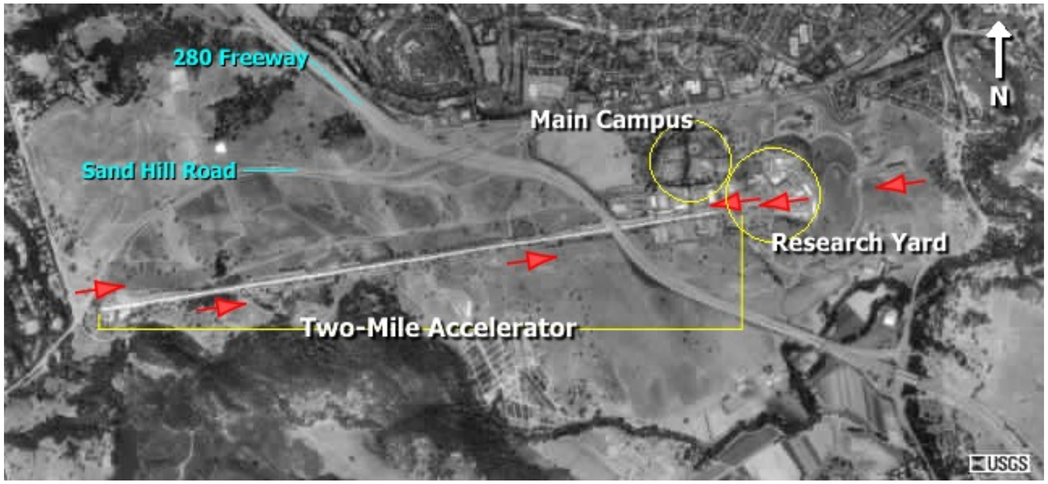
Physics 294H

- Professor: Joey Huston
- email:huston@msu.edu
- office: BPS3230
- Homework will be with Mastering Physics (and an average of 1 hand-written problem per week)
  - **Help-room hours: 12:40-2:40 Monday (note change); 3:00-4:00 PM Friday**
  - **36.73 hand-in problem for next Wed**
- Quizzes by iclicker (sometimes hand-written)
- Average on 2nd exam (so far)=71/120
- **Final exam Thursday May 5 10:00 AM – 12:00 PM 1420 BPS**
- Course website: www.pa.msu.edu/~huston/phy294h/index.html
  - lectures will be posted frequently, mostly every day if I can remember to do so

Example

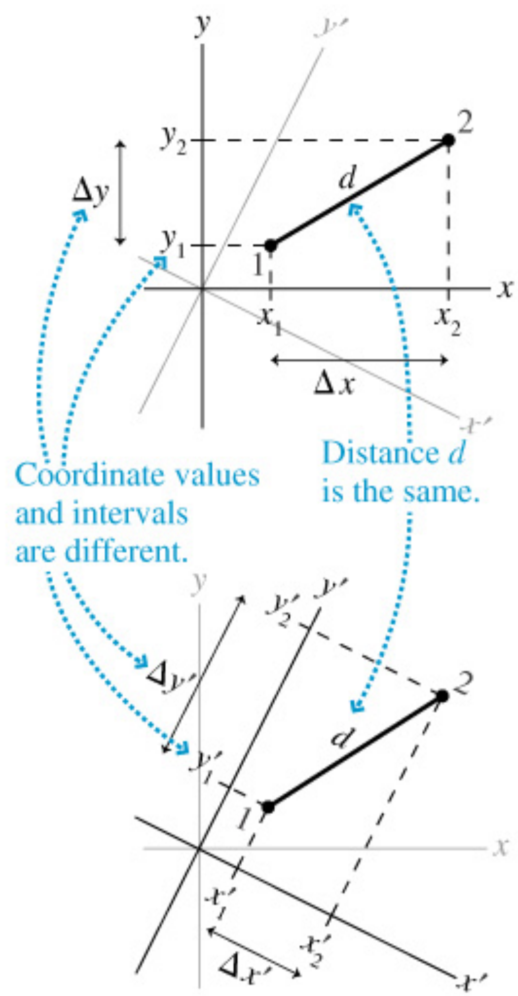
- At SLAC, electrons are accelerated to v=0.99999997c in a 3.2 km long accelerator
- How long is the accelerator from the electron's point of view?



