PHY294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
    - **Help-room hours: <u>12:40-2:40 Monday (note change)</u>; 3:00-4:00 PM Friday**
    - **hand-in problem for Wed Mar. 23: 34.60**
- Quizzes by iclicker (sometimes hand-written)
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
    - lectures will be posted frequently, mostly every day if I can remember to do so
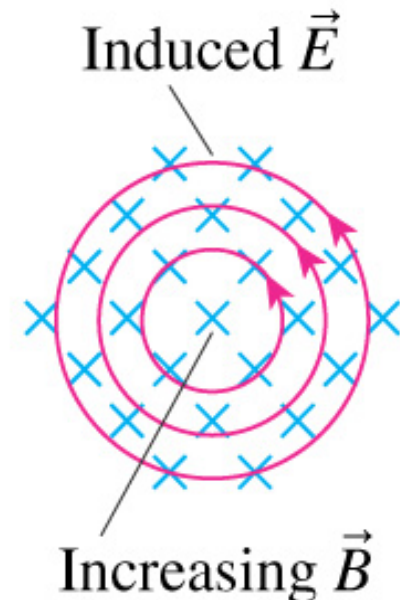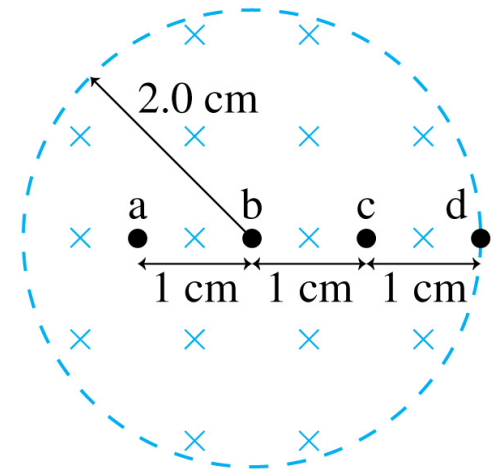
Another example

- The magnetic field is increasing at the rate of 0.10 T/s
- What is the acceleration of a proton at the points indicated?
- The proton will be accelerated because an electric field will be produced by the changing magnetic field

$$\oint_{curve} \vec{E} \cdot \vec{ds} = EL = 2\pi rE$$

$$\Phi_B = AB = \pi r^2 B$$

$$\oint_{curve} \vec{E} \cdot \vec{ds} = 2\pi rE = -\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$$

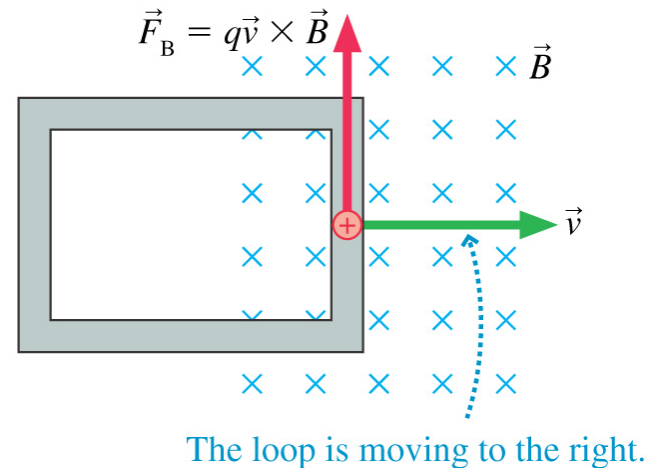$$E = \frac{r}{2}\frac{dB}{dt}$$

Induced $\vec{E}$

Increasing $\vec{B}$

Faraday's law revisited

- Consider a conducting loop moving through a region of magnetic field

- There is a force on the charges in the leading edge of the loop

- But in a frame attached to the loop, the loop is stationary and instead the magnetic field is moving to the left

- The observer on the loop sees both a magnetic field and an electric field
  - $\vec{E'} = \vec{v} \times \vec{B}$

- Both observers see the same force acting on the charges, but the observer in S attributes it to the magnetic field and the observer in S' attributes it to the electric field

- Only in S' is there a changing magnetic field, so only in S' is there an induced electric field

(a) Laboratory frame S

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

$\vec{B}$

$\vec{v}$

The loop is moving to the right.

(b) Loop frame S'

The induced electric field points up.

$$\vec{F}_E = q\vec{E'}$$

$-\vec{v}$

$\vec{E'} = \vec{v} \times \vec{B}$

The magnetic field is moving to the left.
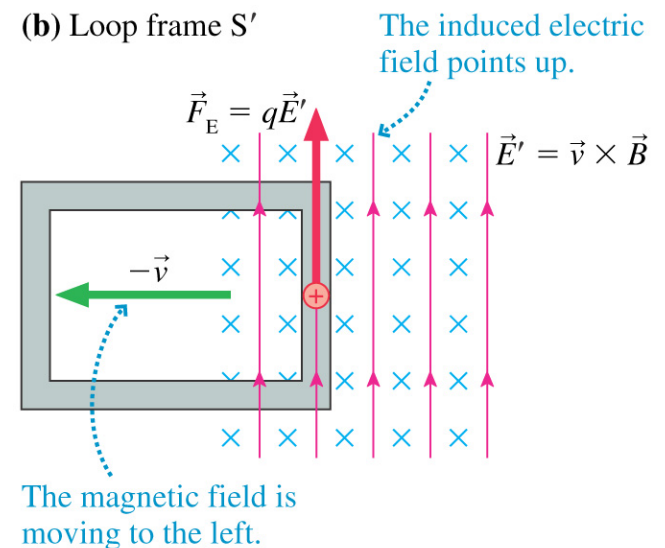
Maxwell's equations

- Maxwell was the first to assemble the 4 equations that describe electromagnetism

