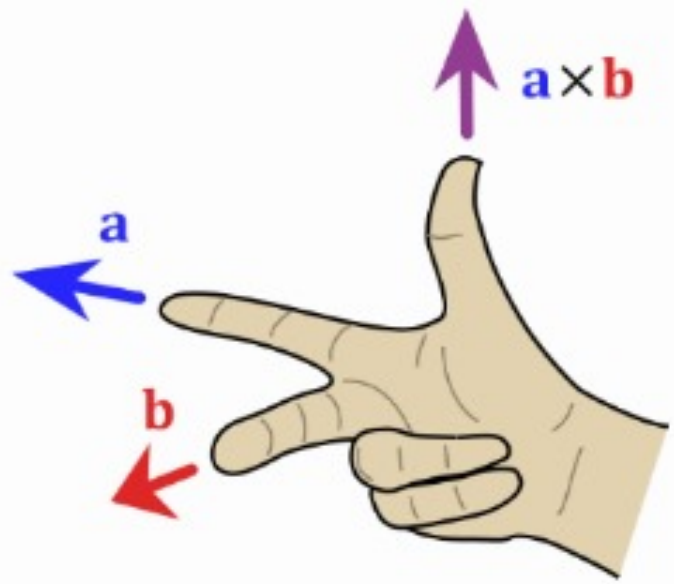
PHY294H

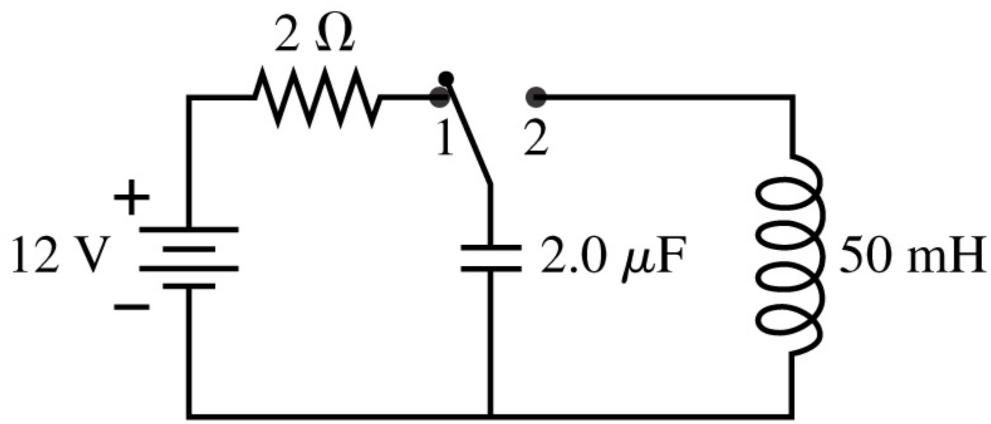
- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Help-room hours: 12:40-2:40 Monday (note change); 3:00-4:00 PM Friday**
  - **hand-in problem for Wed Mar. 16: 33.54**
- Quizzes by iclicker (sometimes hand-written)
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

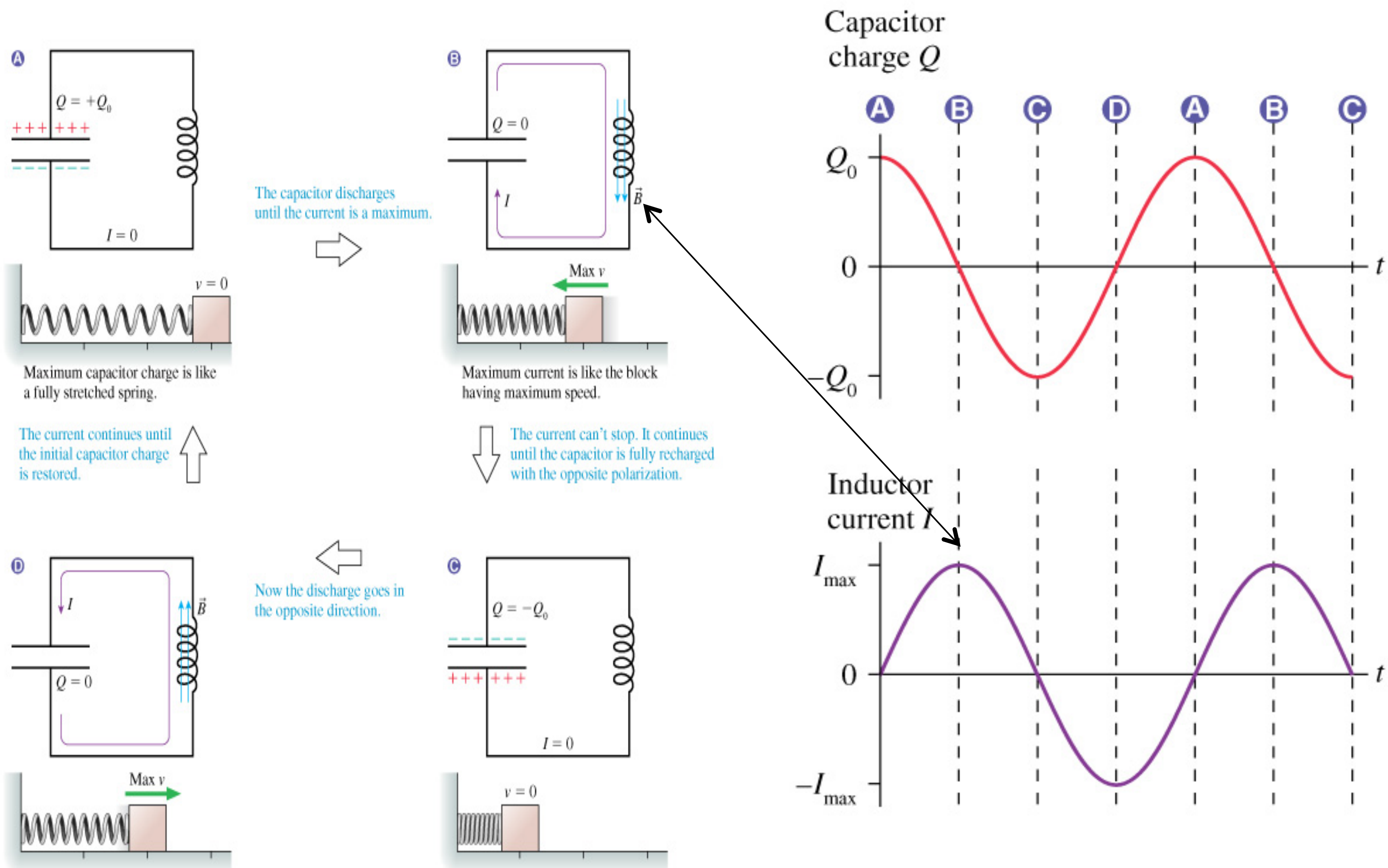


Physics gang sign.

Example

- The switch has been in position 1 for a long time
- Then it is abruptly moved to position 2
  - what is the maximum current through the inductor?
  - when does this maximum current occur?
  - what if the inductor and capacitor changed position?

