Gauss' law is equivalent to Coulomb's law only because Coulomb's law is an inverse square law
- How well is Coulomb's law/ Gauss' law known?

- Joseph Priestly knew that there is no gravitational field within a spherically symmetric mass distribution and speculated that a similar behavior of the gravitational and electric force laws would explain a charged cork ball placed inside the a charged metal container is not attracted to the walls of the container
  - this effect was first seen by Benjamin Franklin who told Priestly

Deviations from inverse square law

- John Robison did experimental tests in 1769 of the distance behavior of the forces between charges
- Robison expressed the uncertainties in his result as a deviation from Coulomb's law
  - $F \alpha 1/r^{2+/-\delta}$

Constraints on δ have improved

Investigator	Date	Maximum δ
Robison	1769	0.06
Cavendish	1773	0.02
Coulomb	1785	0.10
Maxwell	1873	5×10^{-5}
Plimpton and Lawler	1936	2×10^{-9}
Williams, Fawler and Hill	1971	3×10^{-16}

Now verified to better than 1 part per billion from atomic scale to galactic scale.

Electric potential

- We've discussed the similarities before between the force between two charges and the force between two masses

$$\vec{F} = \frac{q_1 q_2}{r^2}\hat{r}$$

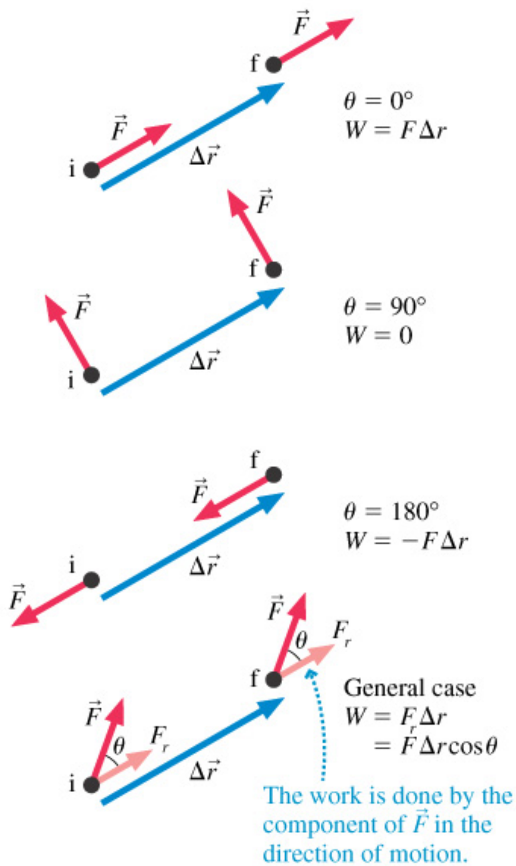
$$\vec{F} = -G\frac{m_1 m_2}{r^2}\hat{r}$$

- Any force that is a function of position only is a conservative force which means that we can associate a potential energy with it
  - $E_{mech} = K + U$
  - $\Delta E_{mech} = \Delta K + \Delta U$

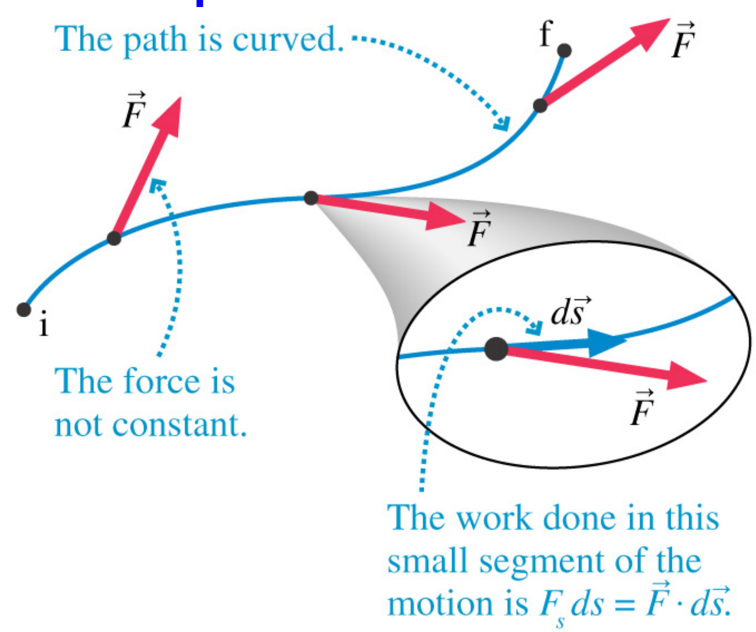
- $K = \Sigma K_i = \Sigma 1/2 m_i v_i^2$
  - sum of all kinetic energies in problem
- U = interaction energy of the system = potential energy
- Most often talk about change in potential energy due to work performed by conservative force
  - $\Delta U = U_f - U_i = - W_{force}$

Work

- Work that a constant force does is
  - $W = \vec{F} \cdot \Delta\vec{r} = F \Delta r \cos\theta$



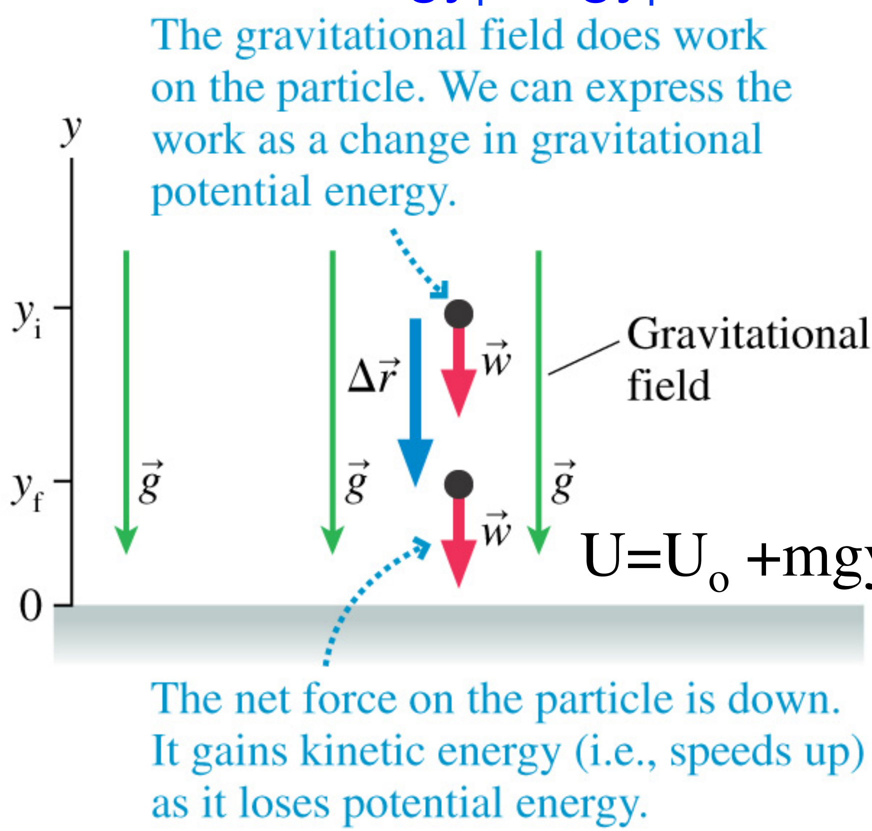
- If $\vec{F}$ or $\Delta\vec{r}$ is not constant, then have to integrate $\vec{F} \cdot d\vec{s}$ over the path travelled