- He presented his paper "On Faraday's Lines of Force" when he was 24

$$\oint_{surface} \vec{E} \cdot \vec{dA} = \frac{Q_{in}}{\varepsilon_o}$$

$$\oint_{surface} \vec{B} \cdot \vec{dA} = 0$$

$$\oint_{curve} \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt}$$

$$\oint_{curve} \vec{B} \cdot \vec{ds} = \mu_o \left(I_{through} + \varepsilon_o \frac{d\Phi_E}{dt} \right)$$

Maxwell's laws (integral form)

$$\oint_{surface} \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\varepsilon_o}$$

$$\oint_{surface} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{curve} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

$$\oint_{curve} \vec{B} \cdot d\vec{s} = \mu_o \left(I_{through} + \varepsilon_o \frac{d\Phi_E}{dt} \right)$$

- Gauss' law for electric fields: charged particles create an electric field
- Gauss' law for magnetic fields: there are no magnetic monopoles
- Faraday's law: an electric field can also be created by a changing magnetic field
- Ampere-Maxwell law: a magnetic field can be created either by an electric current or by a changing electric field

Maxwell's laws (differential form)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \times \vec{E} = \boxed{-\frac{\partial \vec{B}}{\partial t}}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \boxed{\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

- Gauss' law for electric fields: charged particles create an electric field
- Gauss' law for magnetic fields: there are no magnetic monopoles
- Faraday's law: an electric field can also be created by a changing magnetic field
- Ampere-Maxwell law: a magnetic field can be created either by an electric current or by a changing electric field

Of course, Maxwell never wrote them down in this compact form. His original paper had 20 equations, not 4. Didn't use curls or divergences.

That was left to Oliver Heaviside

- **Oliver Heaviside** 18 May 1850 – 3 February 1925) was a self-taught English electrical engineer, mathematician, and physicist who adapted complex numbers to the study of electrical circuits, invented mathematical techniques for the solution of differential equations (equivalent to Laplace transforms), reformulated Maxwell's field equations in terms of electric and magnetic forces and energy flux, and independently co-formulated vector analysis. Although at odds with the scientific establishment for most of his life, Heaviside changed the face of telecommunications, mathematics, and science for years to come.
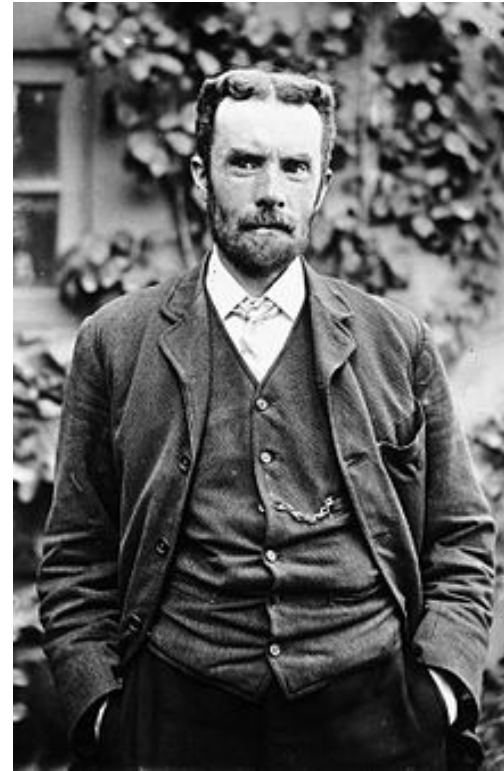
Fun facts: In later life, he started painting his fingernails pink and had granite blocks moved into his house for use as furniture.

Heaviside layer

Up, up, up past the Russell Hotel
Up, up, up, up to the Heaviside layer
Up, up, up past the Russell Hotel
Up, up, up, up to the Heaviside layer

Up, up, up past the Russell Hotel
Up, up, up, up to the Heaviside layer
Up, up, up past the Russell Hotel
Up, up, up, up to the Heaviside layer

