Physics 294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Help-room hours: <u>12:40-2:40 Monday (note change)</u>; 3:00-4:00 PM Friday**
  - **hand-in problem for Wed Mar. 23: 34.60**
  - **Note I revised Homework assignment 9 (due 3/23) adding some problems that were due a week later**
- Quizzes by iclicker (sometimes hand-written)
- 2^{nd} exam next Thursday
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

Circuits with an inductor

- I can write the voltage across the inductor as

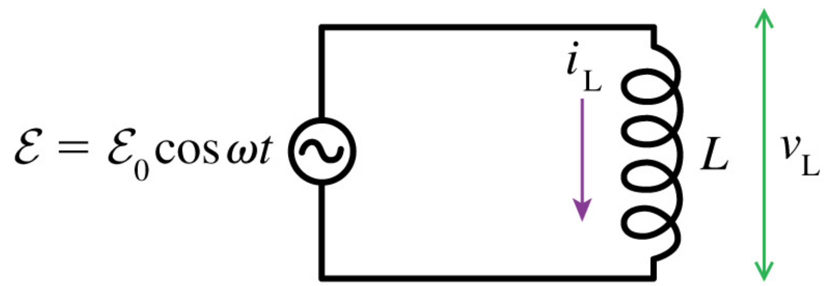
$$v_L = \varepsilon_o \cos\omega t = V_L \cos\omega t$$

- Next calculate the current in the circuit

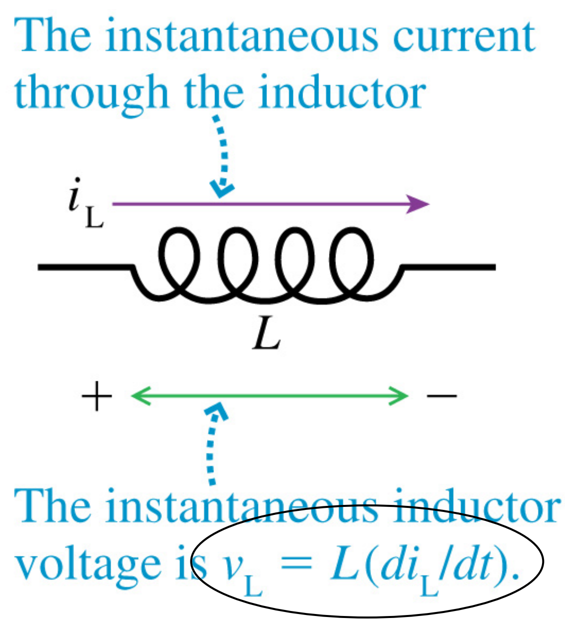
$$di_L = \frac{v_L dt}{L} = \frac{V_L}{L}\cos\omega t dt$$

$$i_L = \frac{V_L}{L}\int \cos\omega t\ dt = \frac{V_L}{\omega L}\sin\omega t = \frac{V_L}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$

(b)



(a)



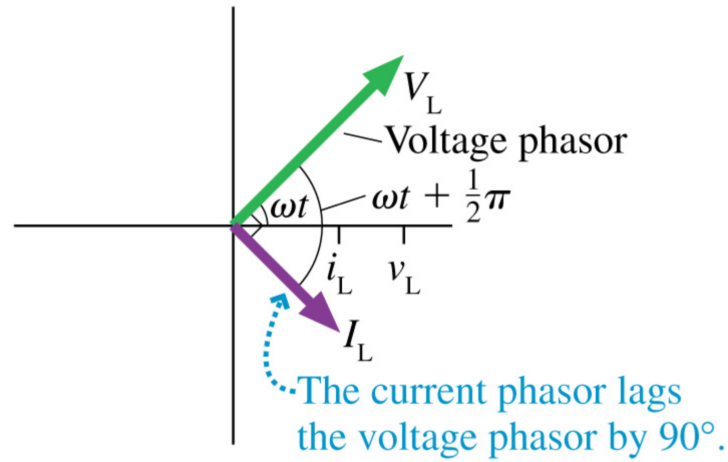
Circuits with an inductor

- The voltage leads the current by $\pi/2$ or 90°
  - does this make sense?

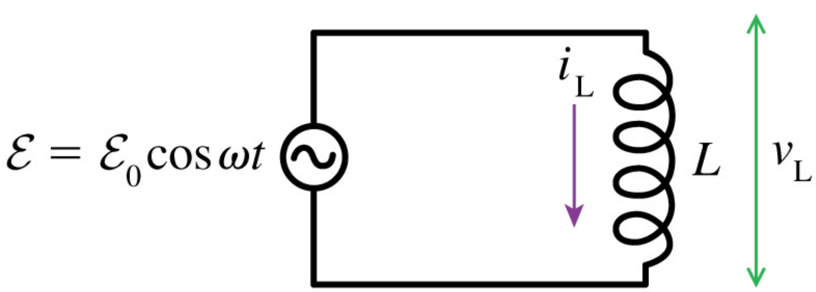
$$v_L = \varepsilon_o \cos \omega t = V_L \cos \omega t$$

$$i_L = \frac{V_L}{\omega L} \cos\left(\omega t - \frac{\pi}{2}\right)$$

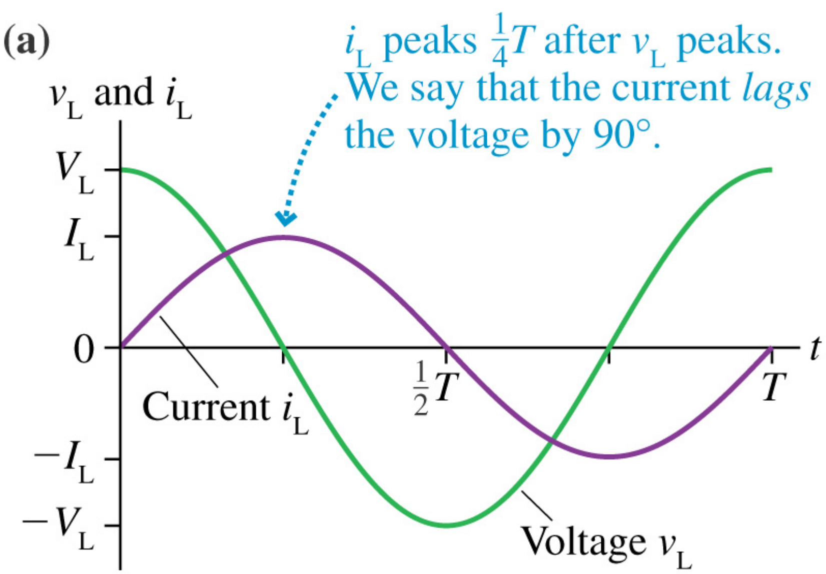
(b)



(b)



(a)