. . .for a conservative force, the work performed is independent of the path

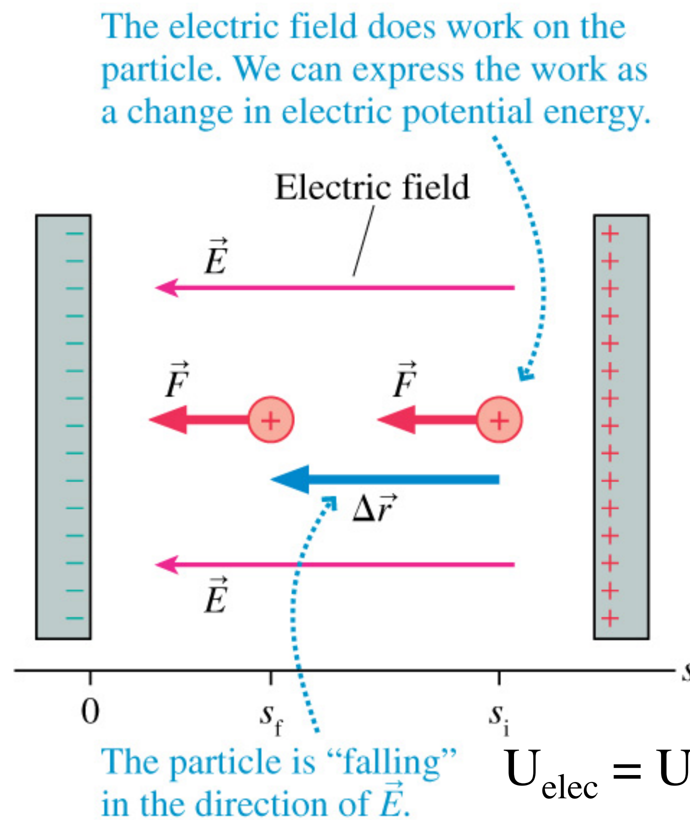
Uniform fields

- $W_{grav} = mg\,\Delta r \cos 0^o$
  $= mgy_i - mgy_f$
- $\Delta U_{grav} = U_f - U_i = -W_{grav}$
  $= mgy_f - mgy_i$



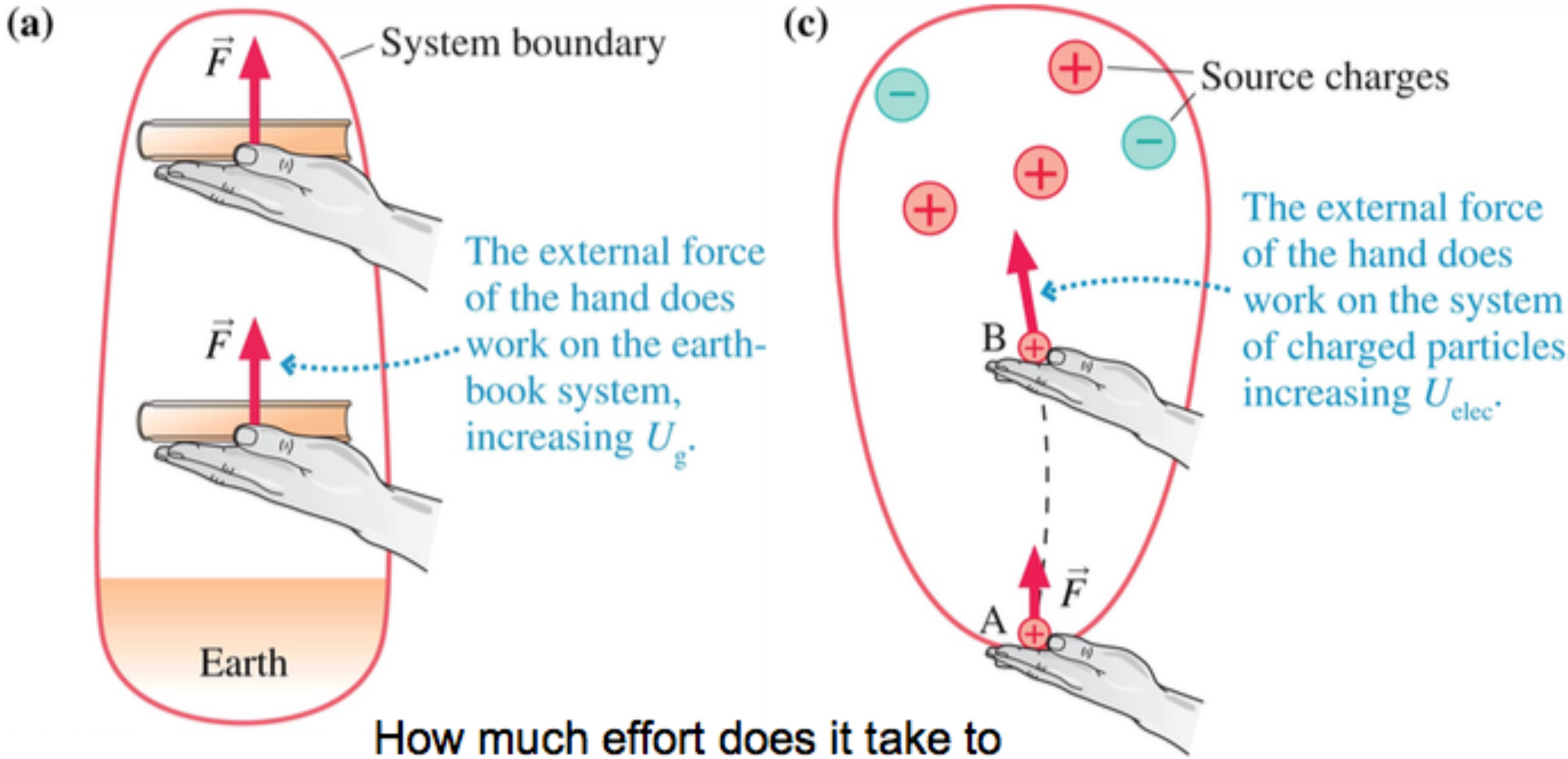
$U = U_o + mgy$

- $W_{elec} = F\Delta r \cos 0^o$
  $= qEs_i - qEs_f$
- $\Delta U_{elec} = U_f - U_i = -W_{elec}$
  $= qEs_f - qEs_i$



$U_{elec} = U_o + qEs$

Potential energy

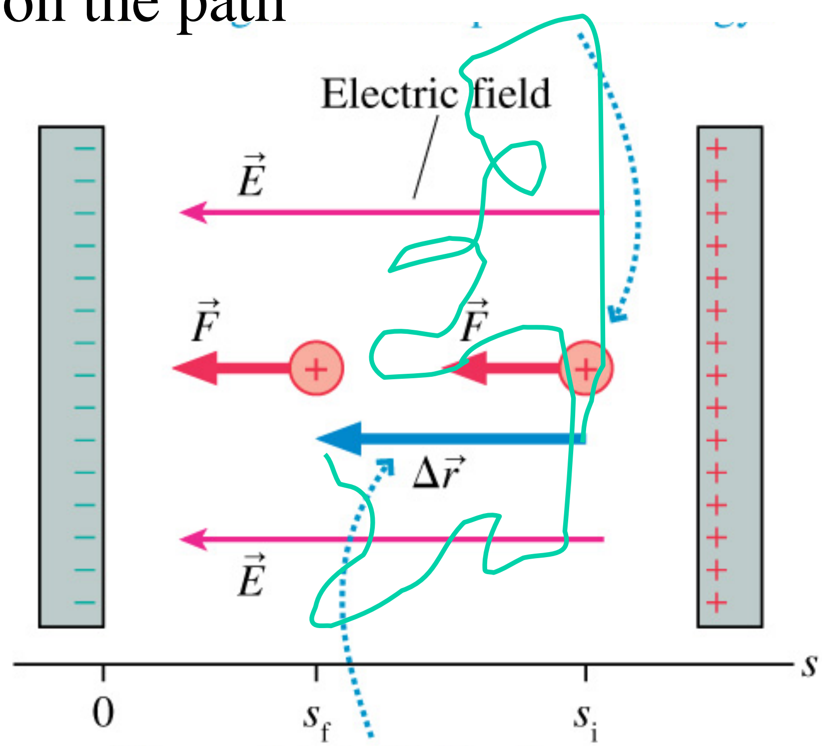


How much effort does it take to move the charge from A to B?