Up, up, up past the Jellicle moon
Up, up, up, up to the Heaviside layer
Up, up, up past the Jellicle moon
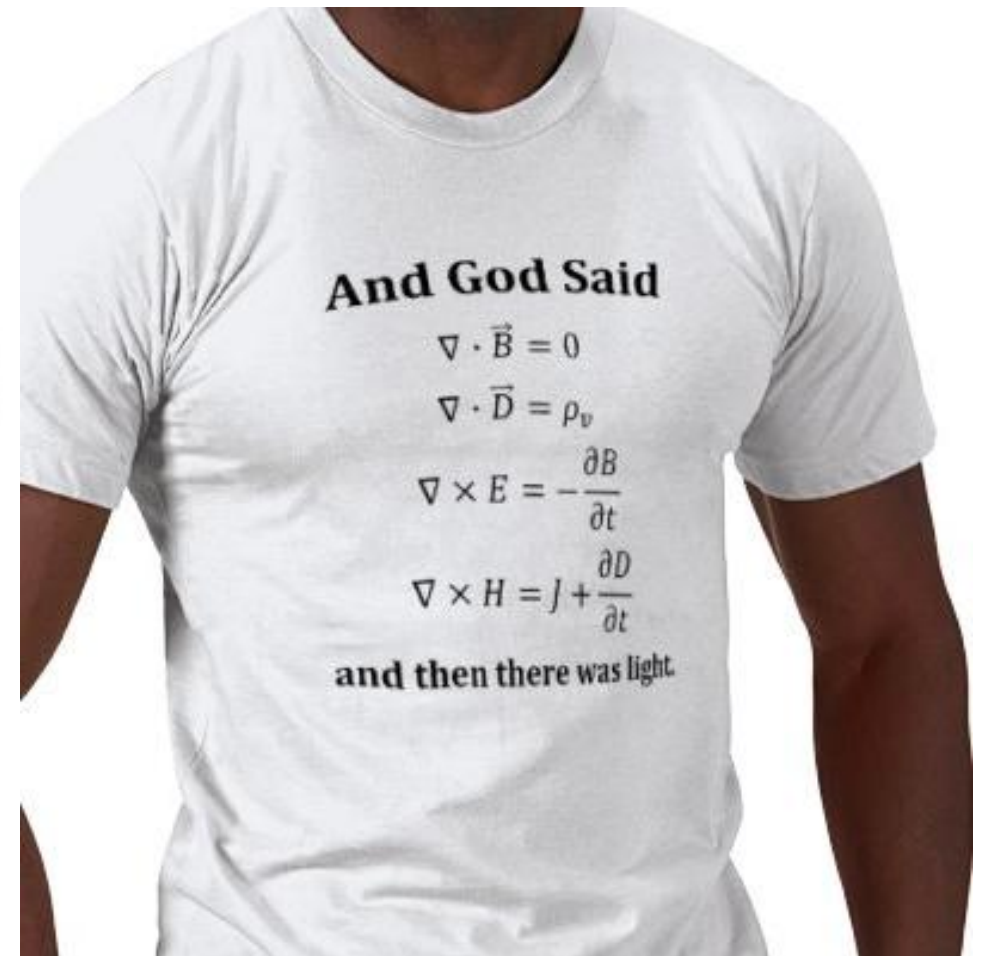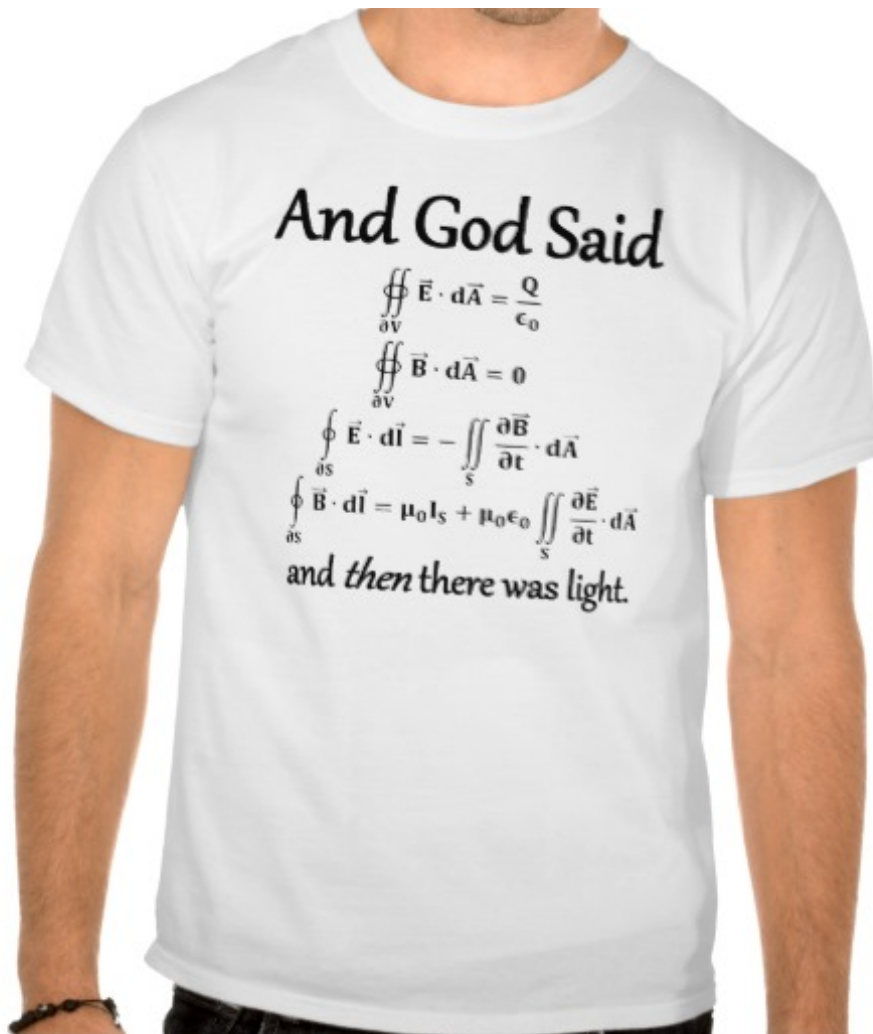Up, up, up, up to the Heaviside layer

The mystical divinity
Of unashamed felinity
Round the cathedral rang 'Vivat'
Life to the everlasting cat

SONGWRITERS
T.S. ELIOT;ANDREW LLOYD WEBBER

- The **Kennelly–Heaviside layer**, is a layer of ionised gas occurring between roughly 90-150 km above the ground, one of several layers in the Earth's ionosphere. It reflects radio waves, and because of this reflection radio waves can be propagated beyond the horizon.

T-shirt form

And God Said

$$\oiint_{\partial V} \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$$

$$\oiint_{\partial V} \vec{B} \cdot d\vec{A} = 0$$

$$\oint_{\partial S} \vec{E} \cdot d\vec{l} = -\iint_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

$$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 I_S + \mu_0 \epsilon_0 \iint_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{A}$$

and *then* there was light.

And God Said

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t}$$

and then there was light.

What do you get when you cross a 🐱 with a 💡?

Some perspective

- There are a total of 11 fundamental equations describing classical physics:

- Newton's first law
- Newton's second law
- Newton's third law
- Newton's law of gravity
- Gauss's law

- Gauss's law for magnetism
- Faraday's law
- Ampère-Maxwell law
- Lorentz force law
- First law of thermodynamics
- Second law of thermodynamics

- **Gauss's law:** Charged particles create an electric field.
- **Faraday's law:** An electric field can also be created by a changing magnetic field.
- **Gauss's law for magnetism:** There are no isolated magnetic poles.
- **Ampère-Maxwell law, first half:** Currents create a magnetic field.
- **Ampère-Maxwell law, second half:** A magnetic field can also be created by a changing electric field.
- **Lorentz force law, first half:** An electric force is exerted on a charged particle in an electric field.
- **Lorentz force law, second half:** A magnetic force is exerted on a charge moving in a magnetic field.