Inductive reactance

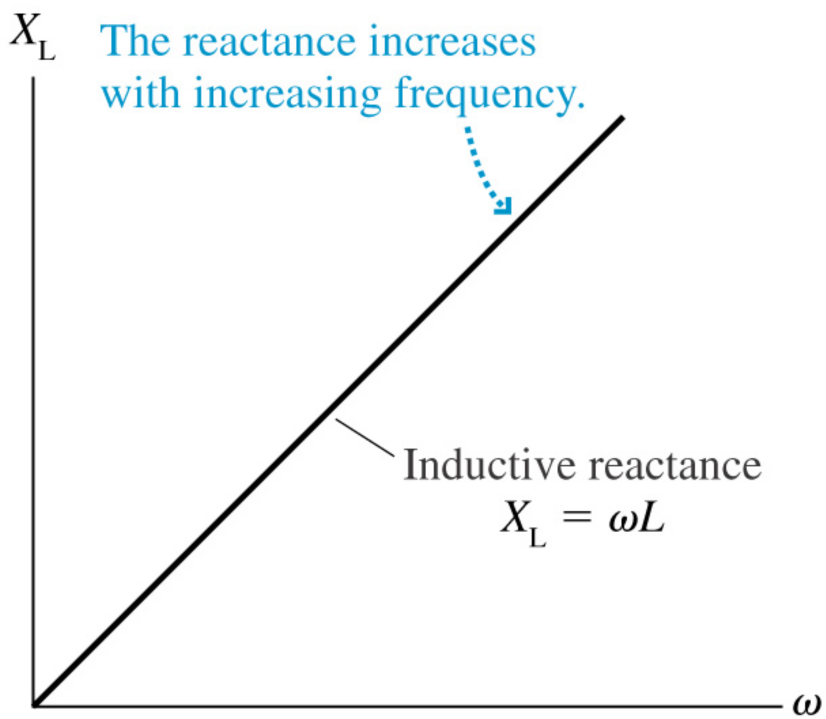
- We can define the inductive reactance as

$$X = \omega L$$

$$I_L = \frac{V_L}{X_L}$$

$$V_L = I_L X_L$$

- The unit of the inductive reactance is Ω



Inductive circuits with a resistor

- Using similar reasoning as for circuits with and R and C, we can write the peak current as

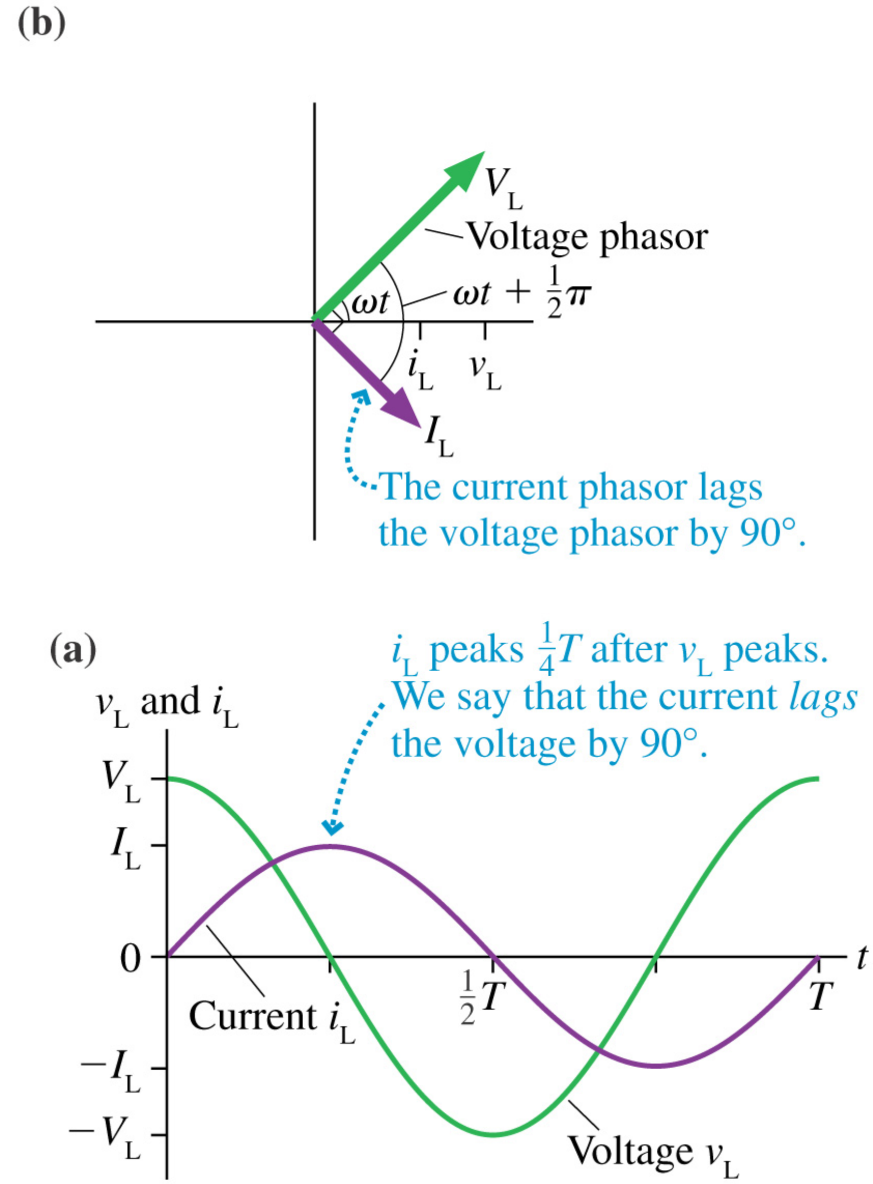
$$I = \frac{\varepsilon_o}{\sqrt{R^2 + X_L^{\,2}}} = \frac{\varepsilon_o}{\sqrt{R^2 + \omega^2 L^2}}$$

- And the peak voltages as

$$V_R = IR = \frac{\varepsilon_o R}{\sqrt{R^2 + X_L^{\,2}}} = \frac{\varepsilon_o R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$V_L = IX_L = \frac{\varepsilon_o X_L}{\sqrt{R^2 + X_L^{\,2}}} = \frac{\varepsilon_o X_L}{\sqrt{R^2 + \omega^2 L^2}}$$

For high frequencies, $V_L \gg V_R$.



iclicker

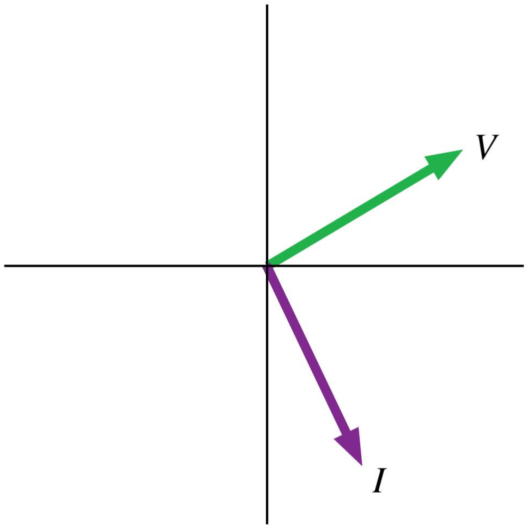
In the circuit represented by these phasors, the current _____ the voltage