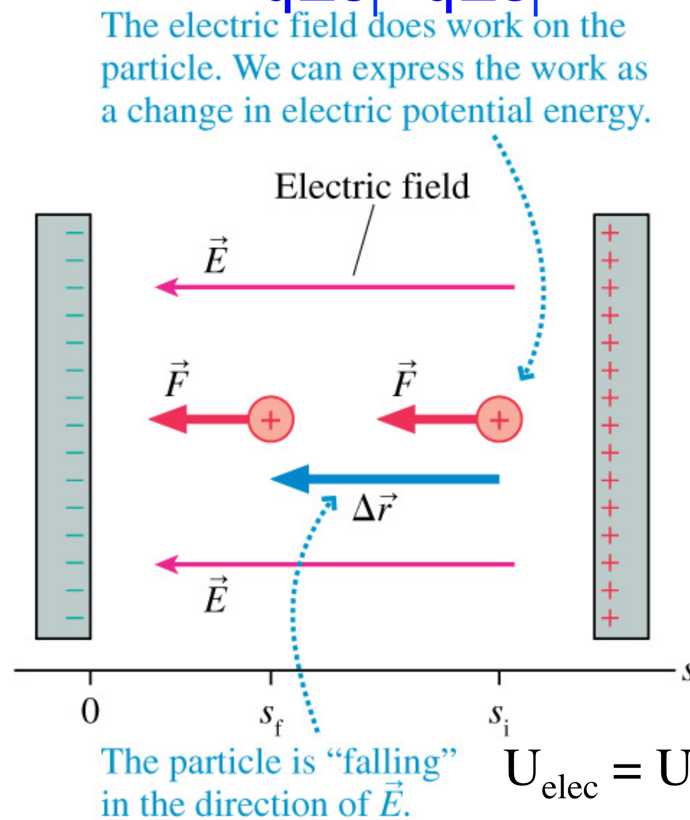
Hard (repulsive): U increases,
Easy (attractive): U decreases

Uniform fields

Note that the work done by the E field, and thus the change in potential energy does not depend on the path

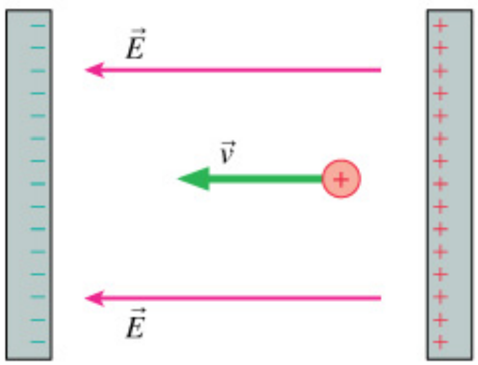


- $W_{elec} = F\Delta r \cos 0^o$
  $= qEs_i - qEs_f$
- $\Delta U_{elec} = U_f - U_i = -W_{elec}$
  $= qEs_f - qEs_i$

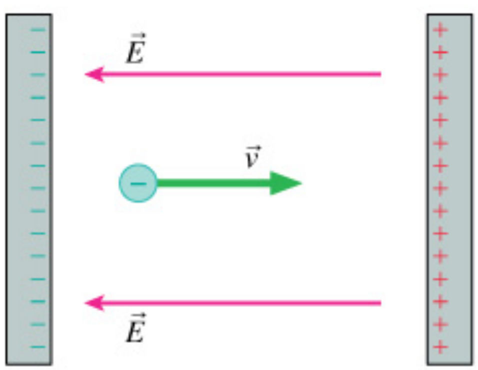


$U_{elec} = U_o + qEs$

Uniform fields



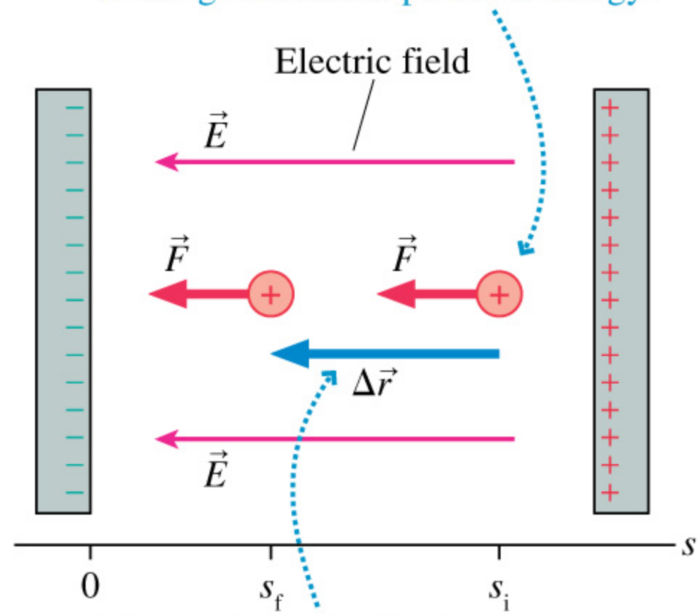
The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.



The potential energy of a negative charge decreases in the direction opposite to $\vec{E}$. The charge gains kinetic energy as it moves away from the negative plate.

- $W_{elec} = F\Delta r \cos 0^{o}$
  $= qEs_i - qEs_f$
- $\Delta U_{elec} = U_f - U_i = -W_{elec}$
  $= qEs_f - qEs_i$

The electric field does work on the particle. We can express the work as a change in electric potential energy.

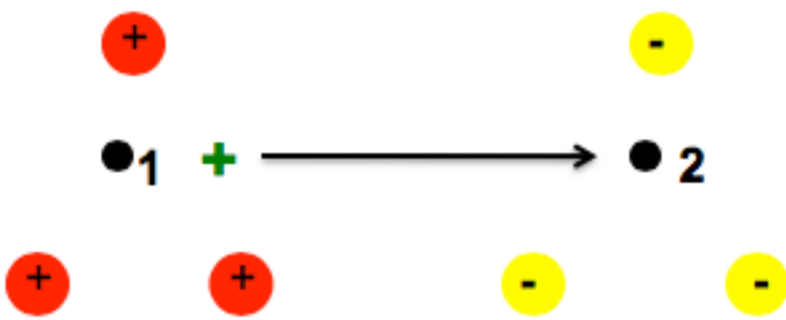


The particle is "falling" in the direction of $\vec{E}$.

$U_{elec} = U_o + qEs$

iclicker question

As you move a positive (+) charge from point 1 to point 2 its electric potential energy (U)



A. Decreases
B. Increases
C. Stays the same
D. Not enough info

iclicker question

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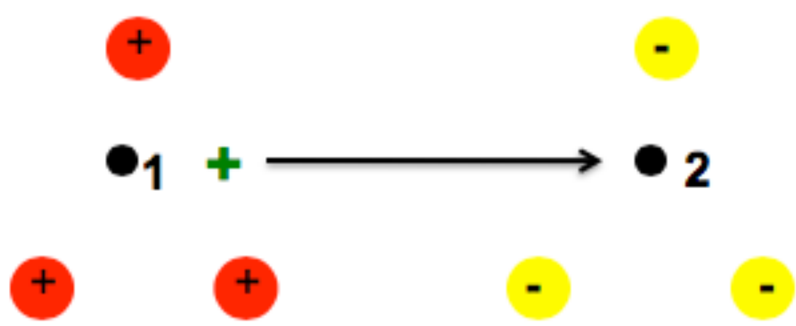