Intervals

- Back to (Galilean) geometry
- Two coordinate systems, one rotated with respect to the other
- Coordinates (x,y;x' ,y') different in two frames but interval is the same
  - $d^2=(\Delta x)^2+(\Delta y)^2=(\Delta x')^2+(\Delta y')^2$
  - this will be true for all such coordinate systems
  - d is called an invariant

Measurements in the *xy*-system



Measurements in the *x'y'*-system

Spacetime intervals

- Consider two events that are separated in time by an interval Δt, and are separated in space by an interval Δx.
- Let us define the **spacetime interval** s between the two events to be:

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2$$

- The spacetime interval s has the same value in all inertial reference frames.
- That is, the spacetime interval between two events is an *invariant.*

EXAMPLE 36.7 Using the spacetime interval

A firecracker explodes at the origin of an inertial reference frame. Then, 2.0 μs later, a second firecracker explodes 300 m away. Astronauts in a passing rocket measure the distance between the explosions to be 200 m. According to the astronauts, how much time elapses between the two explosions?

EXAMPLE 36.7 Using the spacetime interval

MODEL The spacetime coordinates of two events are measured in two different inertial reference frames. Call the reference frame of the ground S and the reference frame of the rocket S′. The spacetime interval between these two events is the same in both reference frames.

EXAMPLE 36.7 Using the spacetime interval

SOLVE The spacetime interval (or, rather, its square) in frame S is

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = (600 \text{ m})^2 - (300 \text{ m})^2 = 270{,}000 \text{ m}^2$$

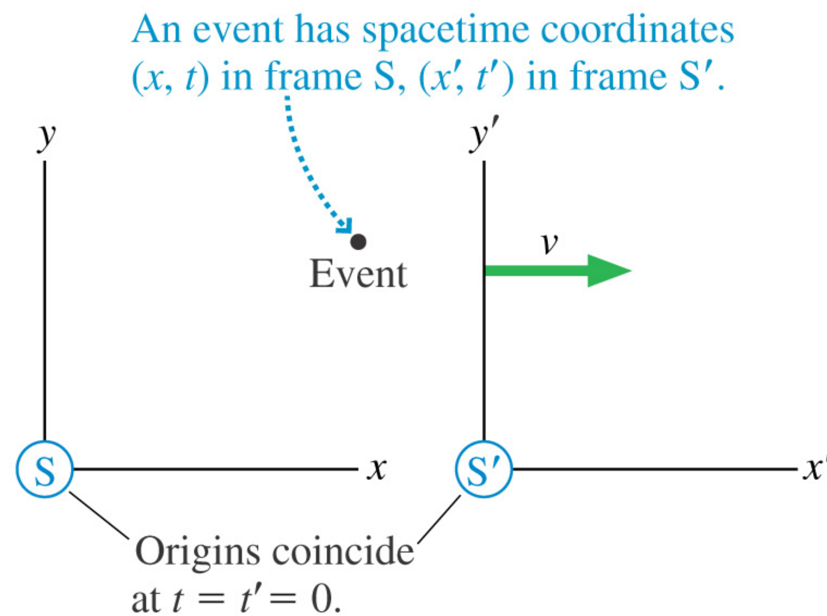
where we used $c = 300 \text{ m}/\mu\text{s}$ to determine that $c\,\Delta t = 600$ m. The spacetime interval has the same value in frame S$'$. Thus

$$s^2 = 270{,}000 \text{ m}^2 = c^2(\Delta t')^2 - (\Delta x')^2$$
$$= c^2(\Delta t')^2 - (200 \text{ m})^2$$

This is easily solved to give $\Delta t' = 1.85\ \mu\text{s}$.