Electromagnetic waves

- We'll be working with Maxwell's equations in free space, i.e. no charges or currents
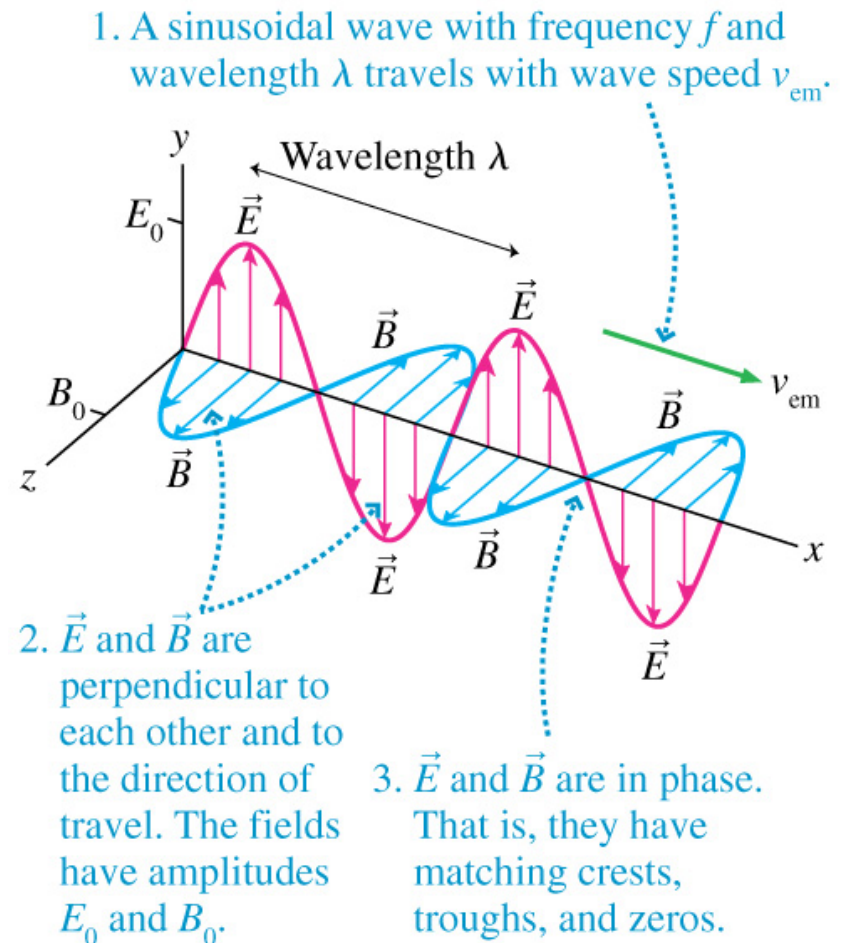
$$\oint_{surface} \vec{E}.\vec{dA} = 0$$

$$\oint_{surface} \vec{B}.\vec{dA} = 0$$

$$\oint_{curve} \vec{E}\cdot\vec{ds} = -\frac{d\Phi_B}{dt}$$

$$\oint_{curve} \vec{B}\cdot\vec{ds} = \mu_o\varepsilon_o\frac{d\Phi_E}{dt}$$

- Assume that the electromagnetic wave has the form shown on the right
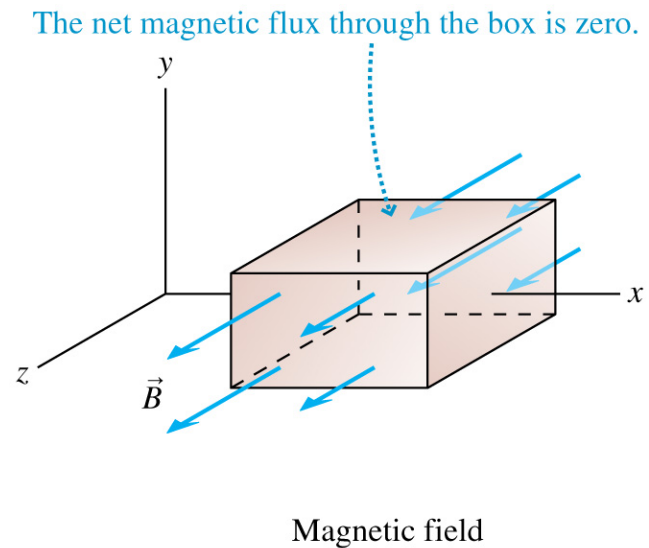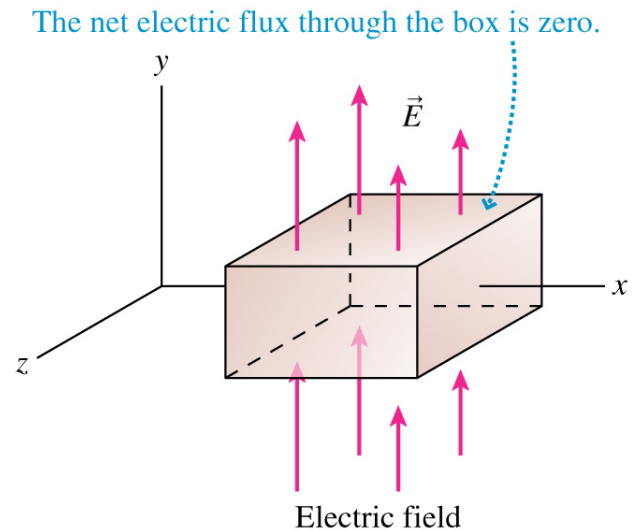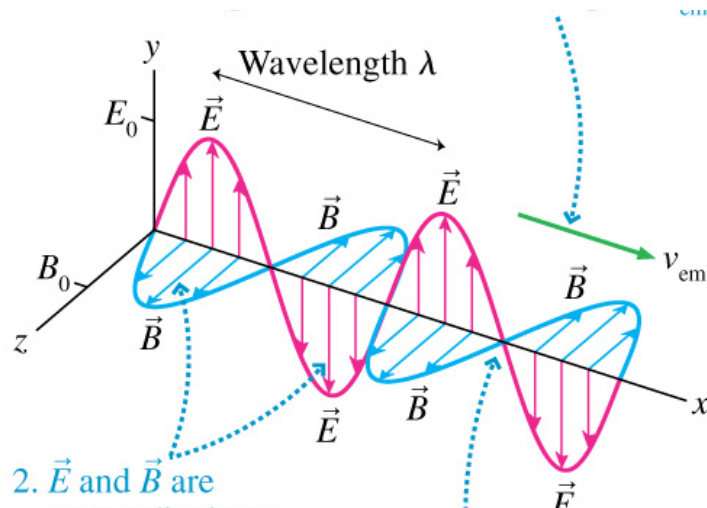 - a plane wave in the y-z plane propagating in the x direction