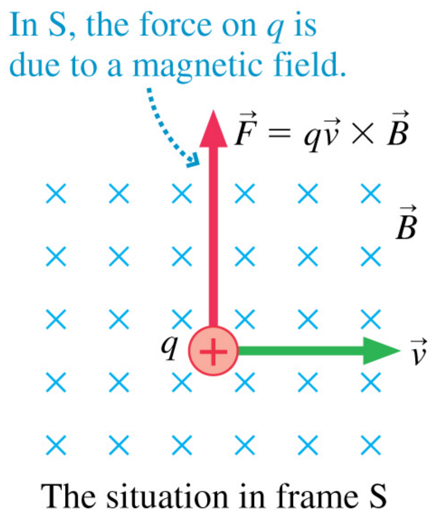


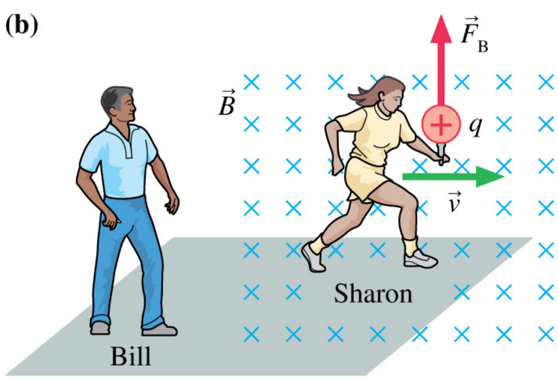


A
Q = +Q0
I = 0
v = 0
Maximum capacitor charge is like a fully stretched spring.
The capacitor discharges until the current is a maximum.
B
Q = 0
I
B
Max v
Maximum current is like the block having maximum speed.
The current can't stop. It continues until the capacitor is fully recharged with the opposite polarization.
C
Q = −Q0
I = 0
v = 0
Now the discharge goes in the opposite direction.
D
I
B
Q = 0
Max v
The current continues until the initial capacitor charge is restored.
Capacitor charge Q
A B C D A B C
Q0
0
−Q0
t
Inductor current I
Imax
0
−Imax
t

Back to Bill and Sharon

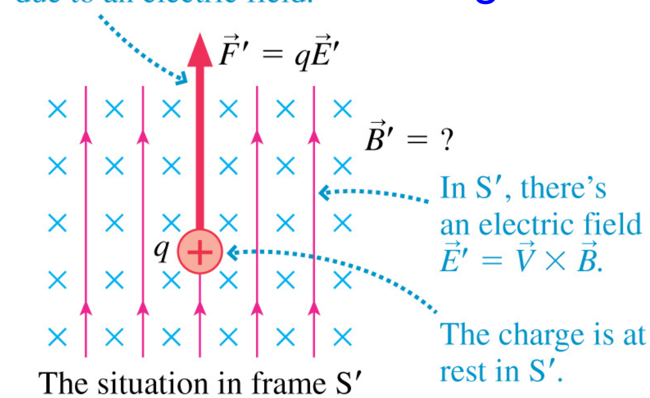
- In S, there is a magnetic force on q
- In S’ , there is no magnetic force





Charge q moves through a magnetic field $\vec{B}$ established by Bill.

- But the forces have to be the same in the two frames of reference
- In S’ , if there is no magnetic force then there must be an electric force on q due to an electric field of size $\vec{V}\times\vec{B}$, in order for Bill and Sharon to agree

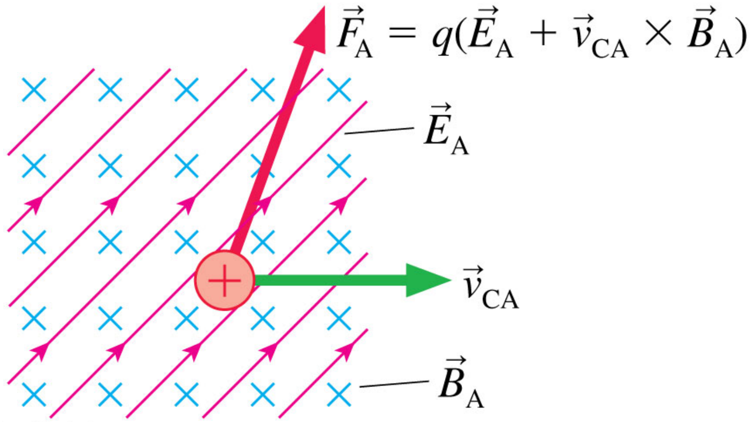


- At least part of Bill’s magnetic field has become an electric field
- Whether a field is seen as electric or magnetic depends on the motion of the reference frame relative to the sources of the field!

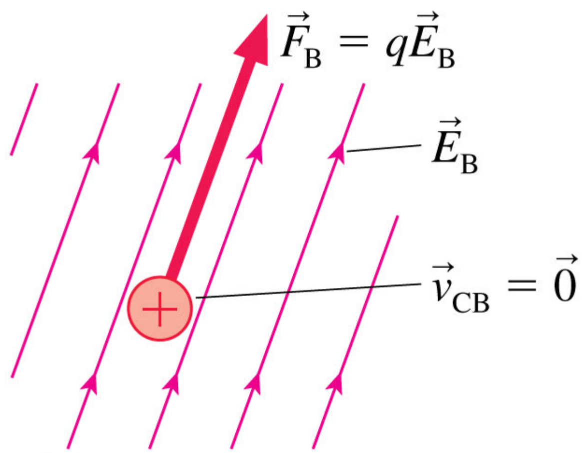
Electric and magnetic forces

- Suppose Bill (the sneaky bastard) decides to create both an electric and magnetic field in his frame of reference (S==A)
- Then the charge carried by Sharon experiences both an electric force and a magnetic force in Bill's frame of reference
- But in Sharon's frame of reference, where the charge is not moving, it experiences only an electric force
- But the magnitude and directions of the forces determined in each frame of reference have to be the same
- So in Sharon's reference frame, part of Bill's magnetic field is transformed into an electric field, and you have Bill's electric field as well

The electric and magnetic fields in frame A =S

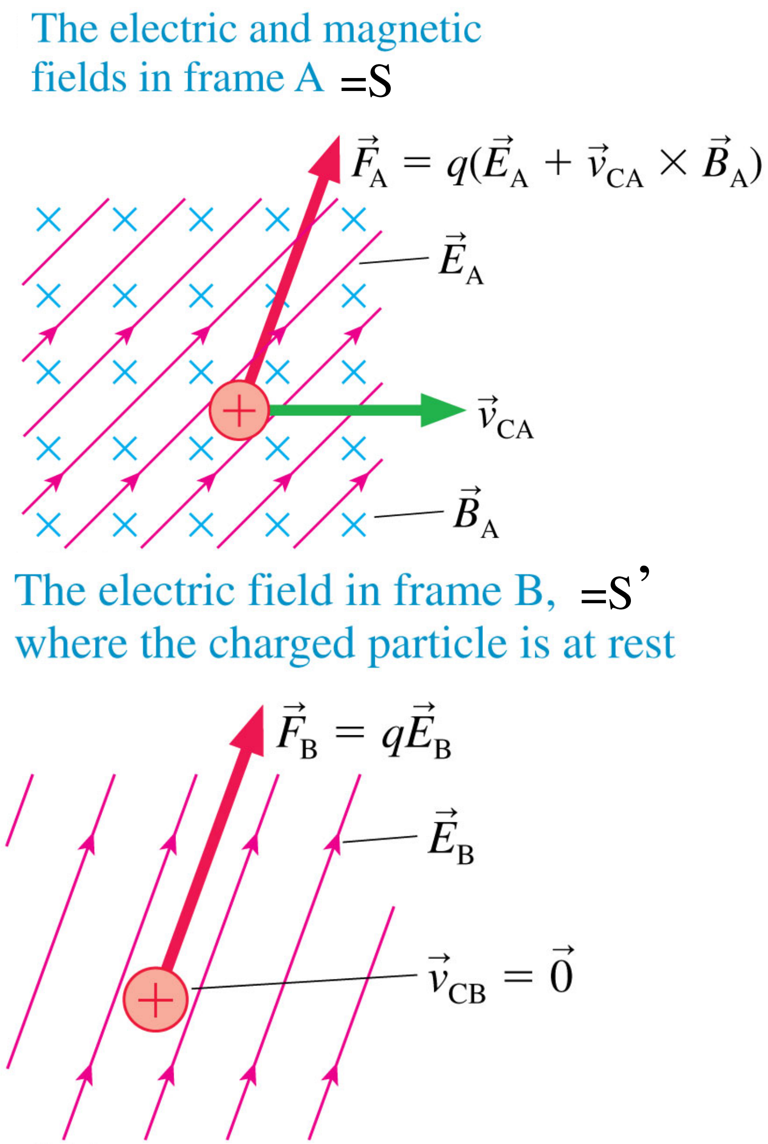


The electric field in frame B, =S' where the charged particle is at rest

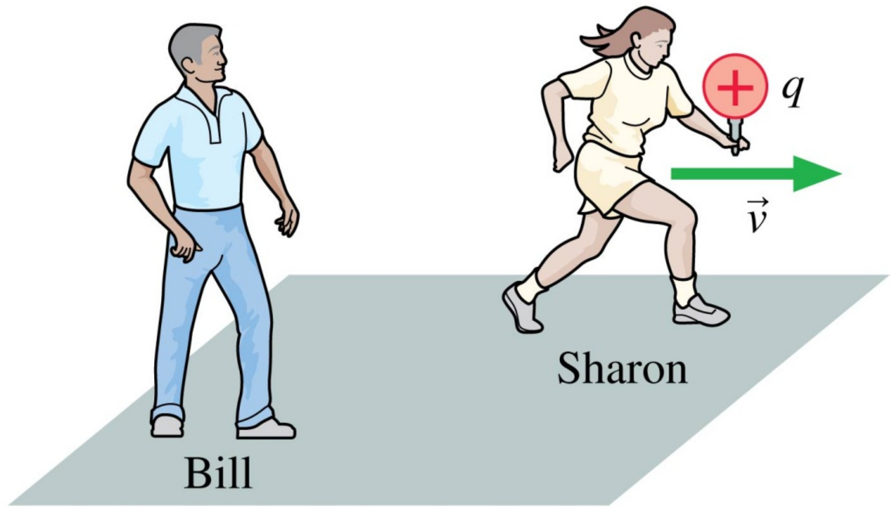


Electric and magnetic forces

- Both Bill and Sharon agree on the force on the charge, but Sharon sees no magnetic force but an electric force from the original electric field and from part of the magnetic field being transformed into an electric field