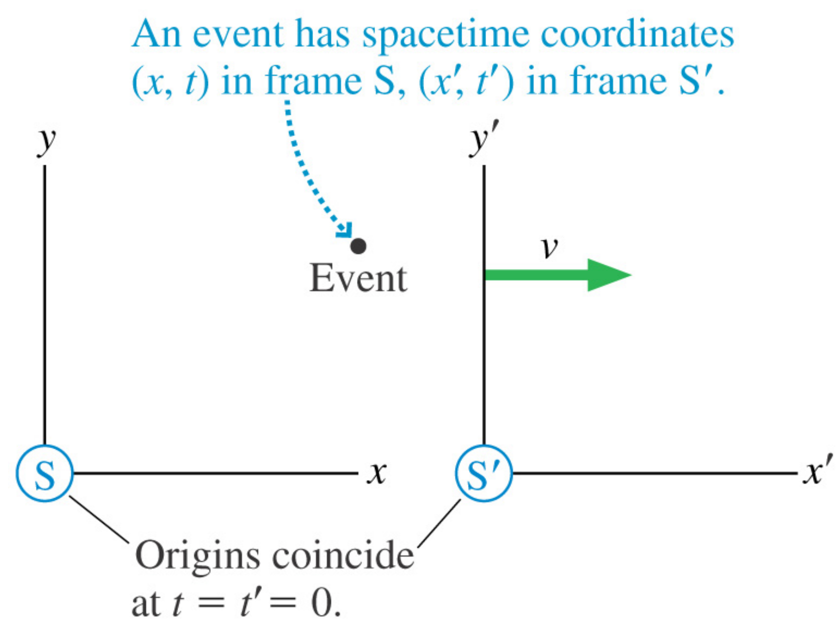
Lorentz transformations

- 2 coordinate systems, S and S'
- Galilean transformations
  - x' =x-vt
  - t=t'
- What about a relativistic form for the transformations?
- Need to
  - agree with Galilean transformations when v<<c
  - transform both spatial and time coordinates
  - ensure that speed of light is the same in all frames of reference



Lorentz transformations

- Try form
  - $x' = \gamma(x - vt)$
  - $x = \gamma(x' + vt)$
  - is this the same γ we met before? We'll see.
- Consider an event where a flash of light is emitted from the origin of both coordinate systems at t=0
- In a second event, the light hits a detector; the coordinates for this event are (x,t) in S and (x',t') in S'
- Since light travels at the same speed in both reference frames, the positions of the second event are x=ct in S and x'=ct' in S'
- Substitute into the equations above

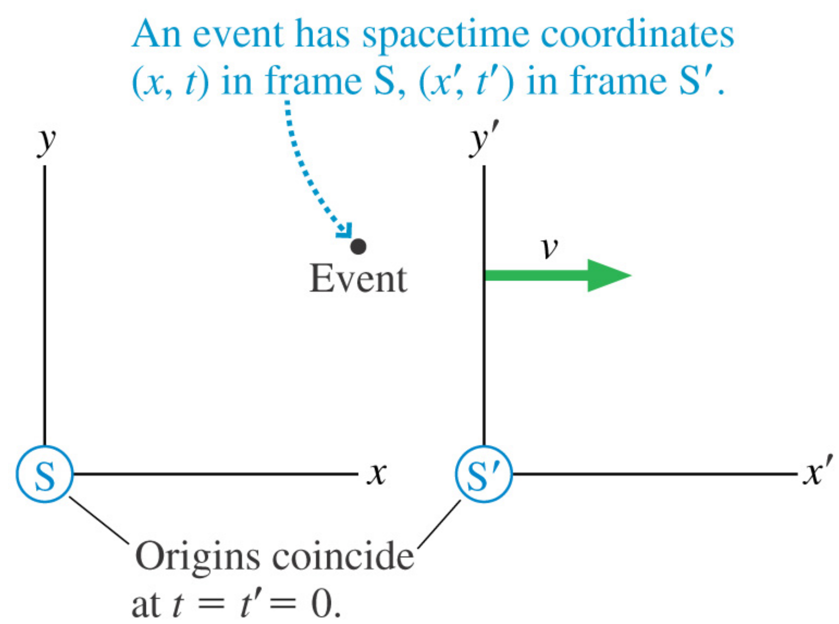


Some algebra

$$ct' = \gamma(ct - vt) = \gamma(c - v)t$$

$$ct = \gamma(ct' + vt') = \gamma(c + v)t'$$

- Solve first for t' and substitute into second



$$ct = \gamma(c + v)\frac{\gamma(c - v)t}{c} = \gamma^2(c^2 - v^2)\frac{t}{c}$$

$$\gamma^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{1 - \frac{v^2}{c^2}}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