1. A sinusoidal wave with frequency f and wavelength λ travels with wave speed v_{em}.

2. $\vec{E}$ and $\vec{B}$ are perpendicular to each other and to the direction of travel. The fields have amplitudes E_0 and B_0.

3. $\vec{E}$ and $\vec{B}$ are in phase. That is, they have matching crests, troughs, and zeros.

Gauss' laws

● This EM wave satisfies Gauss' laws for both the electric and magnetic fields

$$E_x = 0; E_y = E_o \sin(2\pi(\frac{x}{\lambda} - ft)); E_z = 0$$

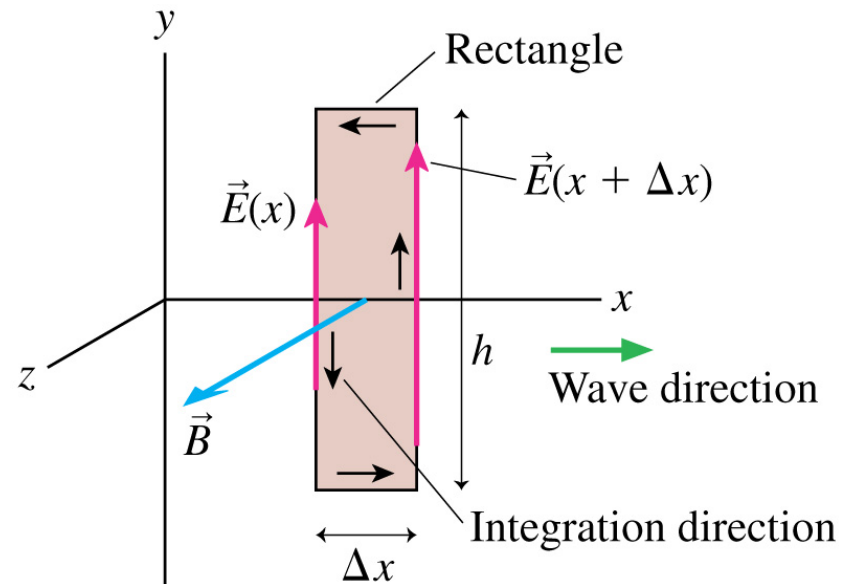$$B_x = 0; B_y = 0; B_z = B_o \sin(2\pi(\frac{x}{\lambda} - ft))$$

The net electric flux through the box is zero.

Electric field

The net magnetic flux through the box is zero.

Magnetic field

Wavelength λ

E_0

B_0

v_{em}

2. $\vec{E}$ and $\vec{B}$ are

Faraday's law

$$\oint_{curve} E\,ds = -\frac{d\Phi_B}{dt}$$

- **Apply Faraday's law to a rectangle in the xy plane**

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}\left(B_z h\Delta x\right) = \frac{\partial B_z}{\partial t}h\Delta x$$

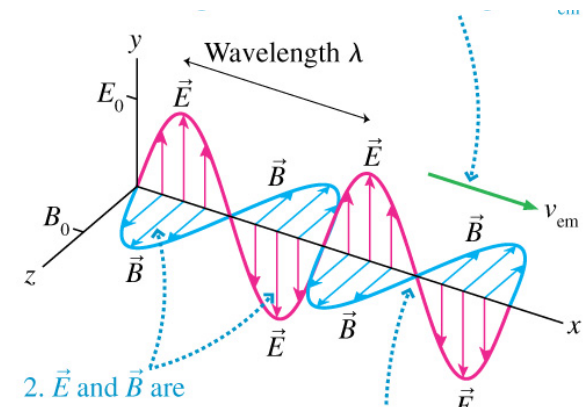- **Integrate $\vec{E}\cdot\vec{ds}$ around the loop in the CCW direction**

$$\oint_{rectangle} \vec{E}\cdot\vec{ds} = -E_y(x)h + E_y(x+\Delta x)h = [E_y(x+\Delta x) - E_y(x)]h$$

- **I can write this as**

$$\oint_{rectangle} \vec{E}\cdot\vec{ds} = \frac{\partial E_y}{\partial x}h\Delta x = -\frac{d\Phi_B}{dt} = -\frac{\partial B_z}{\partial t}h\Delta x$$