Sharon runs past Bill while holding a positive charge q. In Bill's reference frame, there is (or are)



A. Only an electric field.

B. Only a magnetic field.

C. An electric and a magnetic field.

D. No fields.

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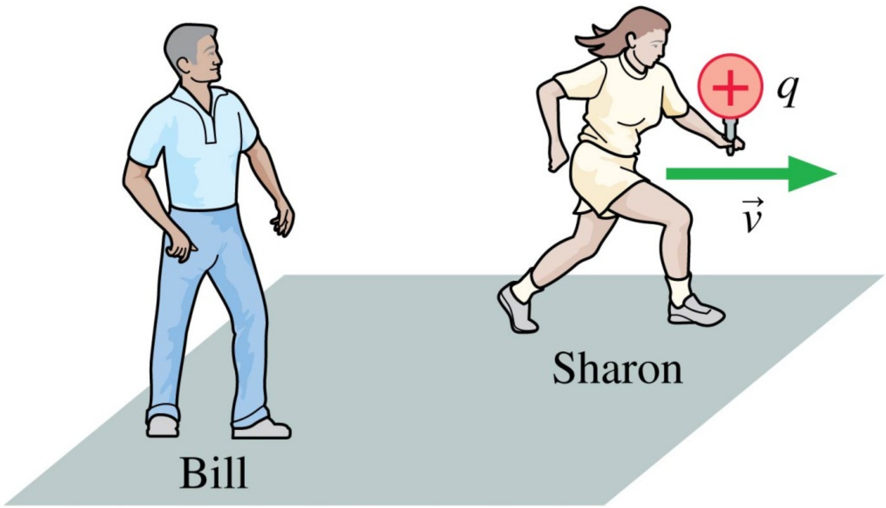

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B. Only a magnetic field.

✔ **C. An electric and a magnetic field.**

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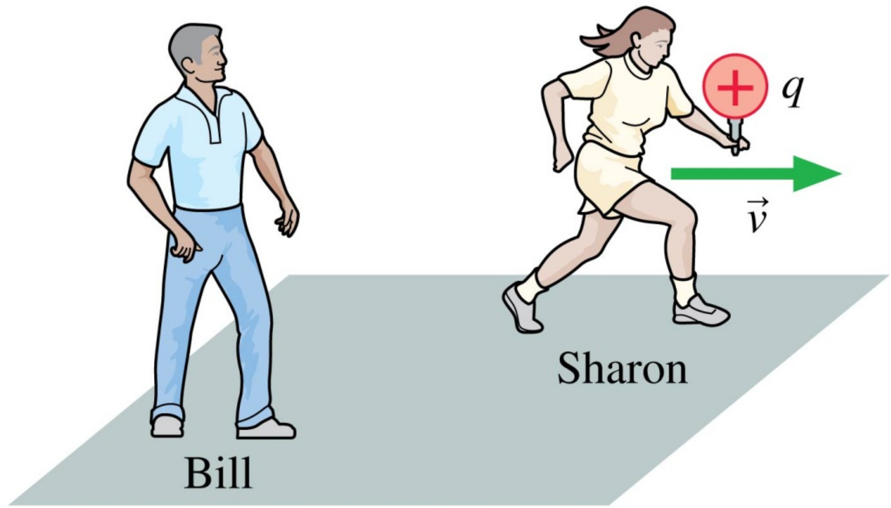

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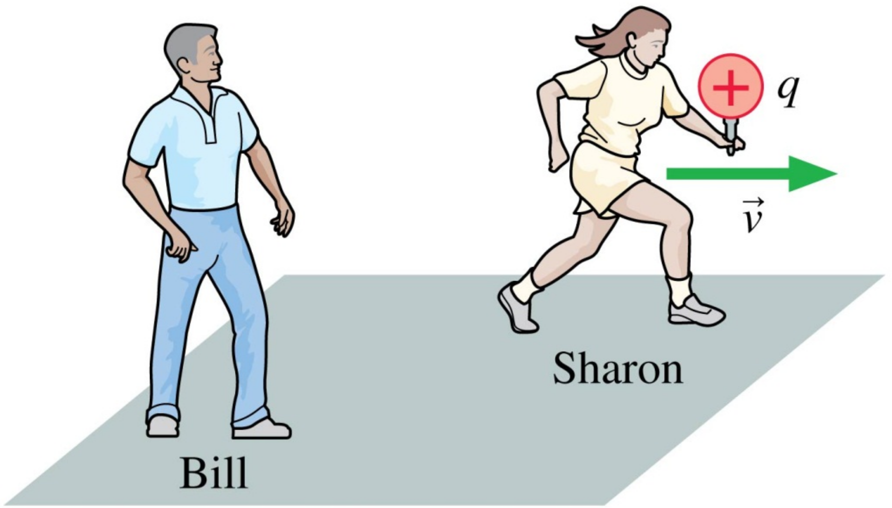


✔ **A. Only an electric field.**

B. Only a magnetic field.

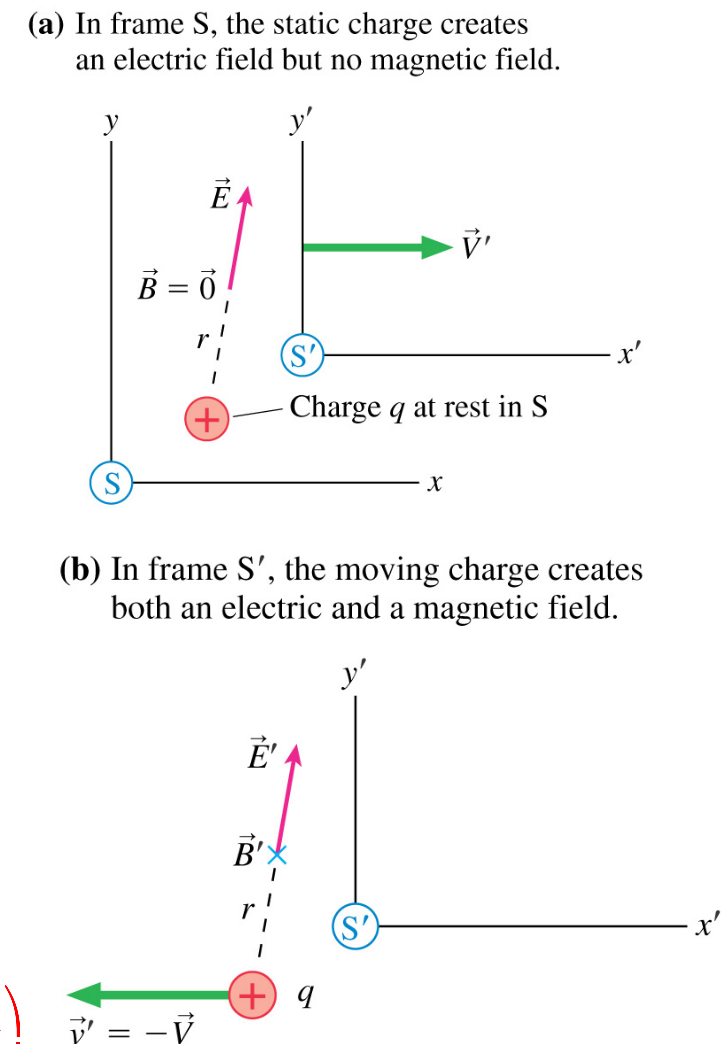
C. An electric and a magnetic field.