Lorentz transformations

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

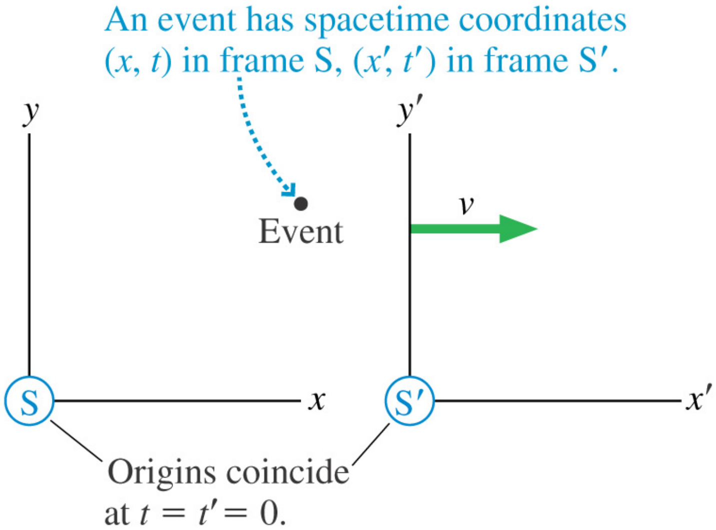
$$x = \gamma(x' + vt')$$

$$y' = y$$

$$z' = z$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

These transformation equations leave Maxwell's equations invariant.



PROBLEM-SOLVING STRATEGY 36.1 Relativity

MODEL Frame the problem in terms of events, things that happen at a specific place and time.

VISUALIZE A pictorial representation defines the reference frames.

- Sketch the reference frames, showing their motion relative to each other.
- Show events. Identify objects that are moving with respect to the reference frames.
- Identify any proper time intervals and proper lengths. These are measured in an object's rest frame.

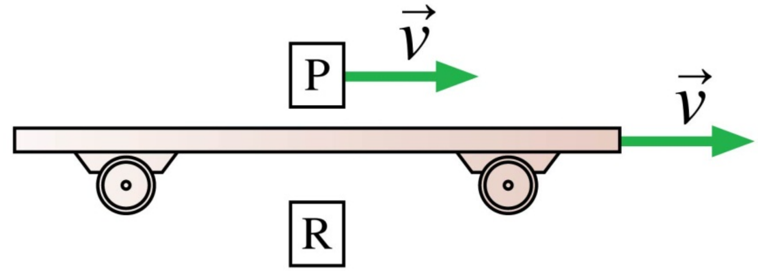
PROBLEM-SOLVING STRATEGY 36.1 Relativity

SOLVE The mathematical representation is based on the Lorentz transformations, but not every problem requires the full transformation equations.

- Problems about time intervals can often be solved using time dilation: $\Delta t = \gamma \Delta \tau$.
- Problems about distances can often be solved using length contraction: $L = \ell/\gamma$.

ASSESS Are the results consistent with Galilean relativity when $v \ll c$?

$\vec{v}$



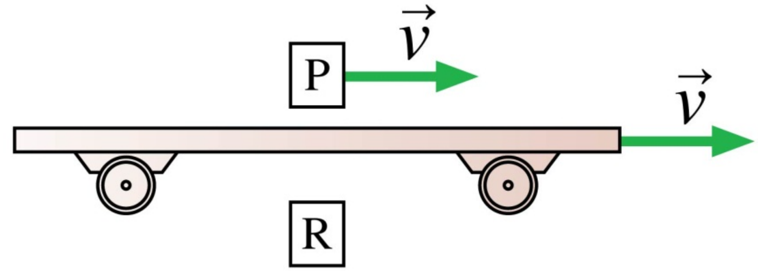
Peggy passes Ryan at velocity v. Peggy and Ryan both measure the length of the railroad car, from one end to the other. The length Peggy measures is ____ the length Ryan measures.

A. longer than

B. at the same as

C. shorter than

Peggy passes Ryan at velocity $\vec{v}$. Peggy and Ryan both measure the length of the railroad car, from one end to the other. The length Peggy measures is ____ the length Ryan measures.