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

Ampere-Maxwell law

$$\oint_{curve} B\, ds = \mu_o \varepsilon_o \frac{d\Phi_E}{dt}$$

- Now let's apply the Ampere-Maxwell law to a rectangle in the xz plane

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}\left(E_y l \Delta x\right) = \frac{\partial E_y}{\partial t} l \Delta x$$
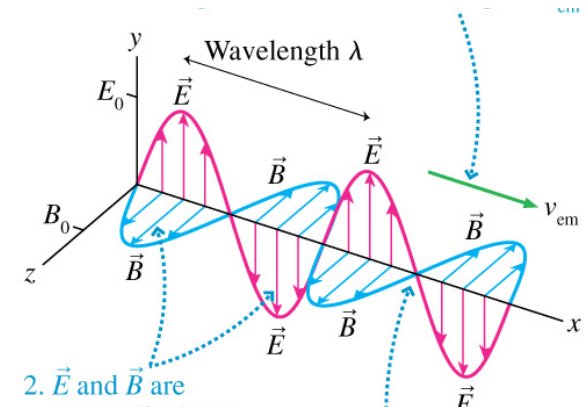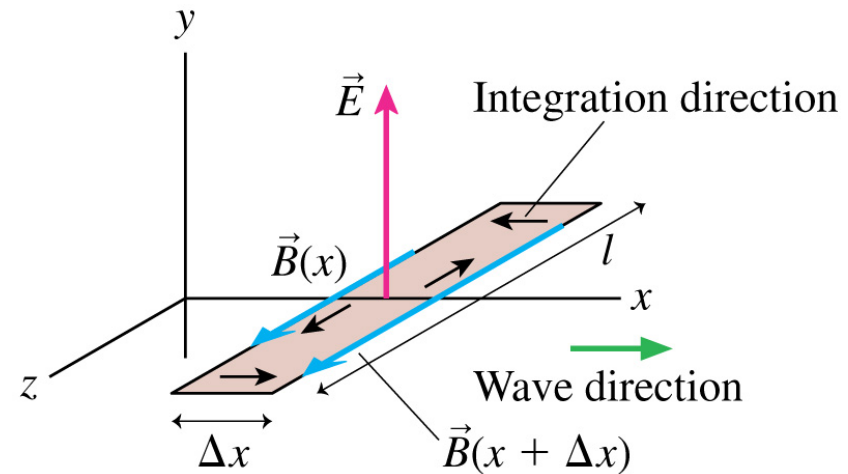
- Integrate $\vec{B}.\vec{ds}$ in the CCW direction

$$\oint_{rectangle} \vec{B}.\vec{ds} = B_z(x)l - B_z(x + \Delta x)l = -\left[B_z(x + \Delta x) - B_z(x)]l\right]$$

- I can write this as

$$\oint_{rectangle} \vec{B}.\vec{ds} = -\frac{\partial B_z}{\partial x} l \Delta x = \varepsilon_o \mu_o \frac{d\Phi_E}{dt} = \varepsilon_o \mu_o \frac{\partial E_y}{\partial t} l \Delta x$$

$$\boxed{\frac{\partial B_z}{\partial x} = -\varepsilon_o \mu_o \frac{\partial E_y}{\partial t}}$$

Wave equations

- Let's play around with these equations

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad \frac{\partial B_z}{\partial x} = -\varepsilon_o \mu_o \frac{\partial E_y}{\partial t}$$
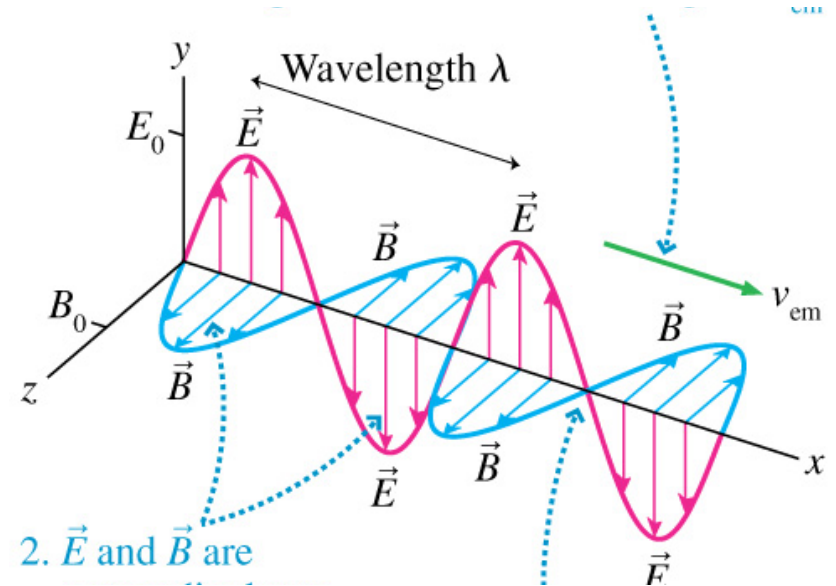
- Take extra derivative

$$\frac{\partial^2 B_z}{\partial t \partial x} = -\varepsilon_o \mu_o \frac{\partial^2 E_y}{\partial t^2} \qquad \frac{\partial^2 B_z}{\partial t \partial x} = -\frac{\partial^2 E_y}{\partial x^2}$$

$$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_o \mu_o \frac{\partial^2 E_y}{\partial t^2}$$

wave equations for E and B fields

$$\boxed{\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_y}{\partial t^2}}$$

$$\boxed{\frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 B_z}{\partial t^2}}$$

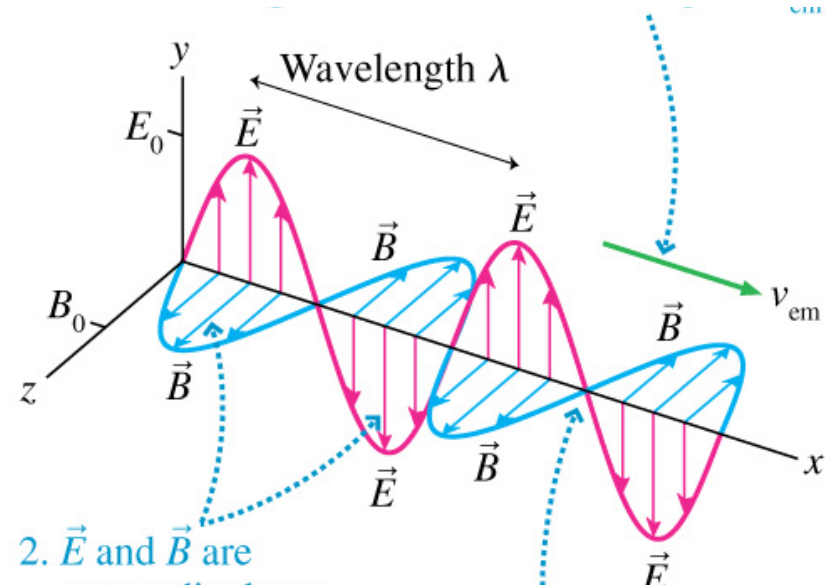Wavelength λ

2. $\vec{E}$ and $\vec{B}$ are

Solutions

● Wave equations

$$\frac{\partial^2 E_y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} \quad \frac{\partial^2 B_z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 B_z}{\partial t^2}$$