A. leads

B. lags

C. is perpendicular to

D. is out of phase with



iclicker

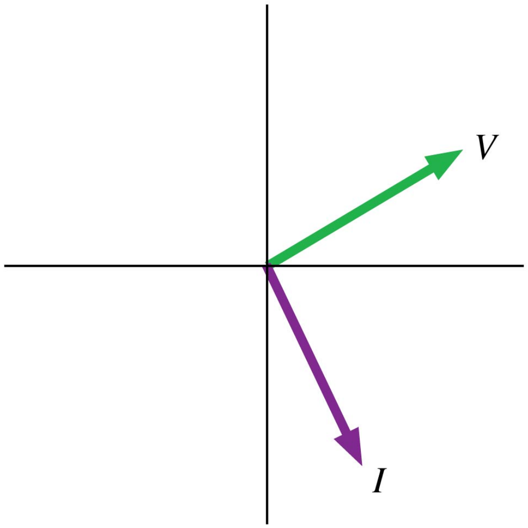
In the circuit represented by these phasors, the current ____ the voltage

A. leads

B. lags

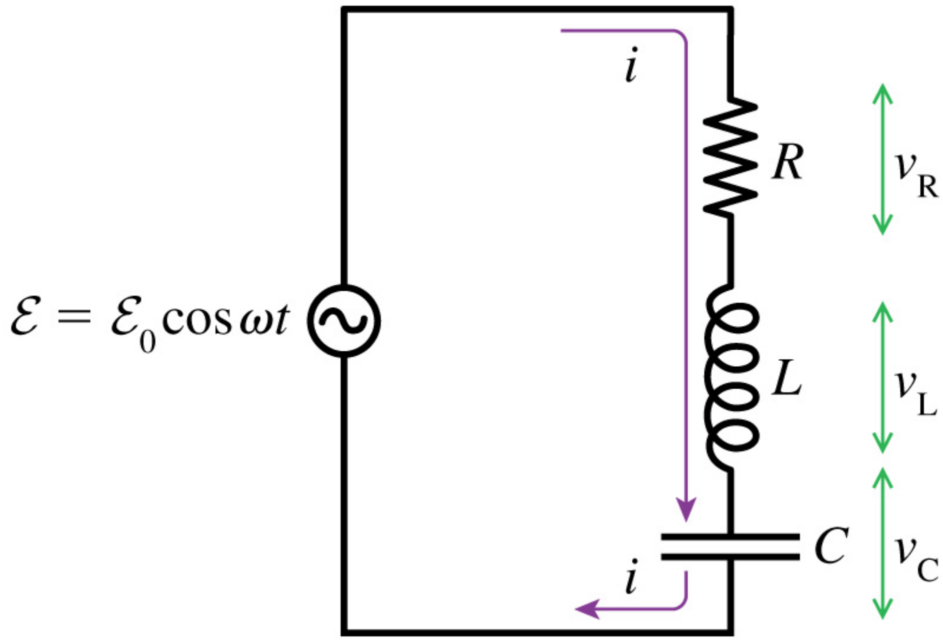
C. is perpendicular to

D. is out of phase with



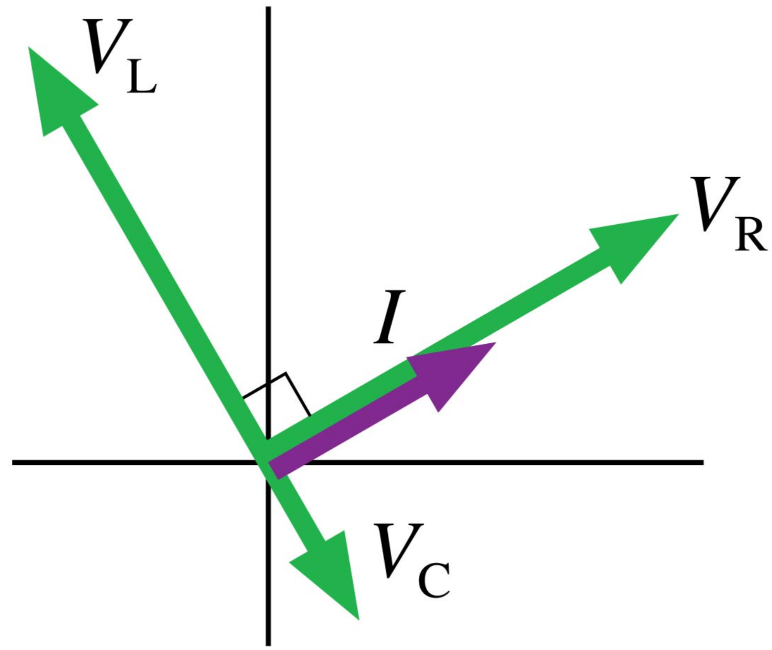
RLC circuits

- Let's go for broke and put all three elements in the same circuit
- So what do we know
  - the instantaneous current through all 3 elements is the same
  - $i = i_R = i_L = i_C$
  - The sum of the instantaneous voltages matches the emf
  - $\varepsilon = v_R + v_L + v_C$



RLC circuits

- Now we have all 3 circuit elements
- Draw phasors for the current, V_R, V_L and V_C
- Note that again the current I and V_R are in phase and V_L and V_C are each 90° out of phase with the current
  - so V_L and V_C are 180° out of phase with each other



RLC circuits

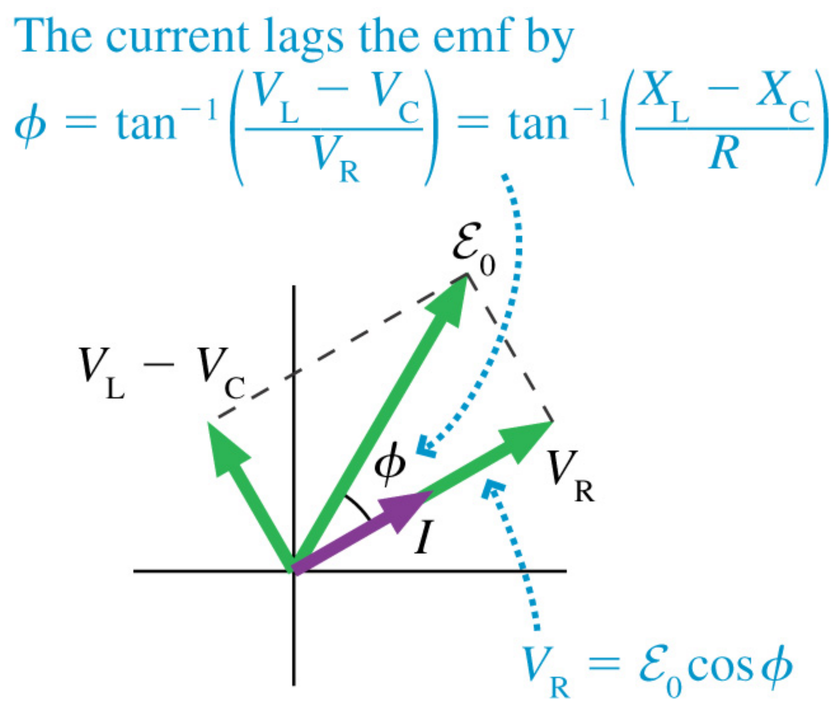
- Write the current as
    - $i = I\cos(\omega t - \phi)$
    - ϕ can be between $+90^o$ and -90^o
    - here we' ve drawn $V_L > V_C$ so ϕ is +; voltage leads I
- I can calculate the current

$$\varepsilon_o = V_R^2 + \left(V_L - V_C\right)^2 = \left[R^2 + \left(X_L - X_C\right)^2\right] I^2$$

$$I = \frac{\varepsilon_o}{\sqrt{R^2 + \left(X_L - X_C\right)^2}} = \frac{\varepsilon_o}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