A. Decreases

B. Increases

C. Stays the same

D. Not enough info

Potential energy of point charges

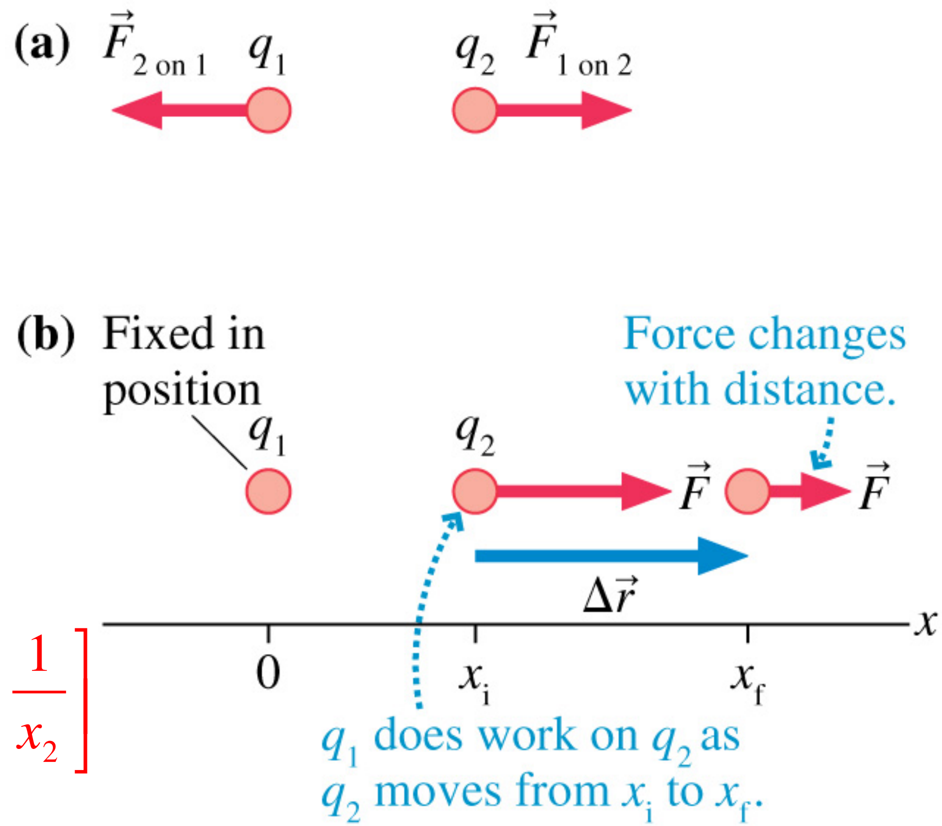
- First, let me calculate the work done by charge 1 on charge 2 while charge 2 moves from point x_1 to point x_2

$$W_{elec} = \int_{x1}^{x2} \vec{F}_{1on2} \bullet \vec{d}x$$

$$W_{elec} = \frac{q_1 q_2}{4\pi\varepsilon_o} \int_{x1}^{x2} \frac{1}{x^2} dx$$

$$W_{elec} = \frac{q_1 q_2}{4\pi\varepsilon_o} \left(-\frac{1}{x} \right)_{x1}^{x2} = \frac{q_1 q_2}{4\pi\varepsilon_o} \left[\frac{1}{x_1} - \frac{1}{x_2} \right]$$

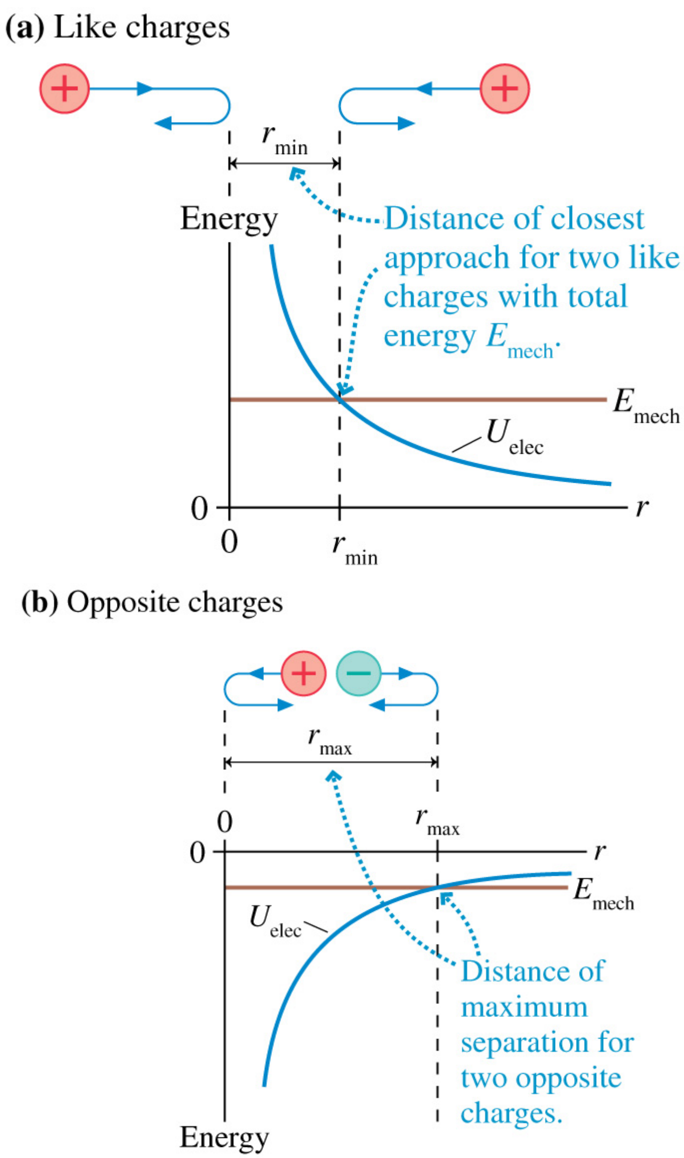
$$\Delta U_{elec} = U_f - U_i = -W_{i \to f} = \frac{q_1 q_2}{4\pi\varepsilon_o} \left[\frac{1}{x_2} - \frac{1}{x_1} \right]$$

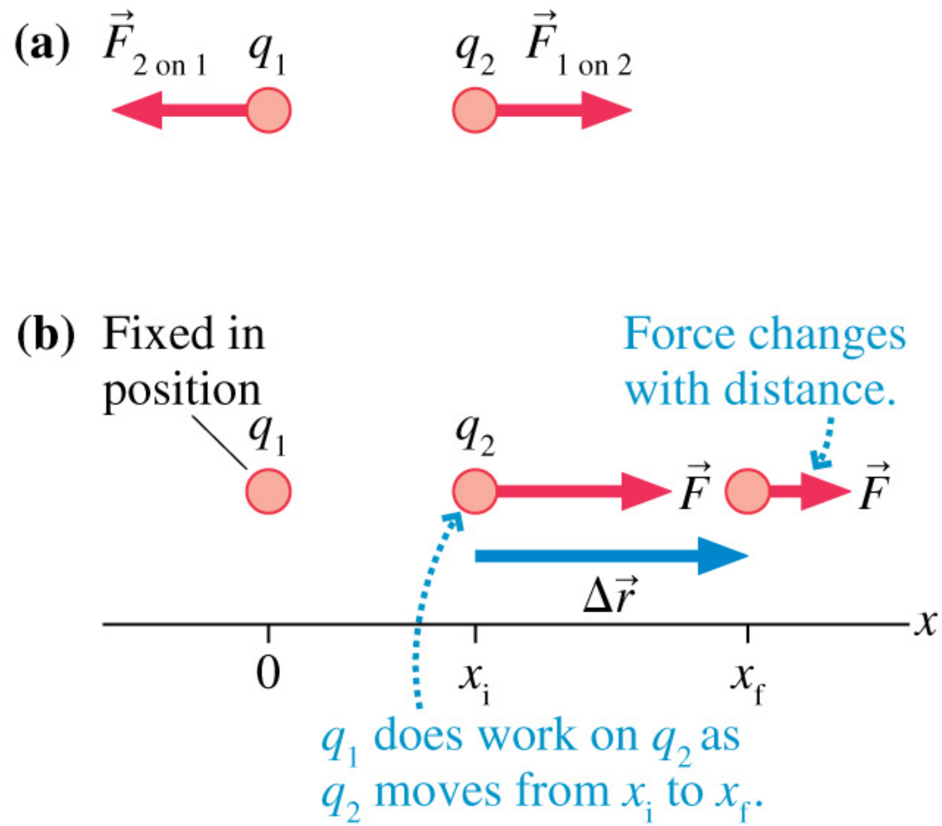


use r instead of x. Set U_{inf}=0

$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \quad \text{(two point charges)}$$

Potential energy of point charges





$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

Note that the potential energy between any two point charges is zero at infinite separation

iclicker question

Rank in order, from largest to smallest, the potential energies U_a to U_d of these 4 pairs of charges. Each + symbol represents the same amount of charge.