● solutions

$$E_x = 0; E_y = E_o \sin(2\pi(\frac{x}{\lambda} - ft)); E_z = 0$$

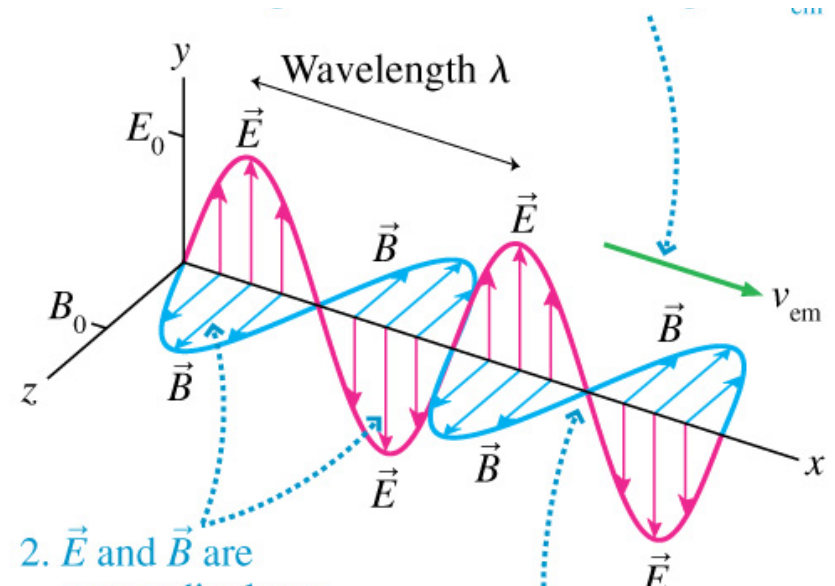$$B_x = 0; B_y = 0; B_z = B_o \sin(2\pi(\frac{x}{\lambda} - ft))$$

E and B fields

$$E_x = 0; E_y = E_o \sin(2\pi(\frac{x}{\lambda} - ft)); E_z = 0$$

$$B_x = 0; B_y = 0; B_z = B_o \sin(2\pi(\frac{x}{\lambda} - ft))$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

y — Wavelength λ

E_0 $\vec{E}$

B_0 $\vec{B}$ $\vec{E}$ v_{em}

z $\vec{B}$

2. $\vec{E}$ and $\vec{B}$ are

$$\frac{2\pi E_o}{\lambda}\cos(2\pi(\frac{x}{\lambda} - ft)) = -2\pi f B_o \cos(2\pi(\frac{x}{\lambda} - ft))$$

$$E_o = \lambda f B_o = cB_o$$

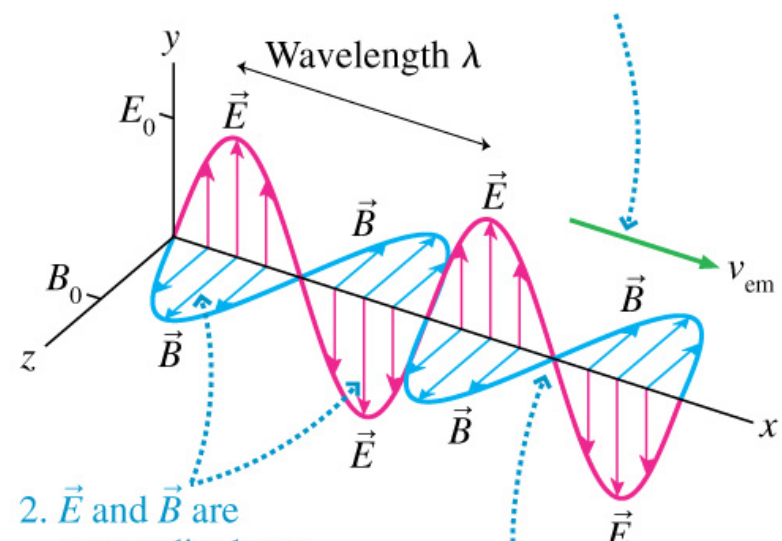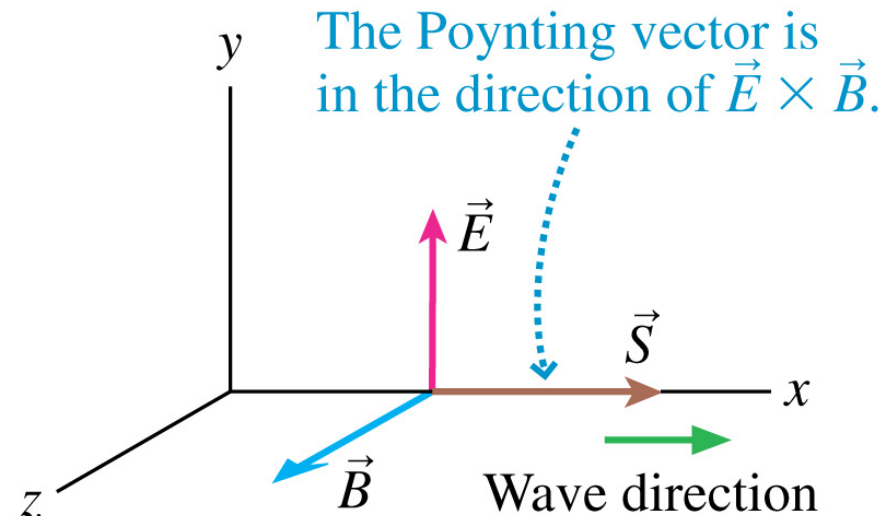The electric and magnetic fields in an EM wave must satisfy this relationship at all times.