- Define the impedance Z

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$



$$I = \frac{\varepsilon_o}{Z}$$

Resonance

- Let's look again at the formula for the impedance

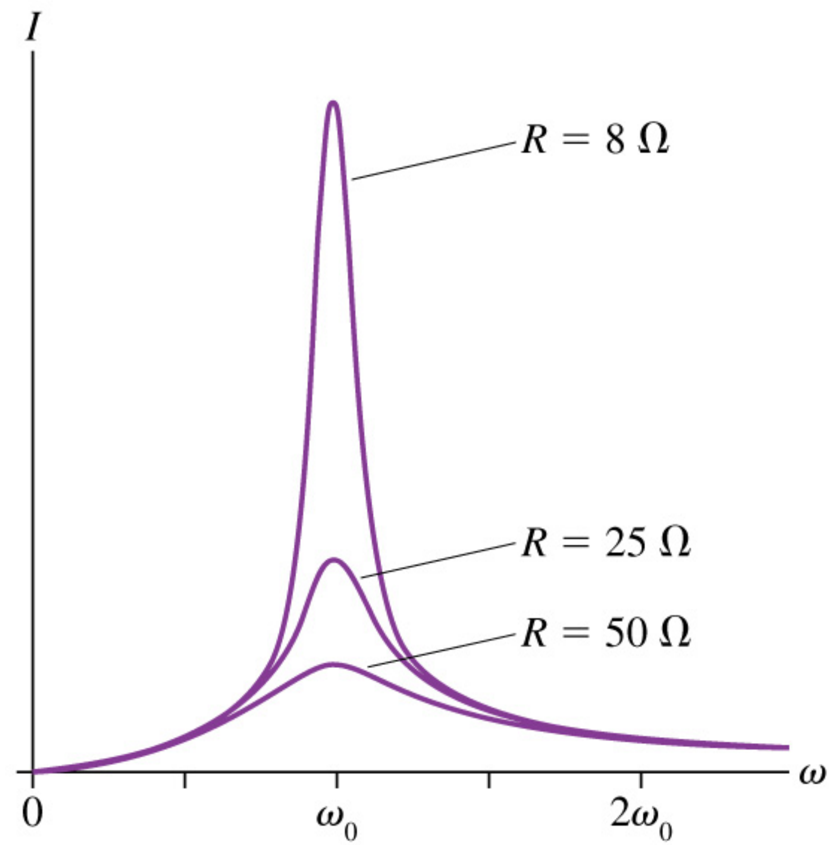
$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

- Note that it has a minimum when $X_L = X_C$

$$Z = R$$

- The current has its maximum value at that frequency

$$I = \frac{\varepsilon_o}{Z} = \frac{\varepsilon_o}{R}$$

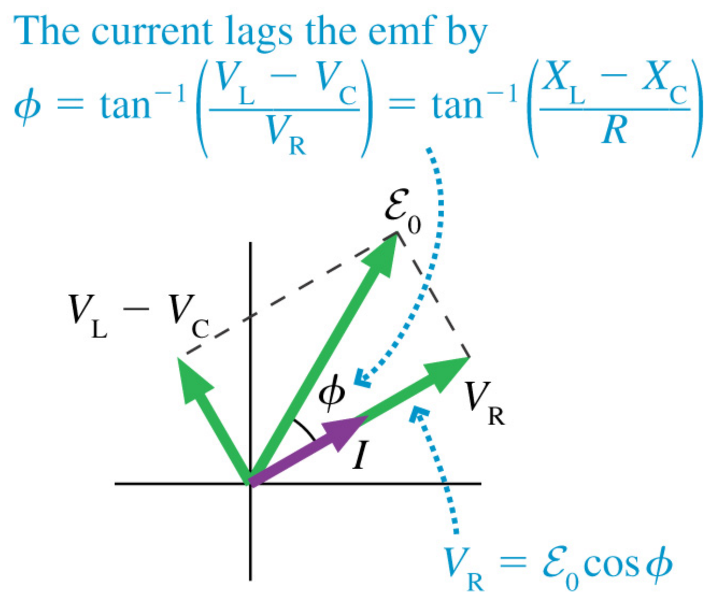


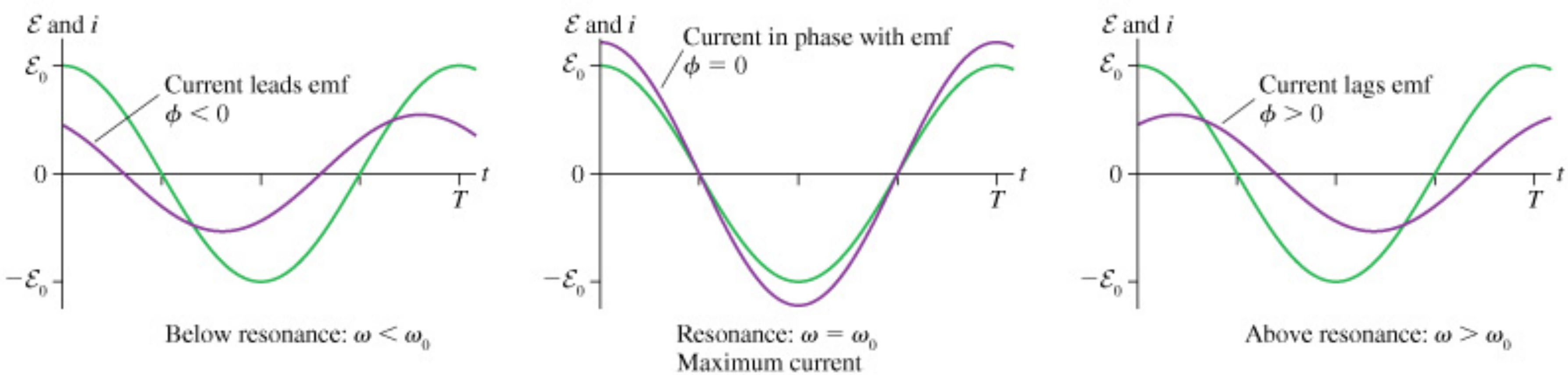
$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega_o = \frac{1}{\sqrt{LC}}$$

Phase angle for RLC circuit

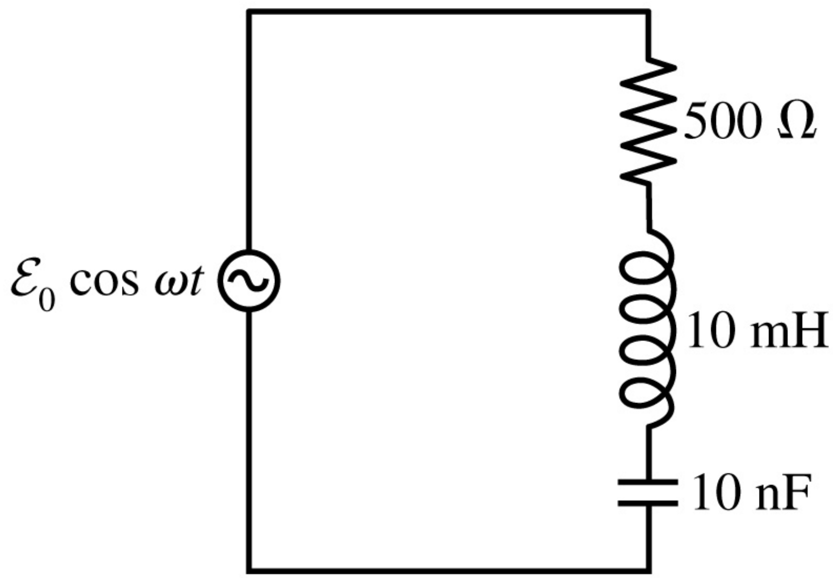
i=Icos(ωt-ϕ)