$$U_{elec} = \frac{Kq_1q_2}{r}$$

A. $U_a = U_b > U_c = U_d$
B. $U_b = U_d > U_a = U_c$
C. $U_a = U_c > U_b = U_d$
D. $U_d > U_c > U_b > U_a$
E. $U_d > U_b = U_c > U_a$

iclicker question

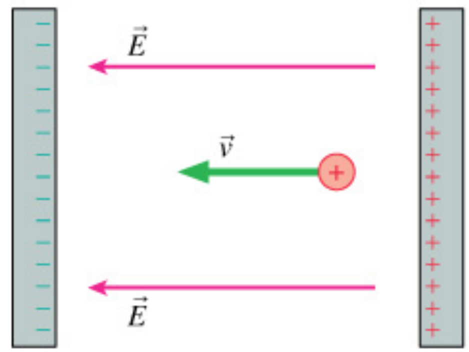
Rank in order, from largest to smallest, the potential energies U_a to U_d of these 4 pairs of charges. Each + symbol represents the same amount of charge.

$$U_{elec} = \frac{Kq_1q_2}{r}$$

A. $U_a = U_b > U_c = U_d$
B. $U_b = U_d > U_a = U_c$
C. $U_a = U_c > U_b = U_d$
D. $U_d > U_c > U_b > U_a$
E. $U_d > U_b = U_c > U_a$

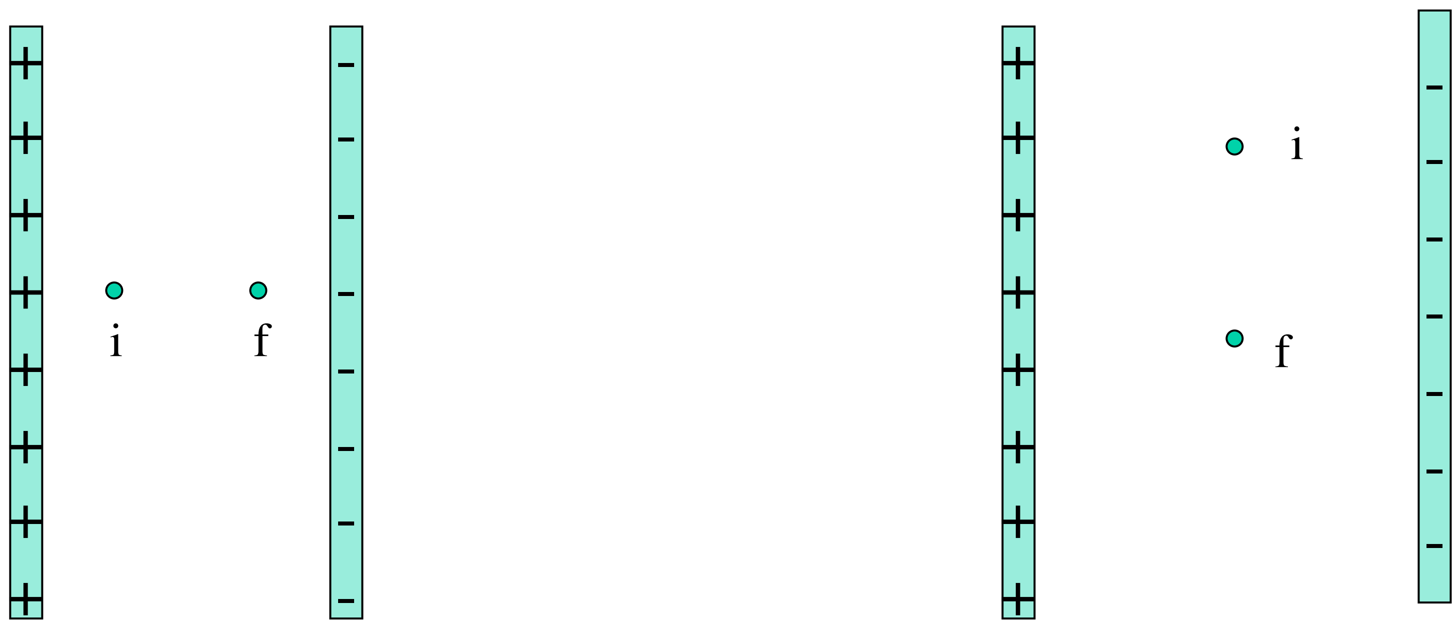
Example

- Suppose that the E field inside the capacitor is 50,000 N/C and the spacing in the capacitor is 2 mm
- I release a proton from rest from the position of the positive plate
  - what's the work done by the electric field by the time it gets to the negative plate?
  - what is the proton's speed?



The potential energy of a positive charge decreases in the direction of $\vec{E}$. The charge gains kinetic energy as it moves toward the negative plate.

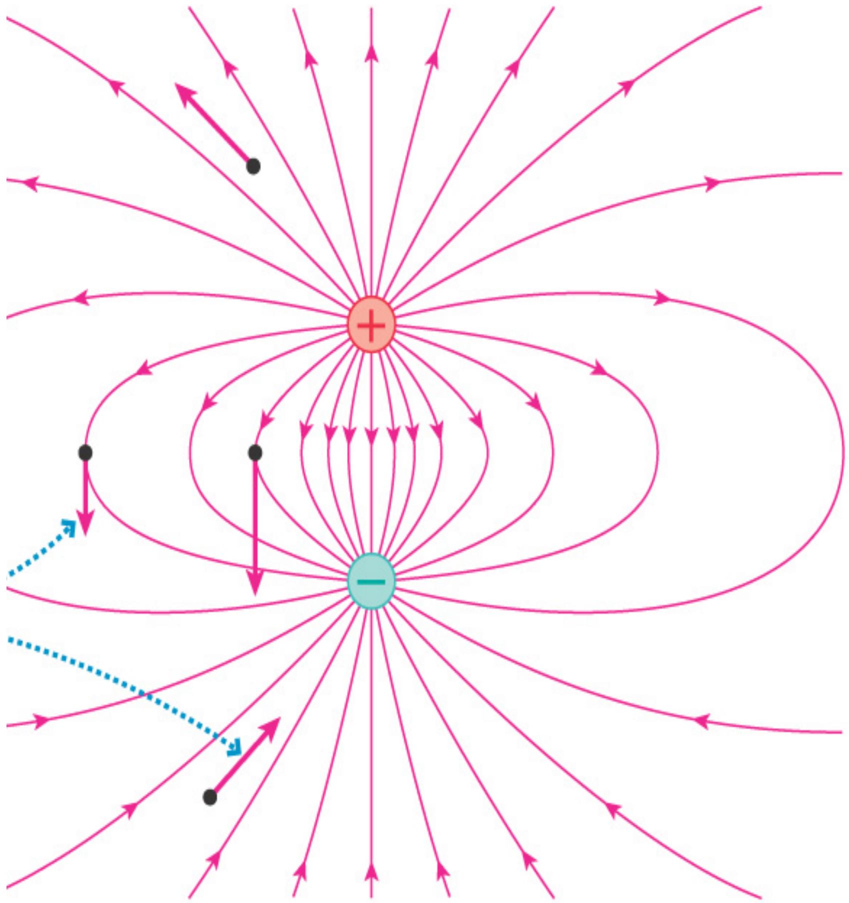
Examples



Suppose I move a positive charge from i to f in each of the two cases above. Is ΔU positive, negative or zero? What about a negative charge? Is the field doing positive or negative work? (Note that it can be easy to confuse the work done by the field and the work done by an external agent.)

Another example

- What if I move a positive charge along a field line, in the direction of the field?
  - is ΔU + or -?
- What about a negative charge?

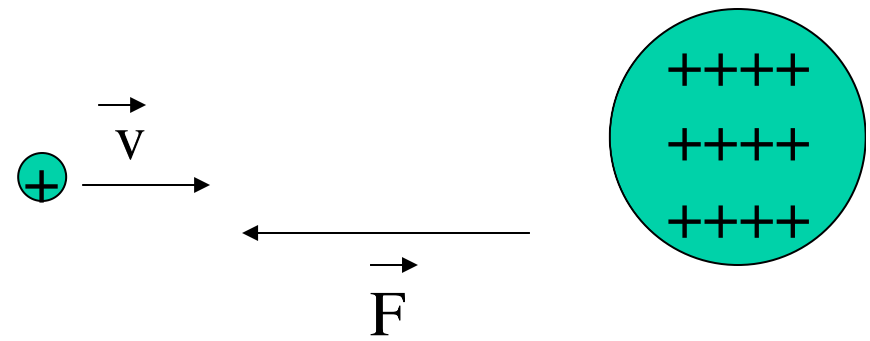


Example

- Suppose a small positive charge is shot towards a larger fixed charge
- What happens to its speed?