D. No fields.

No moving charges in Sharon's frame

What about the magnetic field?

- Consider a charge q at rest in frame S? The electric field is given by the standard formula and the magnetic field B is zero $\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$
- What about in frame S' moving with velocity V'? The electric fields are the same
$$\vec{E}' = \vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}$$
- But there is also a magnetic field since in S' the charge is moving away with a velocity -V



$$\vec{B}' = \frac{\mu_o}{4\pi}\frac{q}{r^2}\vec{v}' X \hat{r} = -\frac{\mu_o}{4\pi}\frac{q}{r^2}\vec{V} X \hat{r} = -\varepsilon_o\mu_o\vec{V} X\left(\frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r}\right)$$

So while there is only an electric field in S, there is both an electric field and magnetic field in S'

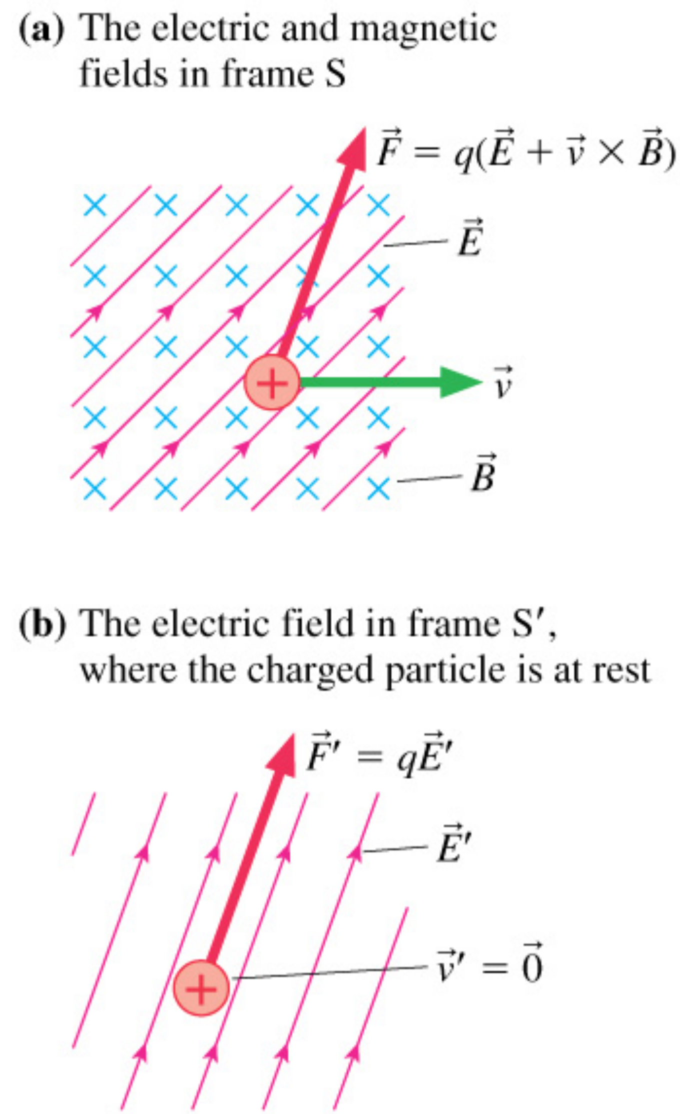
Fields

- The size of the electric and magnetic fields depends on the frame of reference

$$\vec{E}' = \vec{E} + \vec{V} X \vec{B}$$

$$\vec{B}' = \vec{B} - \varepsilon_o \mu_o \vec{V} X \vec{E}$$

- This last term is the Biot-Savart law
  - the magnetic field of a moving point charge is just the Coulomb electric field of a stationary point charge transformed into the moving reference frame



Galilean field transformation equations

- No longer can we think of electric and magnetic fields as being separate things
- There's only one electromagnetic field whose manifestation depends on our frame of reference
- Consider units of the term $\mu_o\varepsilon_o$
  - (Tm/A)(C^2/Nm^2)
  - but 1 T=1 N/Am and 1 A = 1 C/s
  - so units of $\mu_o\varepsilon_o$ = s^2/m^2
  - value of 1/ sqrt($\mu_o\varepsilon_o$)=3X10^8 m/s
- Coincidence
  - I think not

$$\vec{E}' = \vec{E} + \vec{V} X \vec{B}$$

$$\vec{B}' = \vec{B} - \frac{1}{c^2} \vec{V} X \vec{E}$$

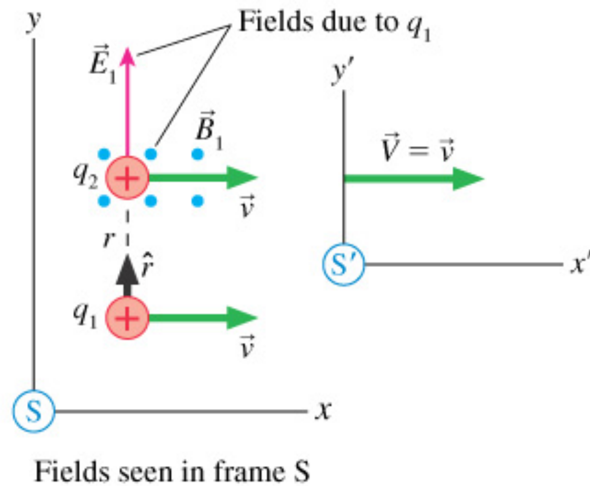
$$\vec{E} = \vec{E}' - \vec{V} X \vec{B}$$

$$\vec{B} = \vec{B}' + \frac{1}{c^2} \vec{V} X \vec{E}$$

works for v<<c

Problems revisited

- Consider the electric and magnetic fields produced by two charges moving with a velocity v in S and at rest in S'

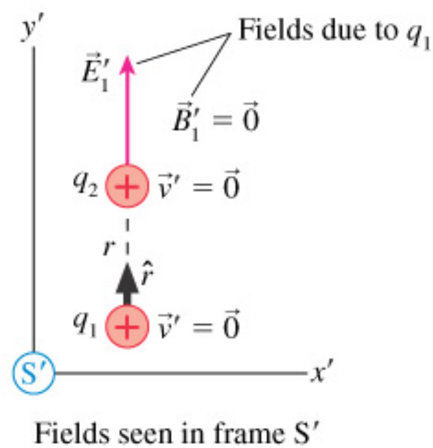


Fields seen in frame S

$$\vec{B}_1' = \vec{B} - \frac{1}{c^2}\vec{V}X\vec{E}_1 = \frac{\mu_o}{4\pi}\frac{q_1 v}{r^2}\hat{k} - \frac{1}{c^2}\left(v\hat{i}X\frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}\hat{j}\right)$$

$$= \frac{\mu_o}{4\pi}\frac{q_1 v}{r^2}\left(1 - \frac{1}{\varepsilon_o\mu_o c^2}\right)\hat{k} = 0$$

…as expected



Fields seen in frame S'

$$\vec{E}'_1 = \vec{E}_1 + \vec{V}X\vec{B}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}\hat{j} + v\hat{i}X\frac{\mu_o}{4\pi}\frac{q_1 v}{r^2}\hat{k}$$

$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}(1 - \varepsilon_o\mu_o v^2)\hat{j} = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}\left(1 - \frac{v^2}{c^2}\right)\hat{j}$$

…not as expected

Problems revisited

- Consider the electric and magnetic fields produced by two charges moving with a velocity v in S and at rest in S'

$$\vec{B}_1' = \vec{B} - \frac{1}{c^2}\vec{V}X\vec{E}_1 = \frac{\mu_o}{4\pi}\frac{q_1 v}{r^2}\hat{k} - \frac{1}{c^2}\left(v\hat{i}X\frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}\hat{j}\right)$$