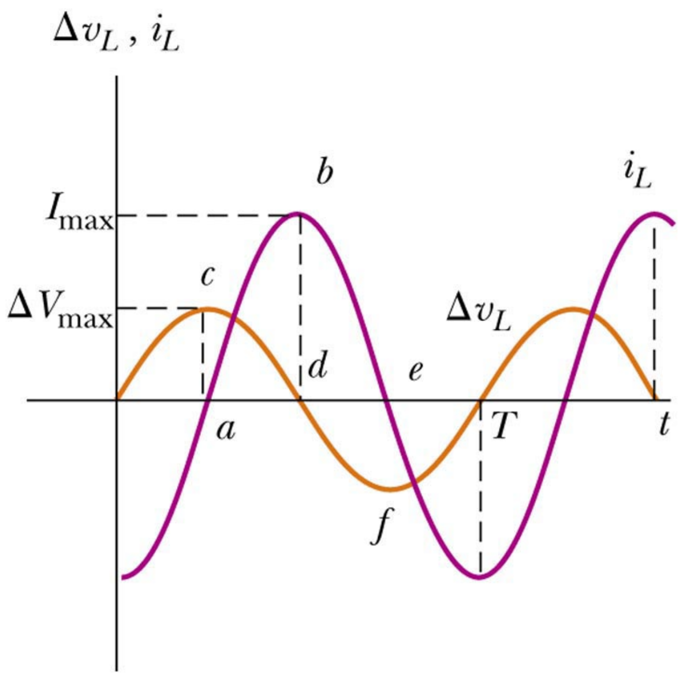
Example

- What is the phase angle when the frequency is 14 khz, 18 khz?

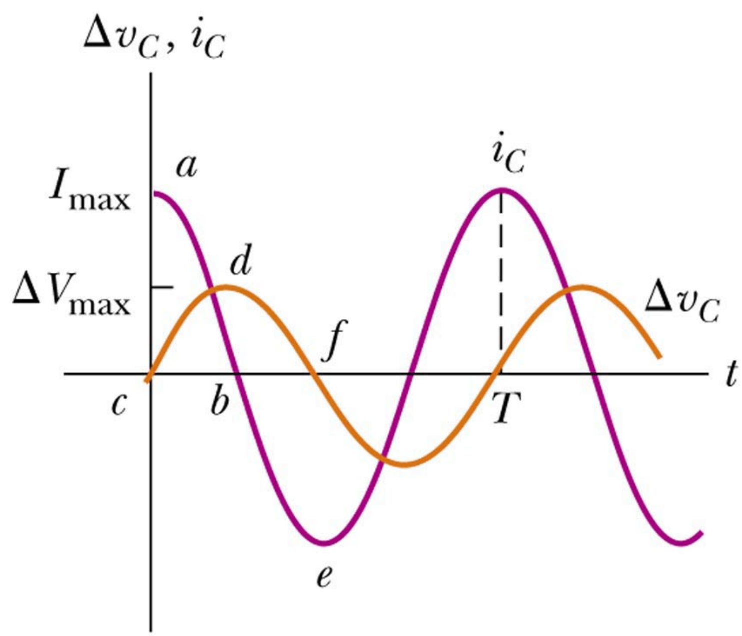


ELI the ICE man

- ELI
- emf leads the current in an inductive circuit



- ICE
- current before the emf in a capacitive circuit



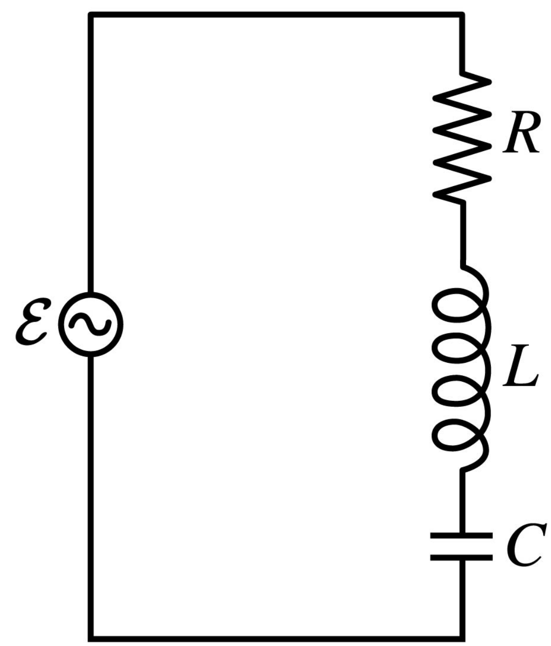
iclicker

If the value of R is increased, the resonance frequency of this circuit

A. Increases.

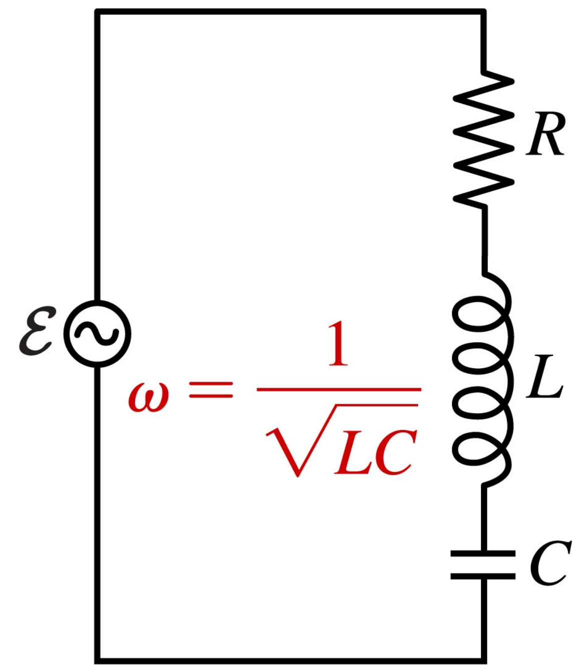
B. Decreases.

C. Stays the same.



The resonance frequency depends on C and L but not on R.