✔ **A. longer than**

B. at the same as

C. shorter than

Peggy measures the proper length because the railroad car is at rest in her frame. Lengths measured in any other reference frame are shorter than the proper length.

- Suppose an 8 m long school bus drives past at 30 m/s. By how much is its length contracted according to an observer standing by the side of the road?
- The proper length (8 m) is in its own rest frame.
- It will be shorter in the frame of the observer.

$$L = \sqrt{1 - \beta^2}\, \ell$$

The binomial approximation

If $x \ll 1$, then $(1+x)^n \approx 1 + nx$.

- The binomial approximation is useful when we need to calculate a relativistic expression for a nonrelativistic velocity $v << c$.

$$\text{If } v \ll c: \begin{cases} \sqrt{1-\beta^2} = (1 - v^2/c^2)^{1/2} \approx 1 - \dfrac{1}{2}\dfrac{v^2}{c^2} \\ \gamma = \dfrac{1}{\sqrt{1-\beta^2}} = (1 - v^2/c^2)^{-1/2} \approx 1 + \dfrac{1}{2}\dfrac{v^2}{c^2} \end{cases}$$

Lorentz velocity transformations

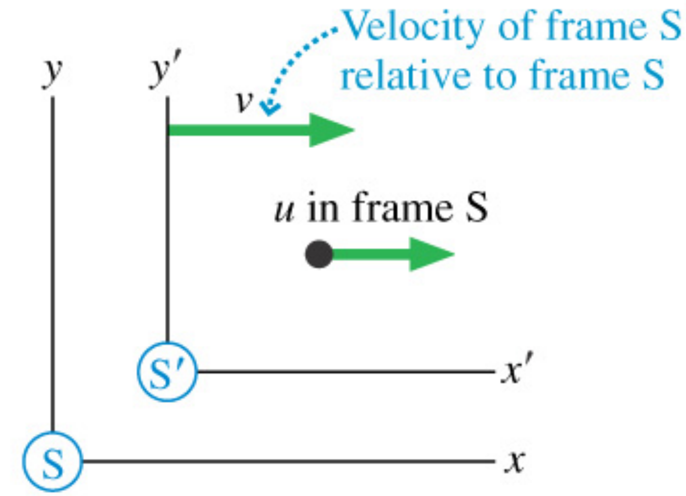
- Frame S' is moving at velocity v with respect to frame S
- A speed u in S corresponds to what speed in S'
- Take derivatives

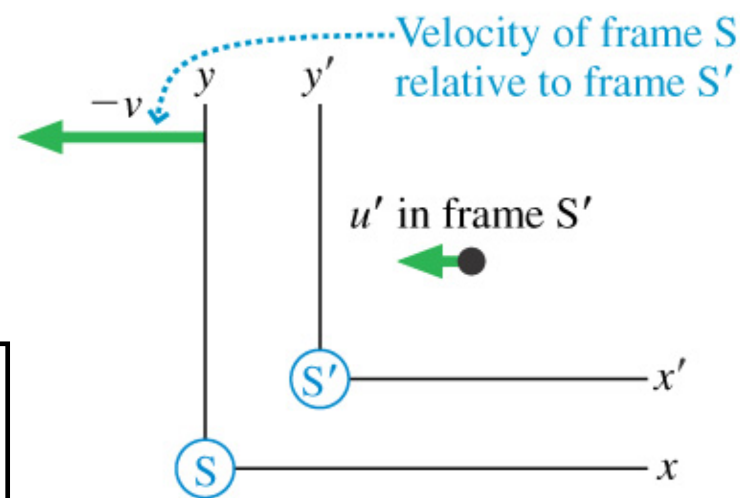
$$u' = \frac{dx'}{dt'} = \frac{d(\gamma(x - vt))}{d(\gamma(t - \frac{vx}{c^2}))}$$

$$u' = \frac{\gamma(dx - vdt)}{\gamma\left(dt - v\frac{dx}{c^2}\right)} = \frac{\frac{dx}{dt} - v}{1 - v\frac{(dx/dt)}{c^2}}$$

$$u = \frac{dx}{dt}$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}} \qquad u = \frac{u' + v}{1 + \frac{u'v}{c^2}}$$