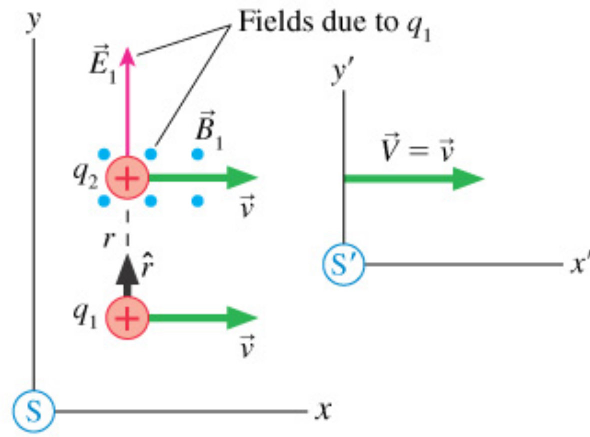
$$= \frac{\mu_o}{4\pi}\frac{q_1 v}{r^2}\left(1 - \frac{1}{\varepsilon_o\mu_o c^2}\right)\hat{k} = 0$$

…as expected

$$\vec{E}'_1 = \vec{E}_1 + \vec{V}X\vec{B}_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}\hat{j} + v\hat{i}X\frac{\mu_o}{4\pi}\frac{q_1 v}{r^2}\hat{k}$$

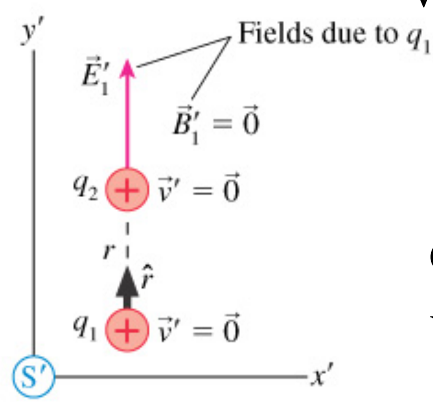
$$= \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}(1 - \varepsilon_o\mu_o v^2)\hat{j} = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r^2}\left(1 - \frac{v^2}{c^2}\right)\hat{j}$$

…not as expected



Fields seen in frame S

wait for relativity to understand v~c

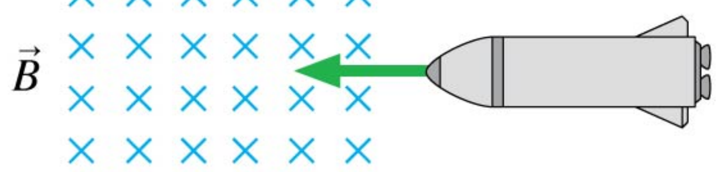


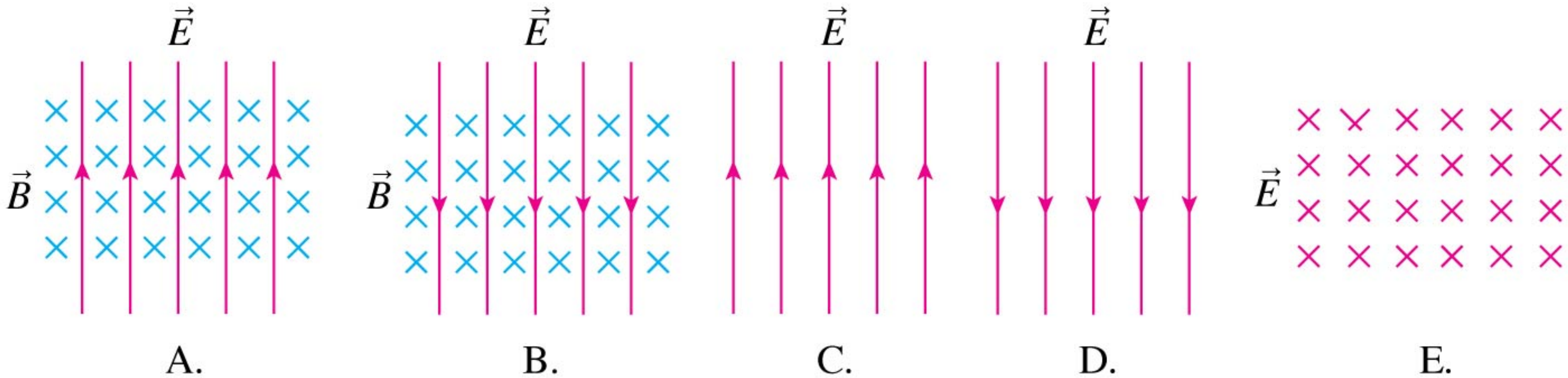
Fields seen in frame S'

close for v<<c

Galilean transformations don't (quite) work

Experimenters on earth have created the magnetic field shown. A rocket flies through the field, from right to left. Which are the field (or fields) in the rocket's reference frame?





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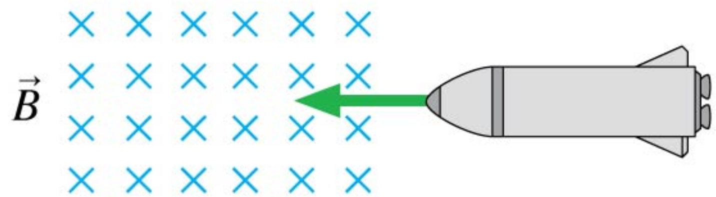
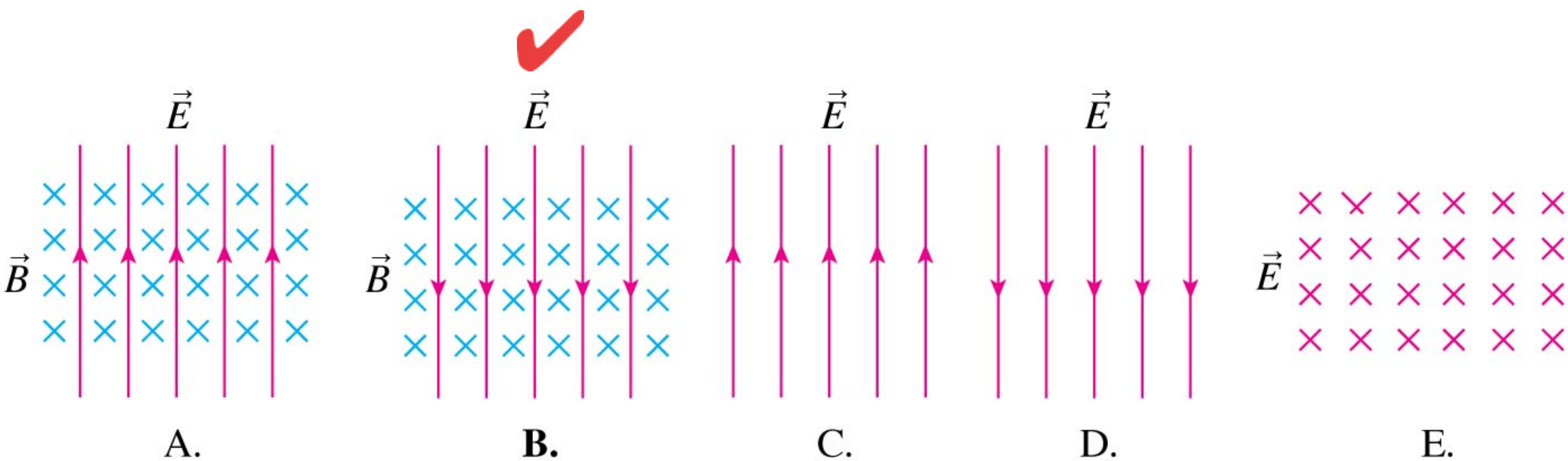