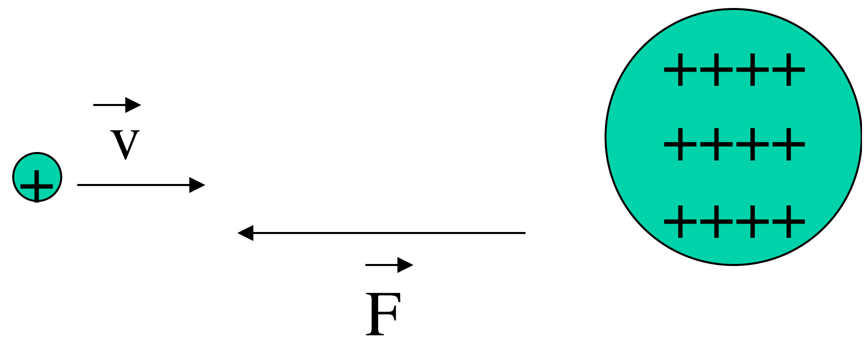
Example

- Suppose a small positive charge is shot towards a larger fixed charge
- What happens to its speed?
  - there's a force acting on the small charge causing a negative acceleration (it's slowing down)
  - this force is varying with the distance between the two charges
  - I can calculate the work done by this force by integrating $\vec{F}\cdot d\vec{r}$
  - …or I can calculate the change in potential energy

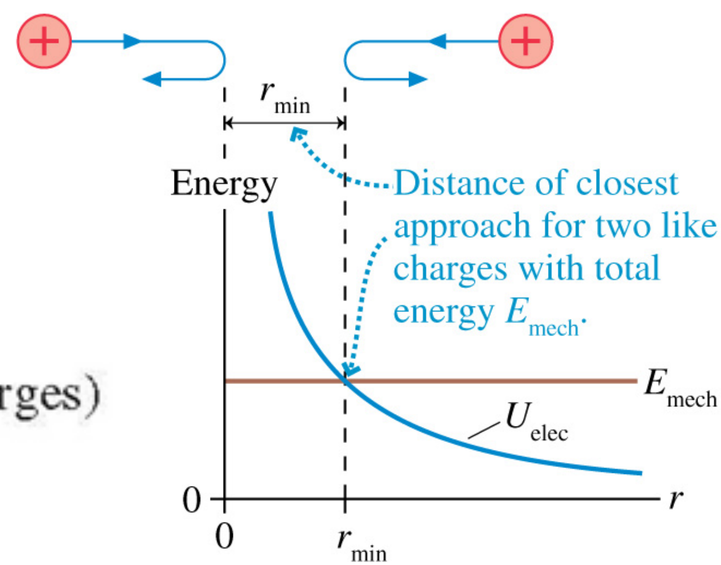


Example

- Suppose a small positive charge is shot towards a larger fixed charge
- How close does it come?
  - let's suppose the small charge is a proton and it has the same velocity we used for the last example?
    - 1.38 X 10^5 m/s
  - and that the fixed charge is a carbon nucleus (Z= +6e)



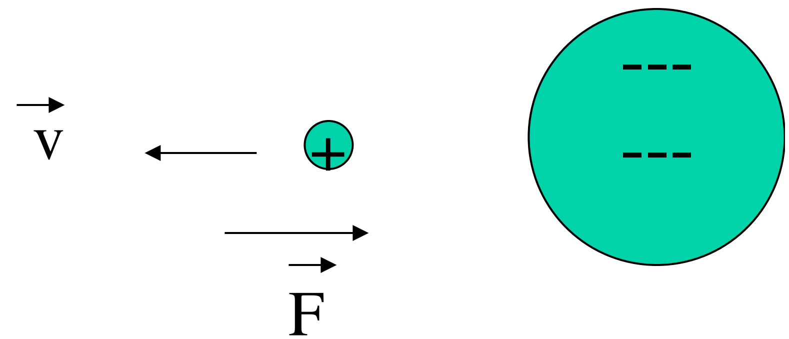
(a) Like charges



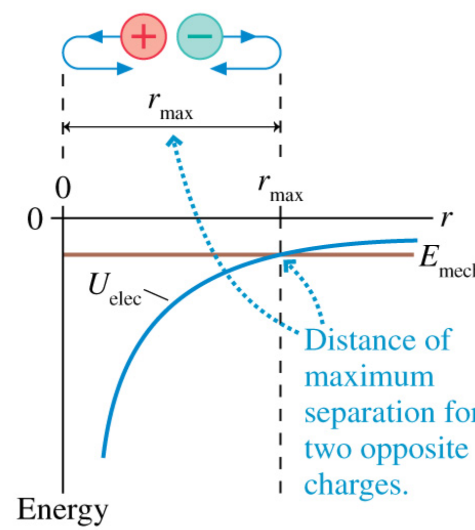
$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

Example

- Suppose instead we have a proton shot away from a large negative charge (equal to -6e) with the initial velocity equal the final velocity for the proton in the previous problem and starting at the distance that the proton had stopped.
- How far does it go before stopping?
  - $U_f - U_i = K_f - K_i$



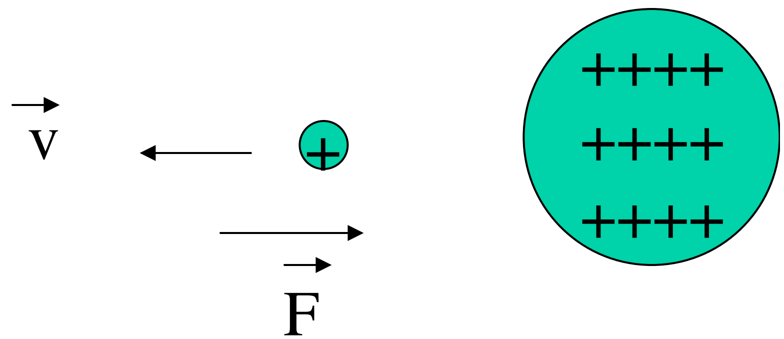
(b) Opposite charges



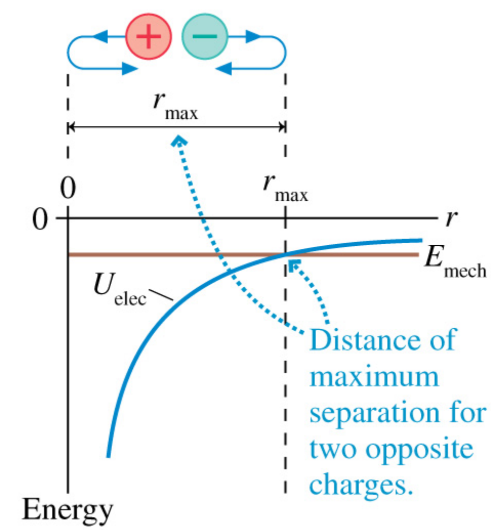
$$U_{\text{elec}} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} \qquad \text{(two point charges)}$$

Example

- Suppose instead we have a small negative charge (an electron) shot away from the positive nucleus (with the initial velocity equal the final velocity for the proton in the previous problem and starting at the distance that the proton had stopped).
- How far does it go before stopping?
  - $U_f - U_i = K_f - K_i$



(b) Opposite charges



$$U_{elec} = \frac{Kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$ (two point charge