Transformation of velocities

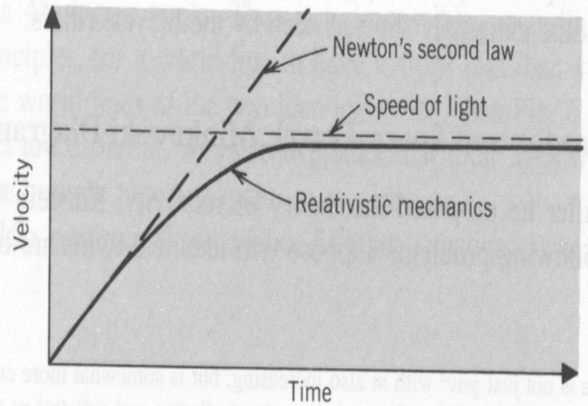


Figure 6.14. Graph of velocity versus time for constant force. In relativistic mechanics, the velocity cannot increase indefinitely, but rather is limited by the speed of light.

Table 6.2. Comparison of Results for Velocity Addition According to Galilean and Lorentz Transformations

u	v	Galilean $u + v$	Lorentz $\left(\frac{u + v}{1 + uv/c^2}\right)$
60 mph	30 mph	90 mph	90 mph
186 mps (0.001c)	18.6 mps (0.0001c)	204.6 mps	204.59998 mps
0.6c	0.3c	0.9c	0.763c
0.5c	0.5c	c	0.800c
0.75c	0.75c	1.5c	0.960c
0.9c	0.6c	1.6c	0.974c
c	0.1c	1.1c	1.000c
c	c	2c	c

Relativistic example

- A man on a (very fast) motorcycle travelling 0.80 c throws a baseball forward (he has a very good arm) with a speed of 0.70 c (from his perspective)
- How fast does the innocent bystander see the ball travelling?

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0.6c	0.3c	0.9c	0.763c
0.5c	0.5c	c	0.800c
0.75c	0.75c	1.5c	0.960c
0.9c	0.6c	1.6c	0.974c
c	0.1c	1.1c	1.000c
c	c	2c	c

From Galilean perspective: 0.80 c +0.70 c =1.5 c
Using Lorentz transformation of velocities:
$[u+v]/[1+uv/c^2] = [0.8c+0.7c]/[1+(.8c)(.7c)/c^2]$
$=0.96$ c

Relativistic momentum

- Total momentum is conserved in any interaction
- (Galilean) formula for momentum (p=mu) doesn't work for high velocities
- Write formula for momentum using $\Delta\tau$ in rest frame of particle

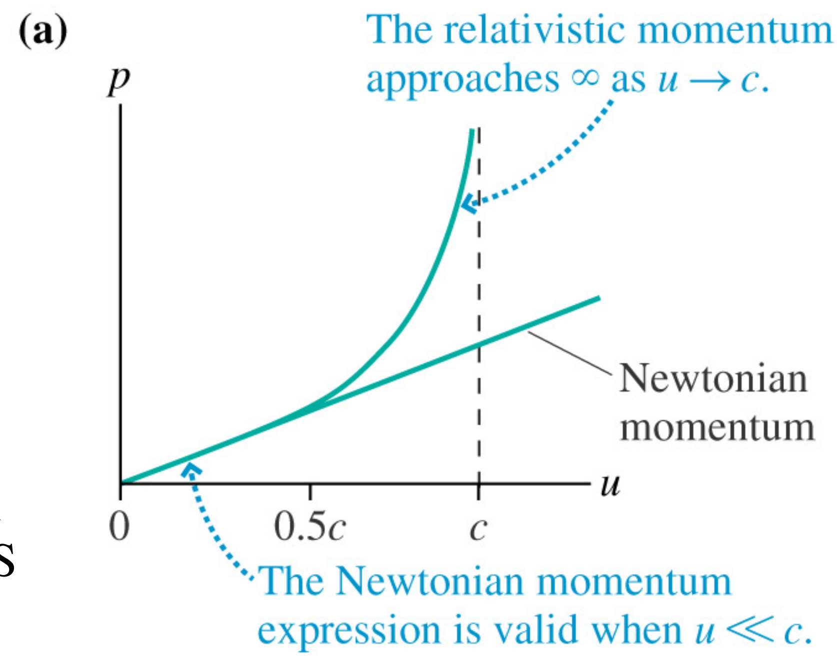
$$p = mu = m\frac{\Delta x}{\Delta \tau}$$

note that we use u for the velocity of a particle since v is already taken

time in rest frame of particle → $\Delta\tau = \sqrt{1-\frac{u^2}{c^2}}\Delta t$ → time in frame S