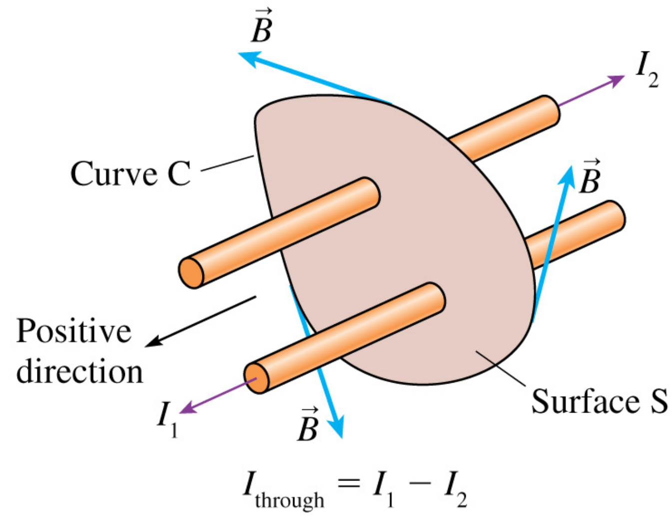


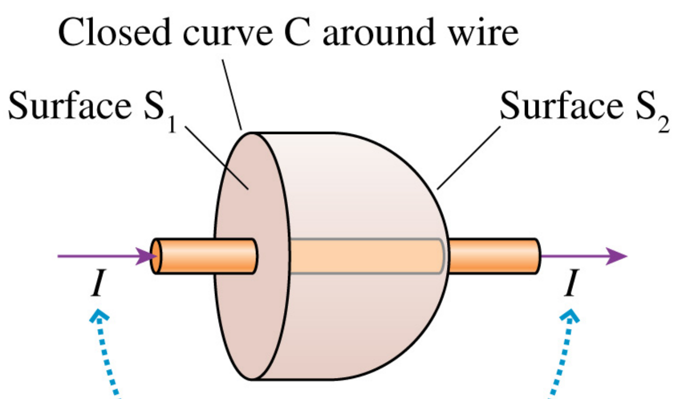
Ampere's law revisited

- Remember the formulation of Ampere's law

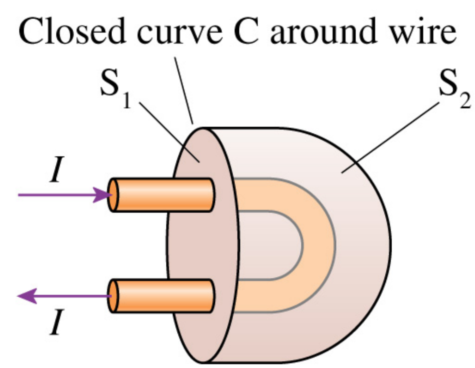
$$\oint_{curve} \vec{B} \cdot \vec{ds} = \mu_o I_{through}$$

- $I_{through}$ refers to any surface bounded by closed curve C
  - e.g. S_1 or S_2





Any currents passing through S_1 must also pass through S_2.



Even in this case, the *net* current through S_1, namely zero, matches the net current through S_2.

What about this case?

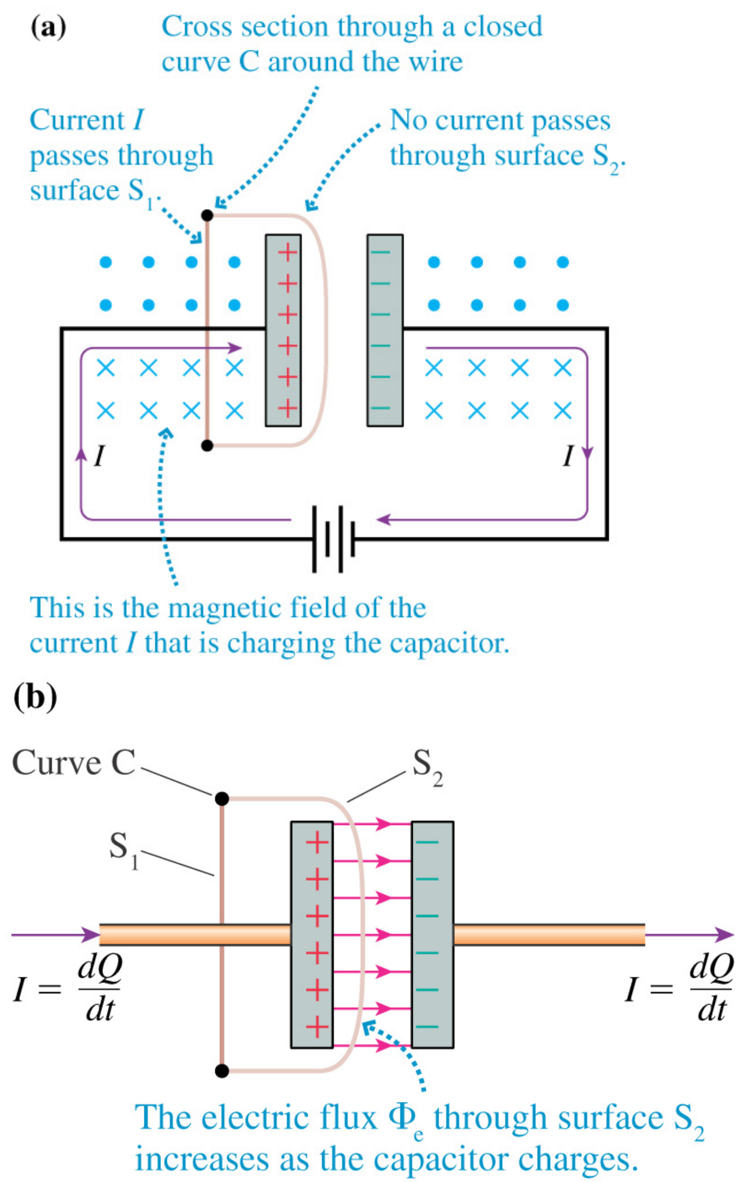
- It's clear that there's a current passing through surface S_1 but not S_2
  - both are bounded by the same curve C
- Maxwell realized that while there is no current passing through S_2, there is an (changing) electric flux

$$\Phi_E = EA$$

$$\Phi_E = \frac{Q}{\varepsilon_o A} A = \frac{Q}{\varepsilon_o}$$

$$\frac{d\Phi_E}{dt} = \frac{1}{\varepsilon_o}\frac{dQ}{dt} = \frac{I}{\varepsilon_o}$$

$$I = \varepsilon_o \frac{d\Phi_E}{dt}$$

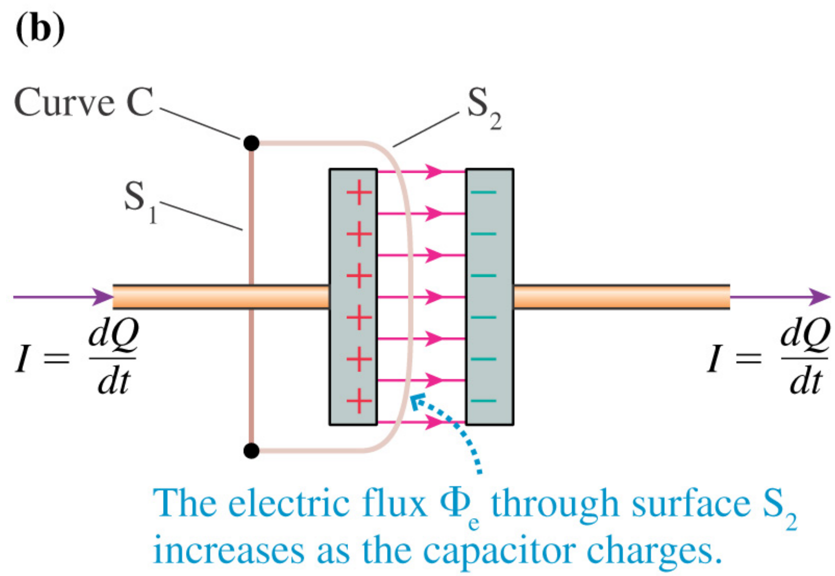


Displacement current

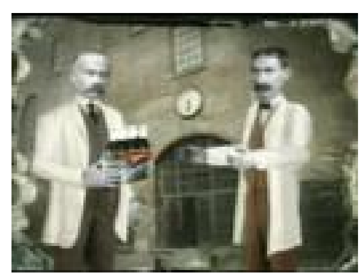
- Now I can re-write Ampere's law as

$$\oint_{curve} \vec{B}\,\vec{ds} = \mu_o \left(I_{through} + \varepsilon_o \frac{d\Phi_E}{dt} \right)$$