The resonance frequency of this circuit is 1000 Hz. To change the resonance frequency to 2000 Hz, replace the capacitor with one having capacitance



A. C/4.

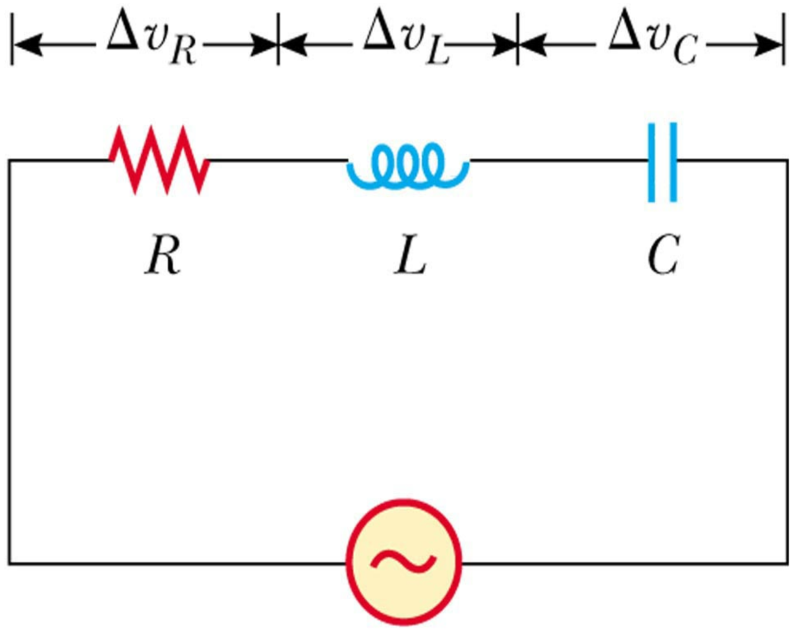
B. C/2.

C. 2C.

D. 4C.

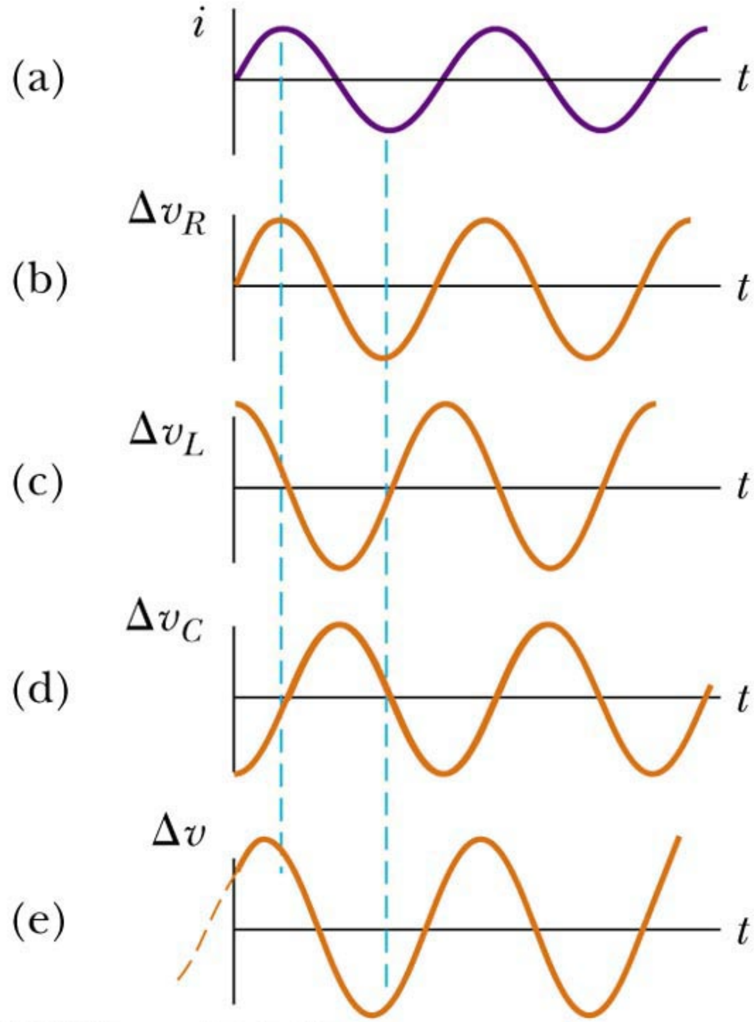
E. It's impossible to change the resonance frequency by changing only the capacitor.

Kirchoff' s loop rule



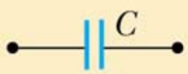
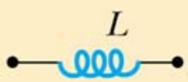
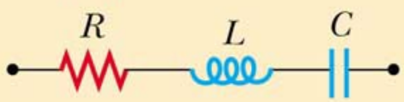
$\Delta v = \Delta v_R + \Delta v_C + \Delta v_L$

instantaneous voltage supplied by generator equals sum of instantaneous voltages across resistor, capacitor and inductor



Handy guide to AC circuits

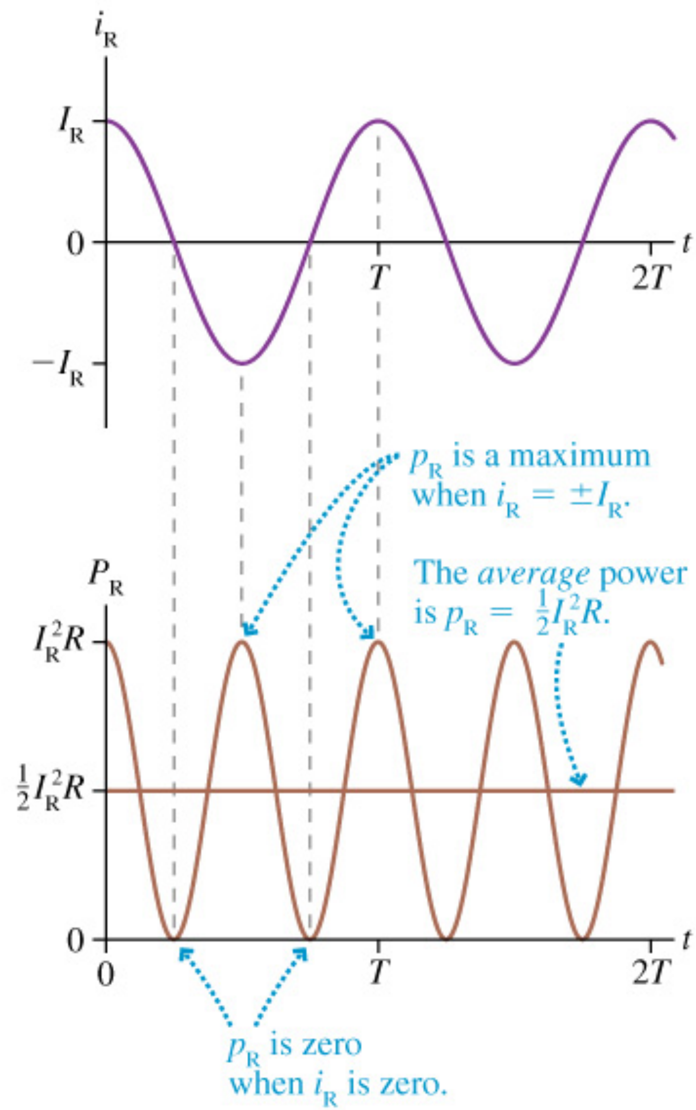
TABLE 21.2 **Impedance Values and Phase Angles for Various Combinations of Circuit Elements**[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the combination of elements (that is, across the dots).