EM waves

- Properties of an EM wave
  - the electric and magnetic fields are perpendicular to the direction of propagation
  - the electric and magnetic fields are perpendicular to each other such that EXB is in the direction of propagation
  - the electric and magnetic fields are in phase
  - the EM wave travels at c
  - E=cB at any point on the wave
- Define the Poynting vector S
  - the Poynting vector points in the direction that the wave is travelling
  - the magnitude of S measures the rate of energy transfer per unit area of the wave

The Poynting vector is in the direction of $\vec{E} \times \vec{B}$.

y

$\vec{E}$

$\vec{S}$

x

$\vec{B}$

Wave direction

z

y

Wavelength λ

E_0

$\vec{E}$

$\vec{B}$

$\vec{E}$

B_0

$\vec{B}$

v_{em}

z

$\vec{B}$

$\vec{B}$

$\vec{B}$

$\vec{E}$

$\vec{E}$

x

$\vec{F}$

2. $\vec{E}$ and $\vec{B}$ are

EM waves

- Poynting vector

$$\vec{S} \equiv \frac{1}{\mu_o} \vec{E} X \vec{B}$$

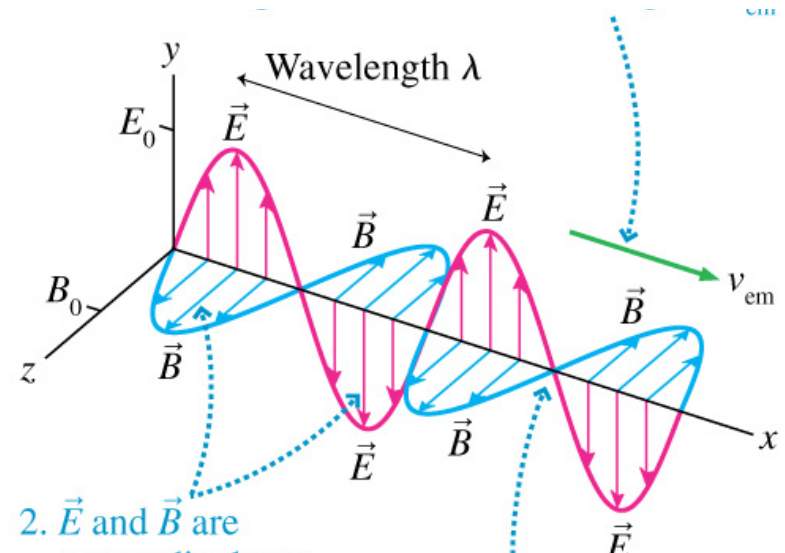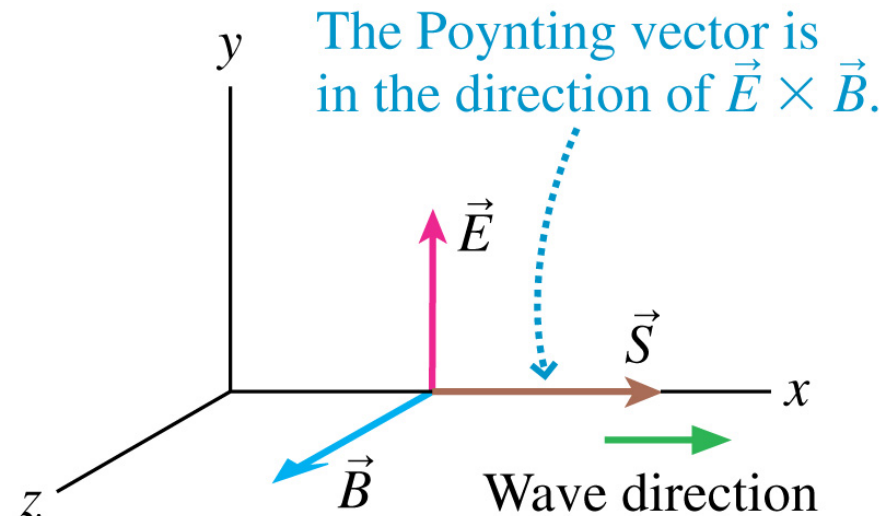$$|S| = \frac{EB}{\mu_o} = \frac{E^2}{c\mu_o} = \frac{cB^2}{\mu_o}$$

- Define the wave's intensity

$$I = \frac{P}{A} = S_{avg} = \frac{1}{2c\mu_o} E_o^2 = \frac{c\varepsilon_o}{2} E_o^2$$

- Energy in electric and magnetic fields

$$u_B = \frac{B^2}{2\mu_o}$$

$$u_E = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}\varepsilon_o c^2 B^2 = \frac{\varepsilon_o B^2}{2\mu_o \varepsilon_o} = \frac{B^2}{2\mu_o}$$

The Poynting vector is in the direction of $\vec{E} \times \vec{B}$.

y

$\vec{E}$

$\vec{S}$

x

$\vec{B}$

Wave direction

z

Wavelength λ

y

E_0

$\vec{E}$

$\vec{B}$

$\vec{E}$

B_0

v_{em}

$\vec{B}$

z

$\vec{B}$

$\vec{E}$

$\vec{B}$

x

$\vec{B}$

$\vec{E}$

$\vec{F}$

2. $\vec{E}$ and $\vec{B}$ are

(same energy density in E and B fields)