displacement current



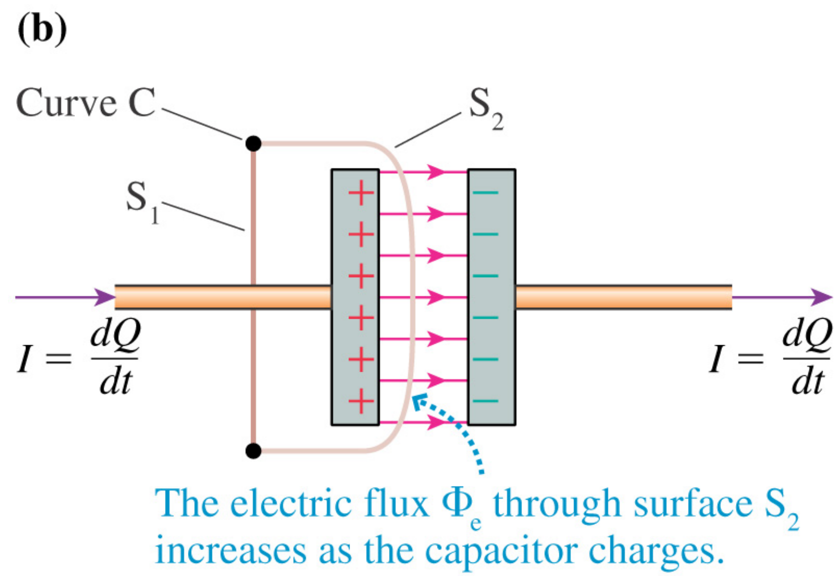
- Note that now I get the same result for each surface
- It's now called the Ampere-Maxwell law
  - and we note an added symmetry: a changing electric field (flux) creates a magnetic field



Brilliant!

Example

- At what rate must the potential difference increase across a 1.0 μF capacitor to create a 1.0 A displacement current in the capacitor?

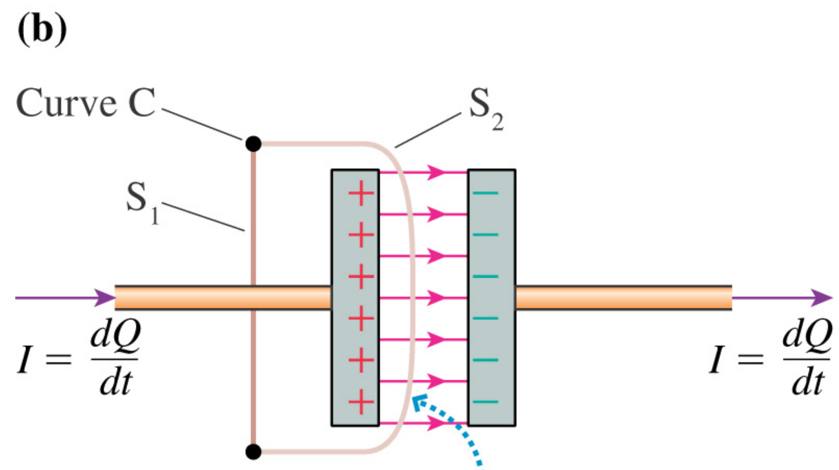


$$I = \varepsilon_o \frac{d\Phi_E}{dt} = \varepsilon_o A \frac{dE}{dt} = \frac{\varepsilon_o A}{d} \frac{dV}{dt} = C \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{I}{C} = \frac{1A}{1.0X10^{-6}F} = 1X10^{6} V/s$$

Example

- Let me re-phrase this. Suppose the potential across a 1.0 μF capacitor is increasing at the rate of 1X10^6 V/s. What must the current charging the capacitor be?
  - 1 A
- …or in other words, the displacement current is equal in value to the real current



The electric flux Φ_e through surface S_2 increases as the capacitor charges.

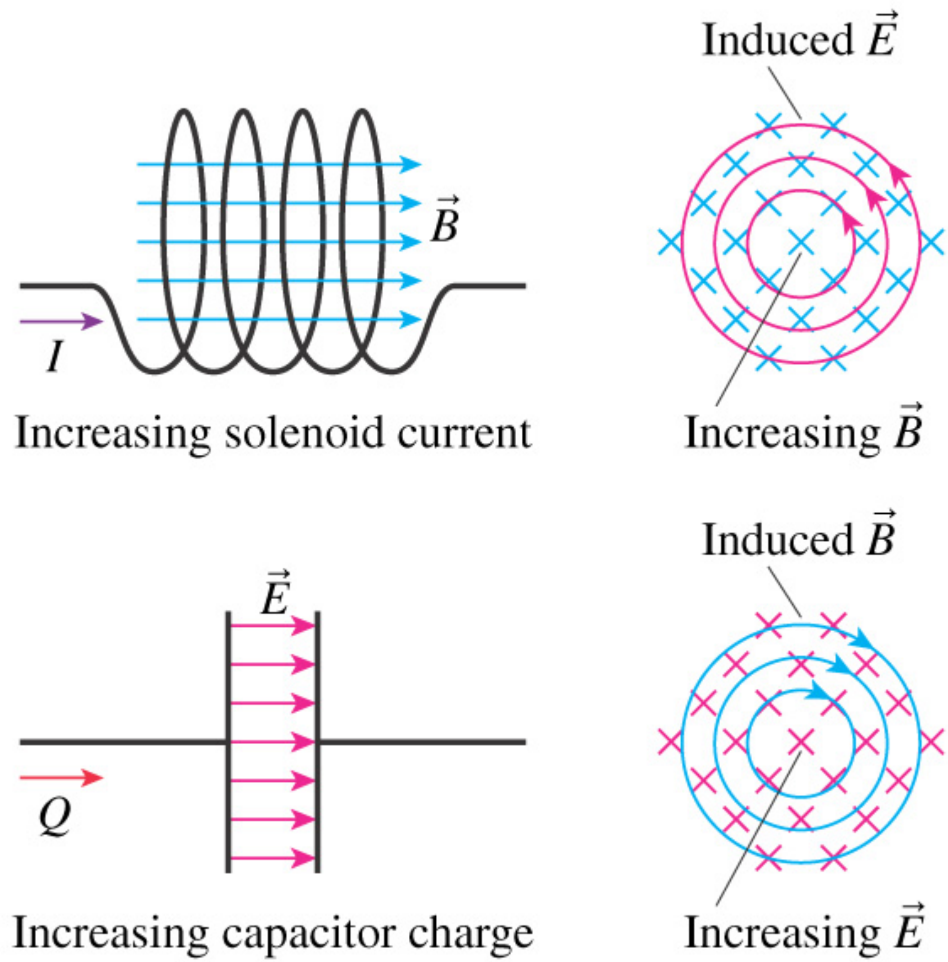
$$V = \frac{Q}{C}$$

$$\frac{dV}{dt} = \frac{1}{C}\frac{dQ}{dt} = \frac{I}{C}$$

$$I = C\frac{dV}{dt} = (1X10^{-6}F)(1X10^{6}V/s)$$

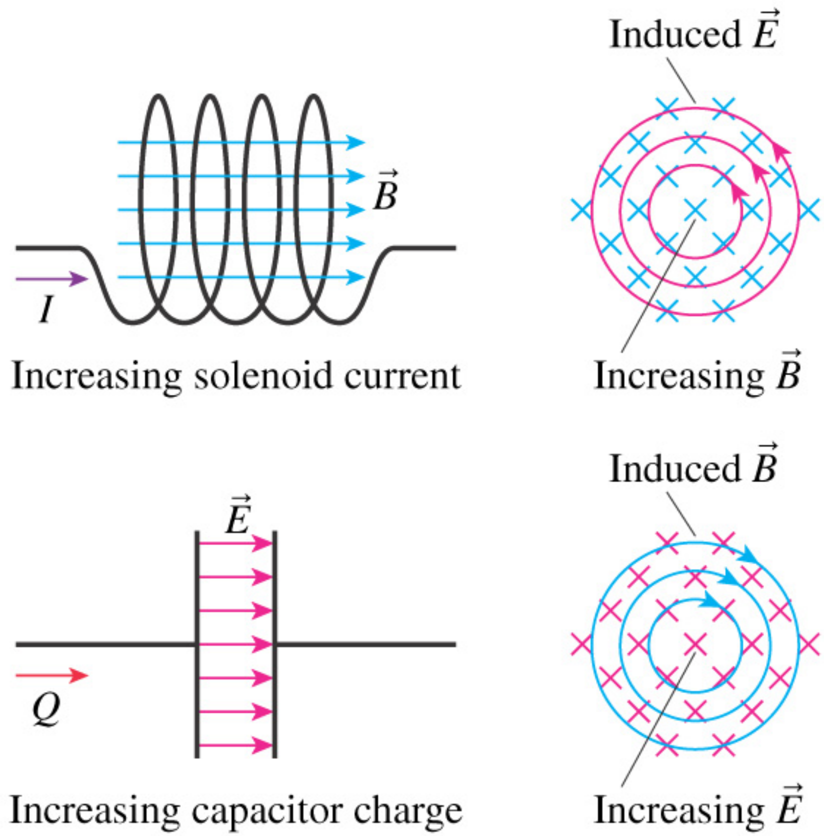
Changing fields

- An increasing magnetic field creates an electric field
- An increasing electric field creates a magnetic field
- Note that the direction of the induced magnetic field is opposite that of the induced electric field
  - Faraday's law has a - sign
  - Ampere-Maxwell law does not