Power in AC circuits

- An AC current by definition changes direction twice each period
- The instantaneous power provided by the emf is
  - $p_{source}=i\varepsilon$
- The power dissipated in the resistor is given by
  - $p_R=i_Rv_R=i_R^2R$
  - $p_R=i_R^2R=I_R^2R\cos 2\omega t$
  - note that the power peaks twice every cycle
- Consider the average power



$$P_R = I_R{}^2R\cos^2\omega t = I_R{}^2R\left[\frac{1}{2}(1+\cos 2\omega t)\right]$$

$$= \frac{1}{2}I_R{}^2R + \cancel{\frac{1}{2}I_R{}^2R\cos 2\omega t}$$

Power in AC circuits

- The average power loss in a resistor is

$$P_R = \frac{1}{2} I_R{}^2 R$$

- or

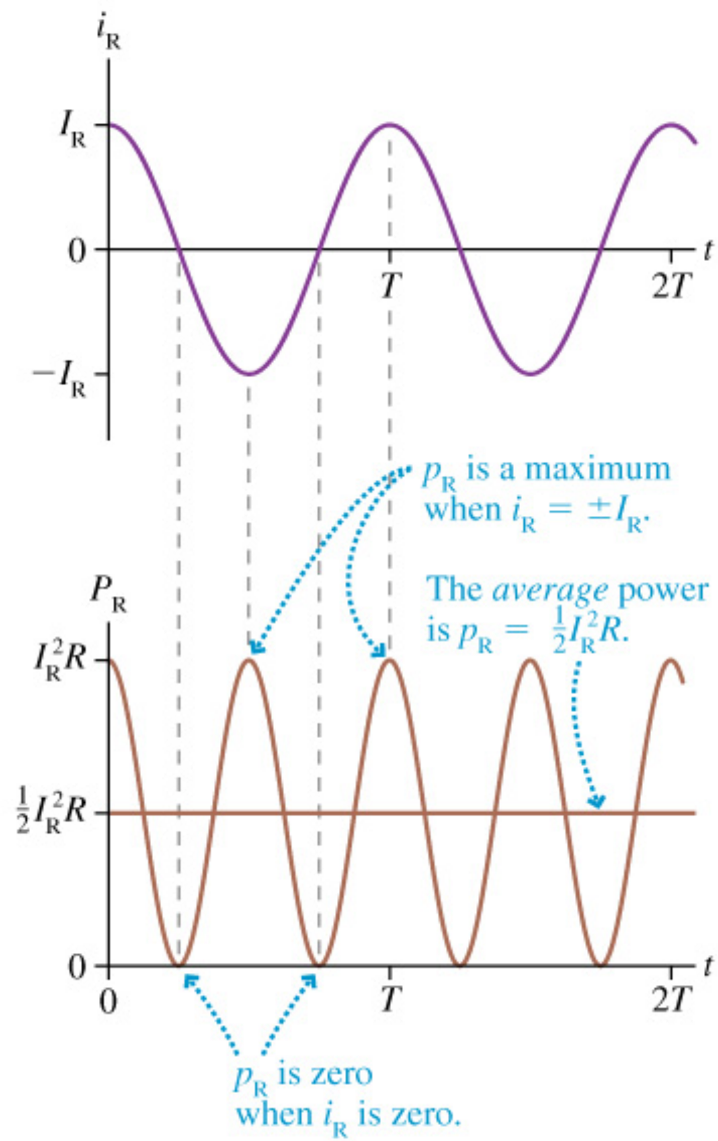
$$P_R = \left(\frac{I_R}{\sqrt{2}}\right)^2 R = \left(I_{rms}\right)^2 R$$

- where I_{rms} is the rms current

$$I_{rms} = \frac{I_R}{\sqrt{2}}$$

- I can also define rms values for the voltage and emf

$$V_{rms} = \frac{V_R}{\sqrt{2}} \qquad \varepsilon_{rms} = \frac{\varepsilon_o}{\sqrt{2}}$$



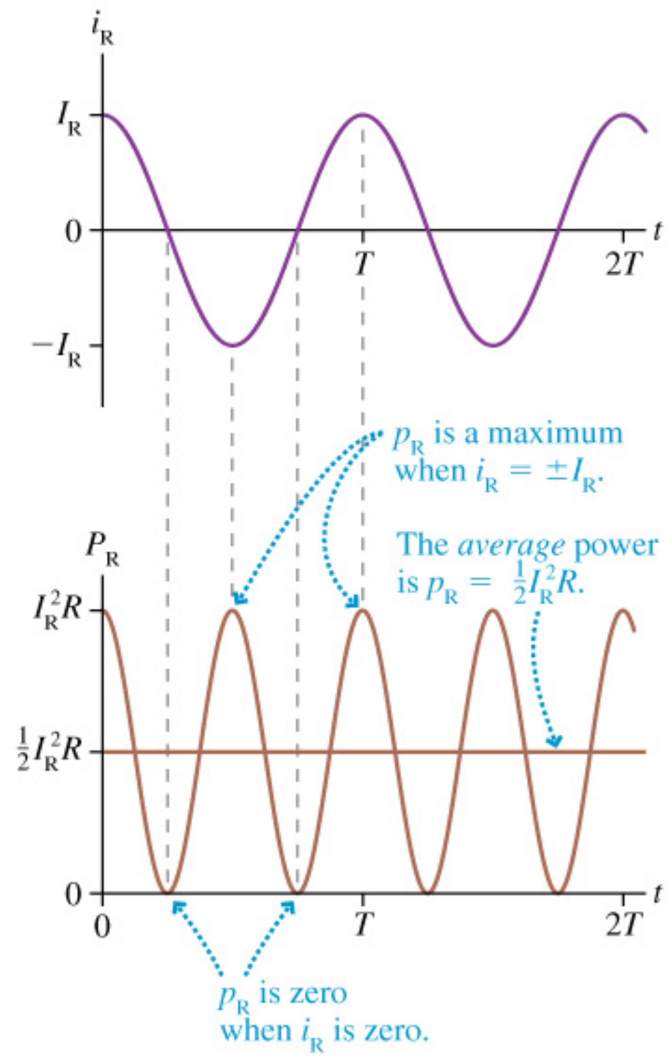
Power

- Can write the average power loss as

$$P_R == \left(I_{rms}\right)^2 R = \frac{\left(V_{rms}\right)^2}{R} = I_{rms} V_{rms}$$