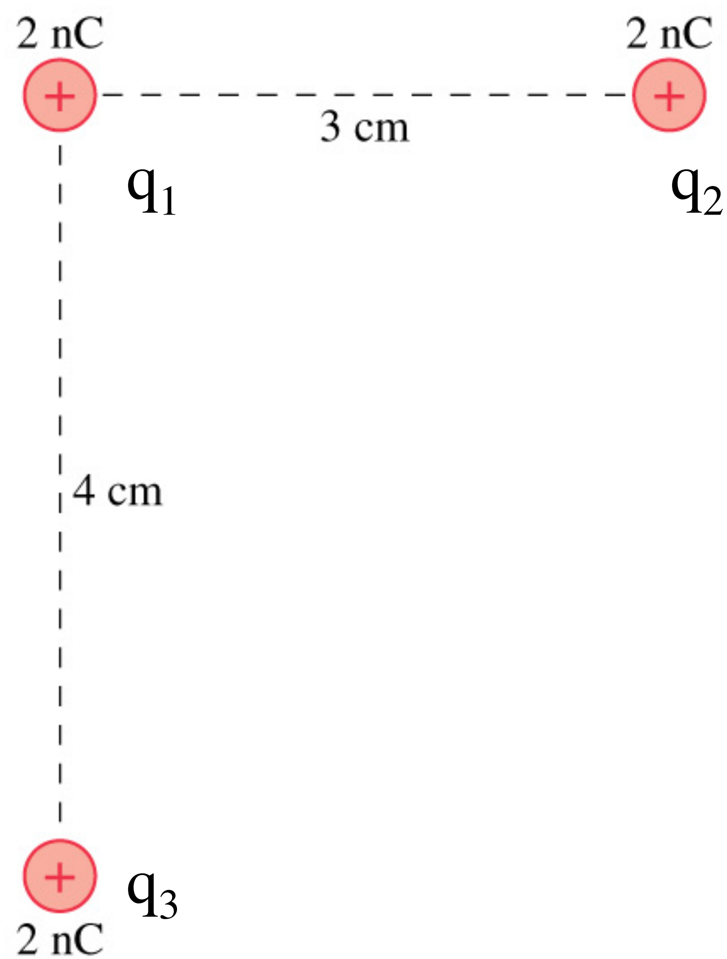
Multiple point charges

- What if I have more than 1 charge? What is the total potential energy?

$$U_{elec} = \sum_{i<j} \frac{1}{4\pi\varepsilon_o} \frac{q_i q_j}{r_{ij}}$$

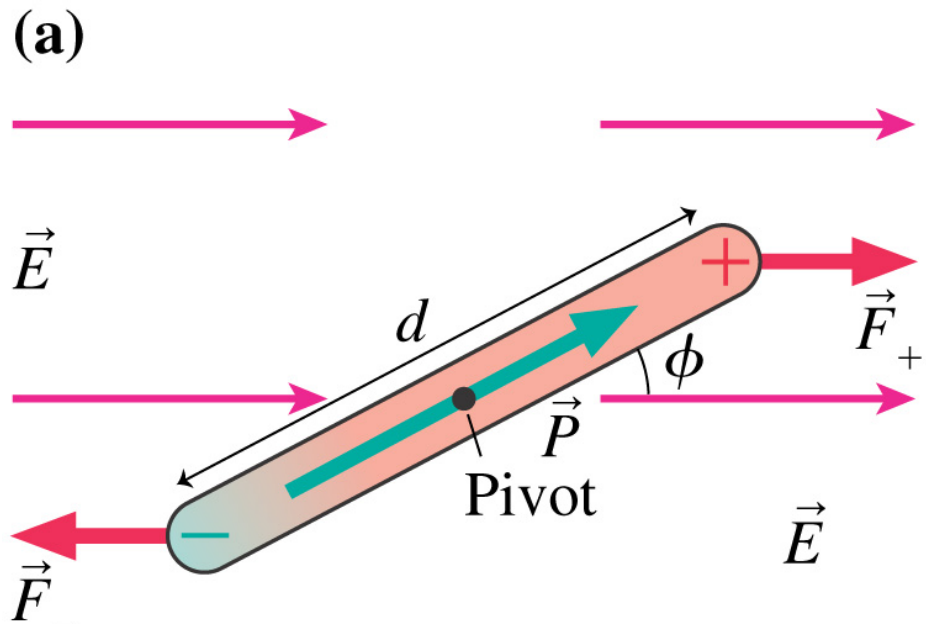
$$U_{elec} = \frac{1}{4\pi\varepsilon_o}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

  - what is the potential energy of these charges when they' re separated by an infinite distance?
  - how much work does it take an external agent to assemble them in the positions shown?
  - $W_{\text{external agent}} = U_f - U_i = -W_{\text{field}}$



Our old friend, the electric dipole

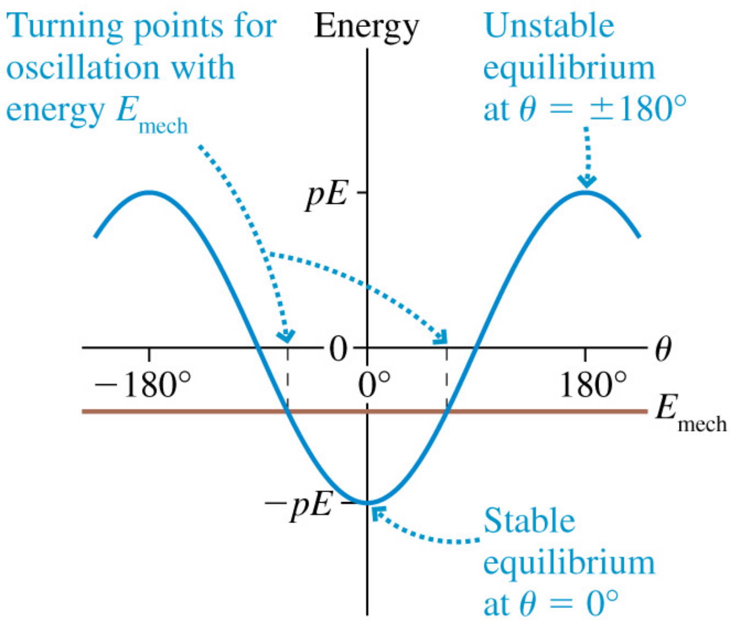
- Earlier we found that a dipole in an electric field experiences a torque that causes the dipole moment $\vec{p}$ to rotate in alignment with the electric field
- What about the work done by the electric field in causing this rotation and the change in the potential energy?