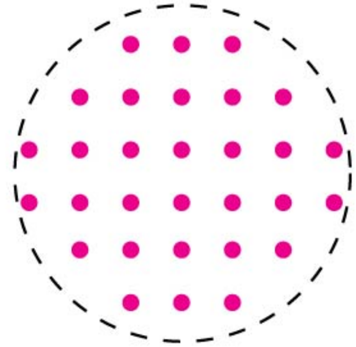


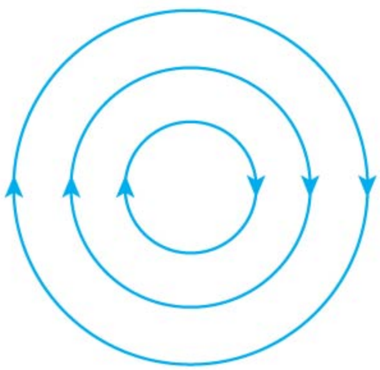
Direction of the induced field

- For an induced electric field, think of Lenz's law
  - any change is going to be opposed
- For an induced magnetic field, think of Lenz's law and then reverse the direction

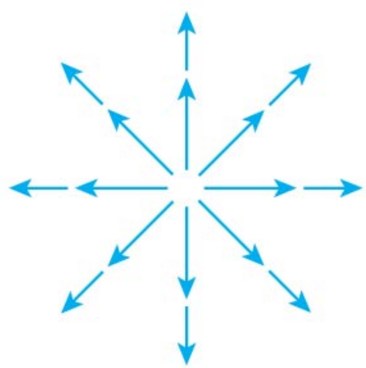


The electric field is increasing.
Which is the induced magnetic field?

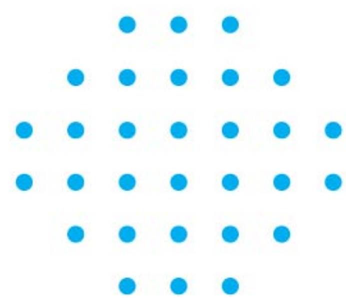




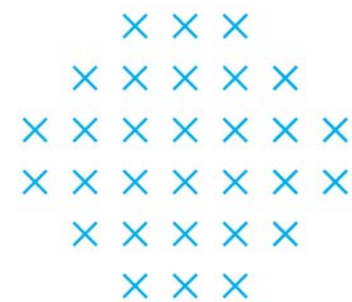
A.



B.



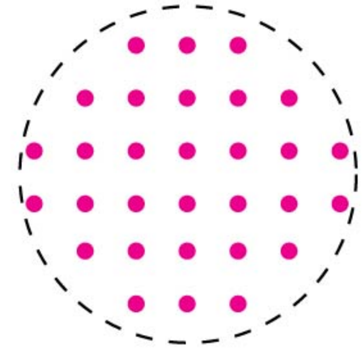
C.

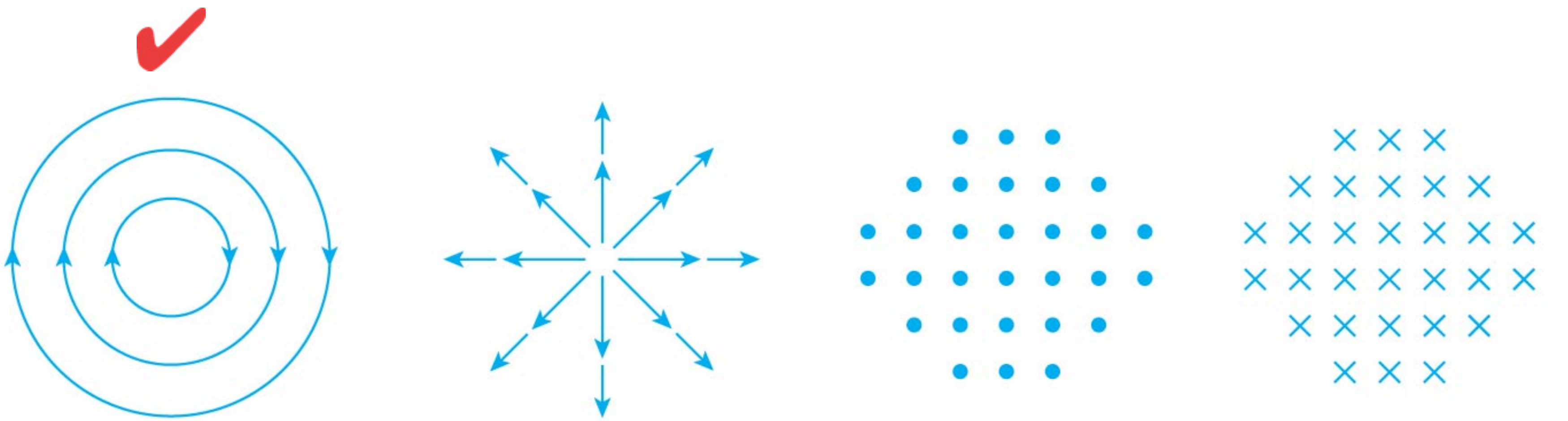


D.

E. There's no induced field in this case.

The electric field is increasing.
Which is the induced magnetic field?





A. B. C. D.

E. There’s no induced field in this case.

Charging capacitor

- A capacitor is being charged
- The electric field is changing
- What is the induced magnetic field inside the capacitor?

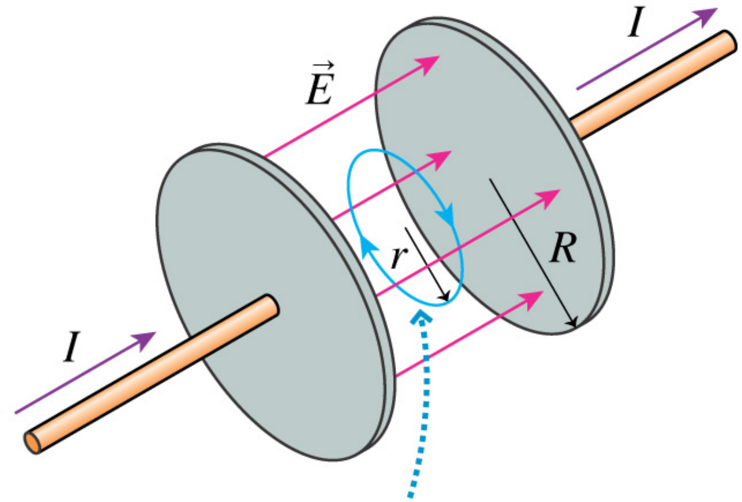
$$\oint_{curve} \vec{B} \cdot d\vec{s} = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

$$= \mu_o \varepsilon_o \frac{d}{dt}\left(\frac{r^2}{R^2} \frac{Q}{\varepsilon_o} \right) = \mu_o \frac{r^2}{R^2} \frac{dQ}{dt}$$

- The magnetic field is everywhere tangent to the circle of radius r

$$2\pi r B = \mu_o \frac{r^2}{R^2} \frac{dQ}{dt}$$

$$B = \frac{\mu_o}{2\pi} \frac{r}{R^2} \frac{dQ}{dt}$$



The magnetic field line is a circle concentric with the capacitor. The electric flux through this circle is $\pi r^2 E$.

$$\Phi_E = \pi r^2 E = \pi r^2 \frac{Q}{\varepsilon_o \pi R^2} = \frac{r^2}{R^2} \frac{Q}{\varepsilon_o}$$