- And average power supplied by emf as

$$P_{source} = I_{rms} \varepsilon_{rms}$$



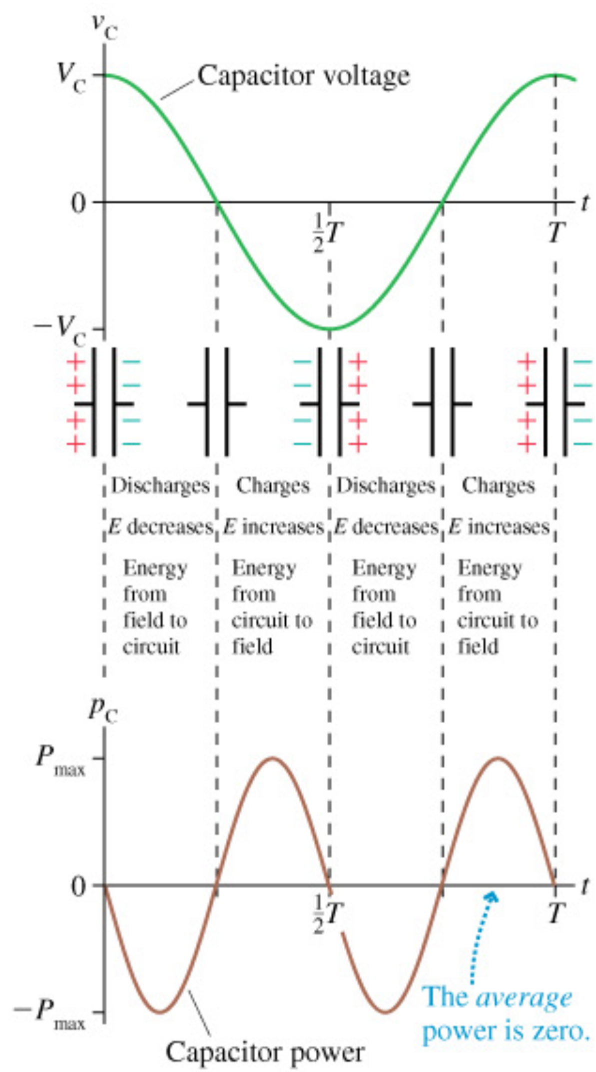
Power in capacitors and inductors

- Consider the instantaneous power in a capacitor
  - $p_C = v_C i_C$
- We can write the current as
  - $i_C = -\omega C V_C \sin\omega t$
- So

$$p_C = v_C i_C = \left(V_C \cos\omega t\right)\left(-wCV_C \sin\omega t\right)$$

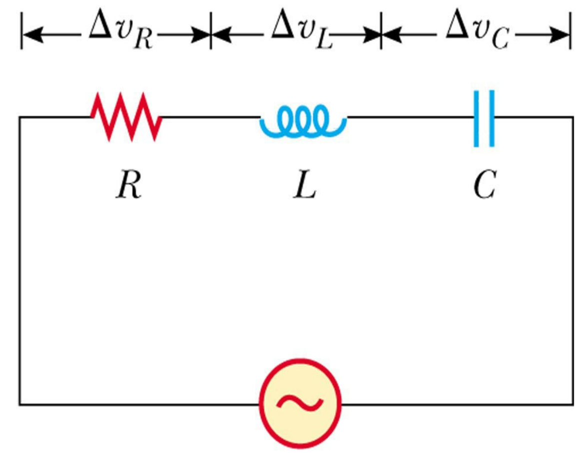
$$= -\frac{1}{2}\omega C V_C^{\;2} \sin 2\omega t$$

- So the average power loss in a capacitor is zero
- Same with an inductor
- Energy is tranferred in and out but there is no loss



Power factor

- In an RLC circuit, energy is supplied by the emf and is dissipated in the resistor
- But the current is not necessarily in phase with the emf voltage and this has consequences for the power



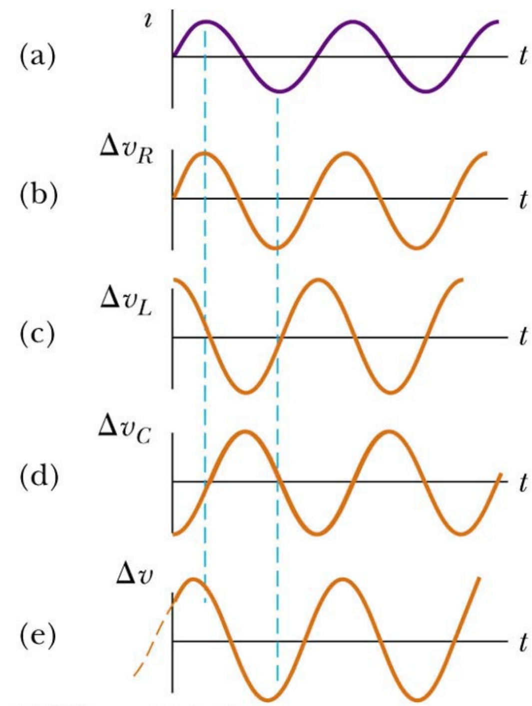
$$p_{source} = i\varepsilon = \left(I\cos\left(\omega t - \phi\right)\right)\left(\varepsilon_o \cos\omega t\right)$$

$$= I\varepsilon_o \cos\omega t \cos\left(\omega t - \phi\right)$$

$$p_{source} = \left(I\varepsilon_o \cos\phi\right)\cos^2\omega t + \left(I\varepsilon_o \sin\phi\right)\sin\omega t \cos\omega t$$

- The average power over a cycle then is

$$P_{source} = \frac{1}{2} I\varepsilon_o \cos\phi = I_{rms}\varepsilon_{rms}\cos\phi$$



Power factor

$$P_{source} = \frac{1}{2} I \varepsilon_o \cos\phi = I_{rms} \varepsilon_{rms} \cos\phi$$

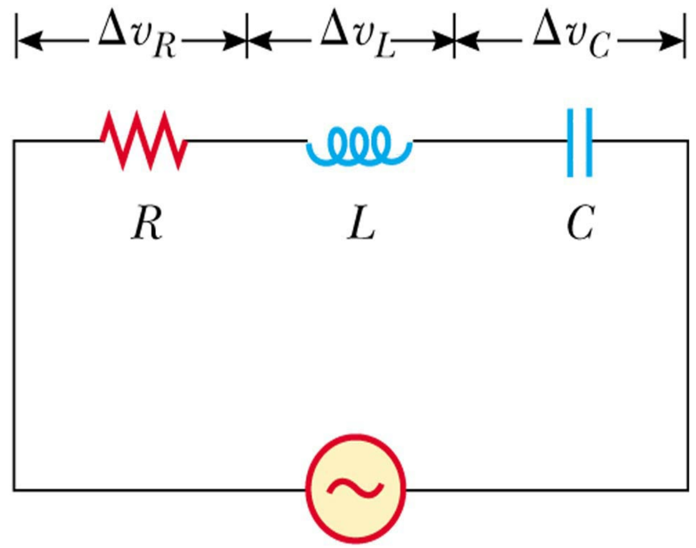
- Noting that $I = I_{max}\cos\phi$, we can write

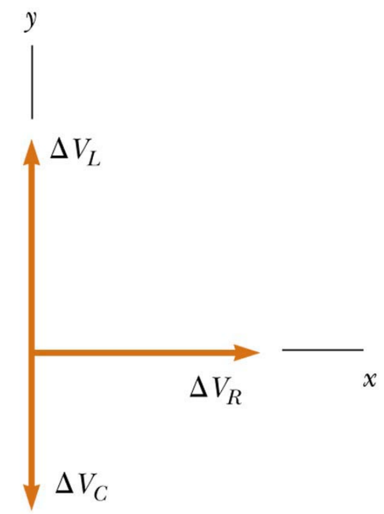
$$P_{source} = P_{\max} \cos^2 \phi$$

  - where $P_{max} = 1/2 I_{max} \varepsilon_o$
- Want to keep $\cos \phi$ close to 1 for efficient use of power

Example

- I have an AC circuit with a generator that supplies an rms voltage of 110 V at 50 hz connected in series with a 0.3 H inductor, a 4.5 μF capacitor and a 280 Ω resistor
- What is the impedance of the circuit?
- What is the rms current through the resistor?
- What is the phase φ?
- What is the power factor?





(a)

Take Fermilab as an example

There's a four mile ring full of superconducting magnets, i.e. it's a huge inductance. What to do if you're an EE?

- Add a big capacitor
  - $\tan \phi = (X_L - X_C)/R$
  - if X_L is big, then make X_C big