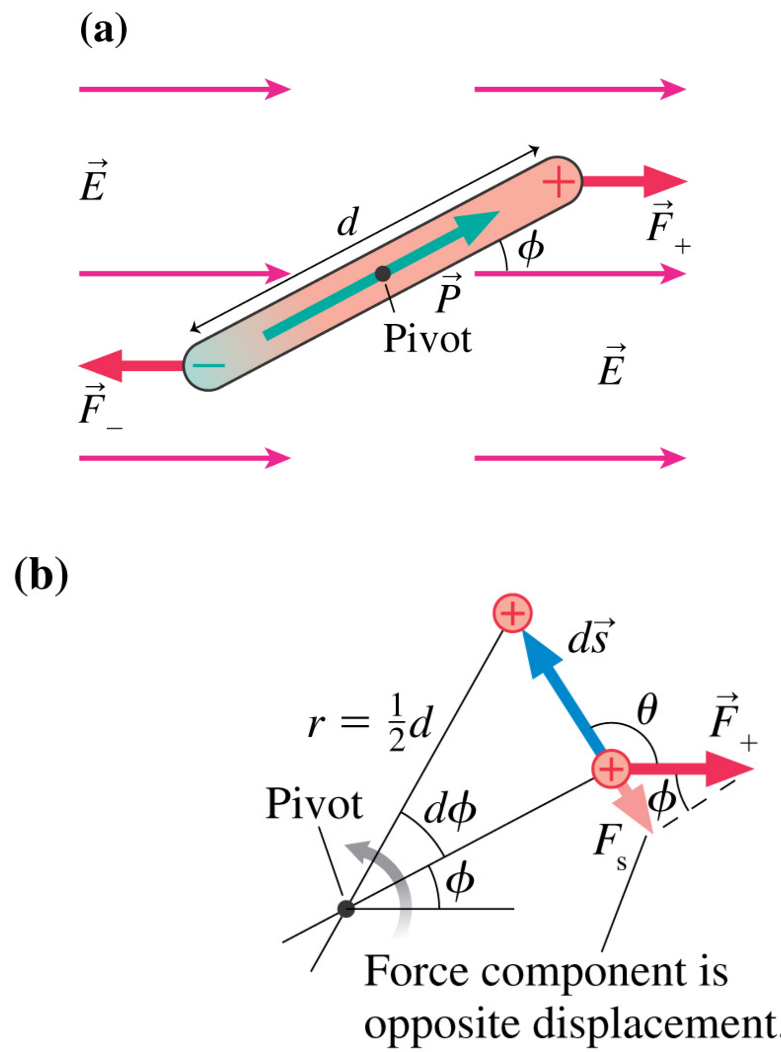


$|p| = qd$

Dipoles

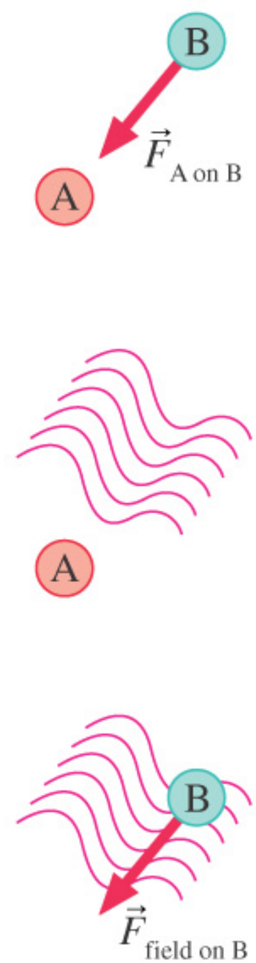
- $\Delta U_{dipole} = U_f - U_i$
  $= -W_{elec}(i\text{->}f)$
  $= -pE\cos\phi_f + pE\cos\phi_i$
- $U_{dipole} = -pE\cos\phi$
  $= -\vec{p}\cdot\vec{E}$





Back to the electric field

- So we' re going to replace the idea of action at a distance by the concept of a field
- Particles don' t interact directly with each other
- They create fields which then interact with the other particles
  - we will need this when we start talking about dynamic situations
- We' ll be dealing with electric and magnetic fields in this course



In the Newtonian view, A exerts a force directly on B.

In Faraday's view, A alters the space around it. (The wavy lines are poetic license. We don't know what the alteration looks like.)

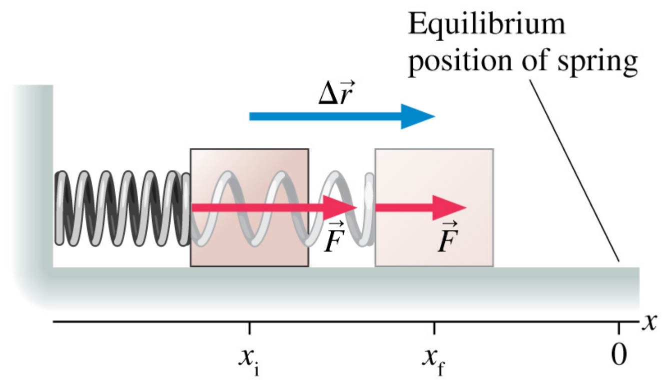
Particle B then responds to the altered space. The altered space is the agent that exerts the force on B.

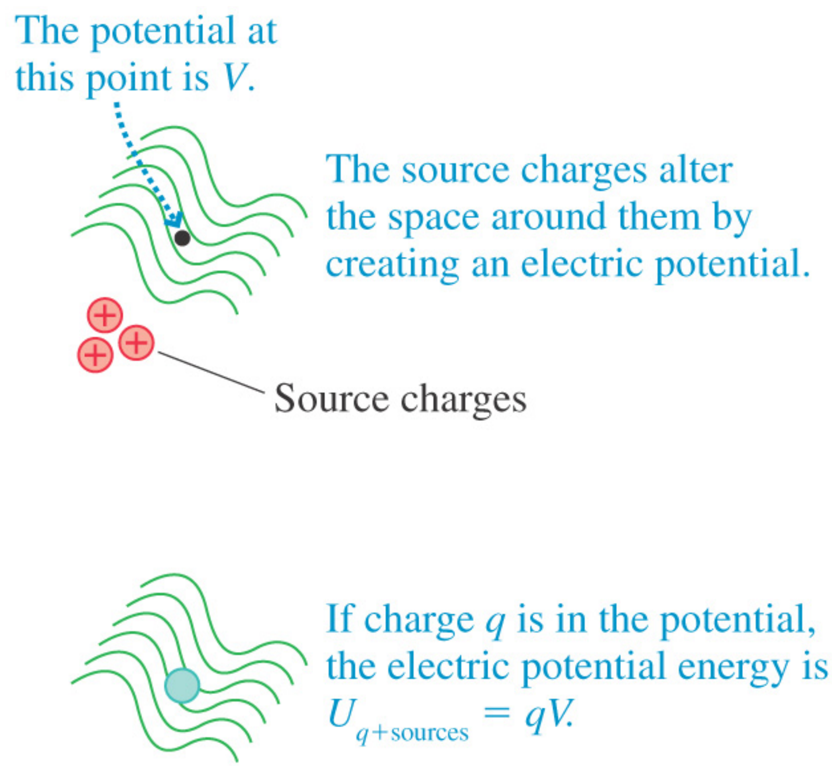
Electric potential: another abstract concept

- It's easy to understand the energy stored in a spring when it's compressed or stretched
- Harder to understand the energy stored in the interaction of charges
- Suppose I have a bunch of source charges interacting with a (test) charge q
- The potential energy is

$$U = \frac{1}{4\pi\varepsilon_o}\frac{q_{source}q}{r}$$

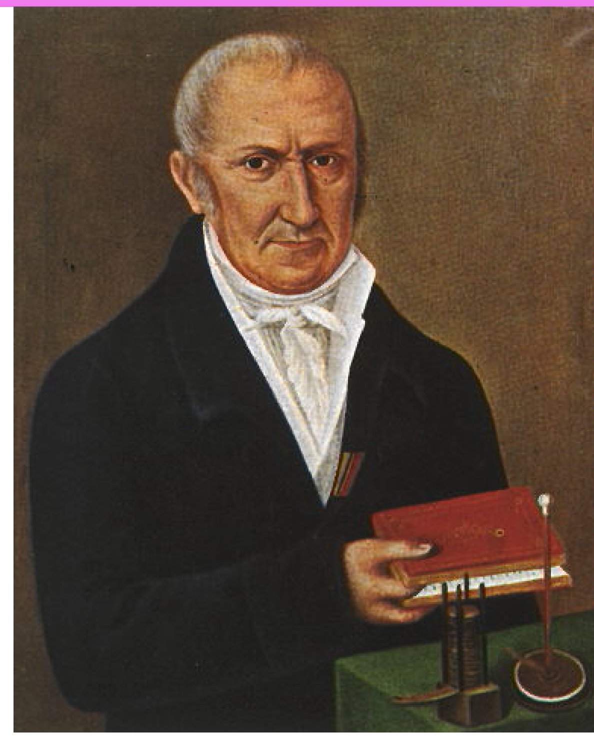
- Let me define a quantity called the electric potential (V) such that U = qV
  - V is the potential created by the source charges
  - U is the potential energy that a charge q has in the potential V

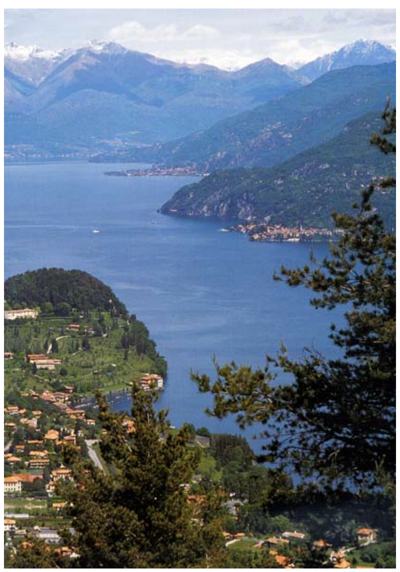




Units of potential

- Unit of electric potential is the joule per coulomb or volt (after Alessandro Volta), inventor of the electric battery
  - 1 V = 1 J/C





Lake Como, where he hung out