- So we can write a relativistic expression for momentum

$$p = mu = m\frac{\Delta x}{\Delta\tau} = m\frac{\Delta x}{\sqrt{1-\frac{u^2}{c^2}}\Delta t} = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}}$$



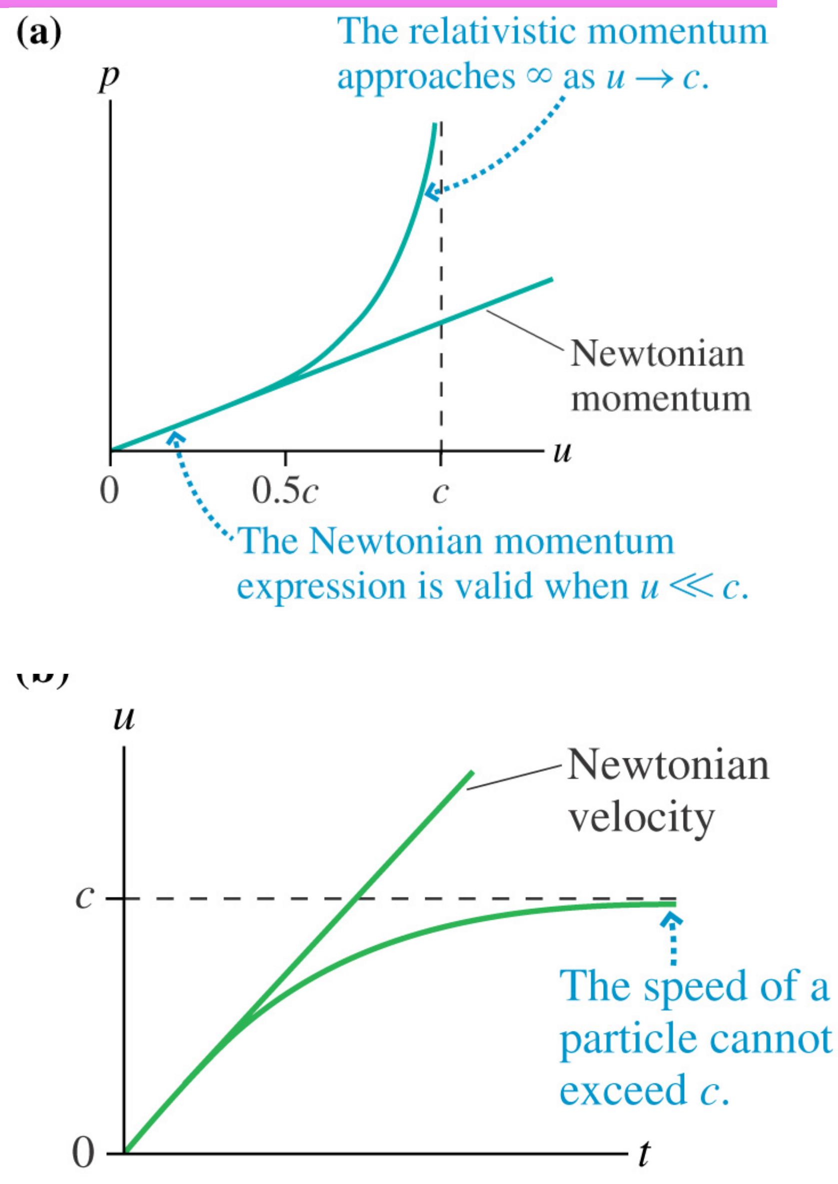
Law of conservation of momentum still holds at relativistic velocities if formula on left is used

Speed limit

- Because the momentum increases to infinity as v->c, no material object can travel at the speed of light without the input of an infinite amount of energy

define $$\gamma_p = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$p = \frac{mu}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma_p mu$$



Relativistic energy

- Need a relativistic form for the energy as well
- Start with the spacetime interval that we discussed before

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = invariant$$

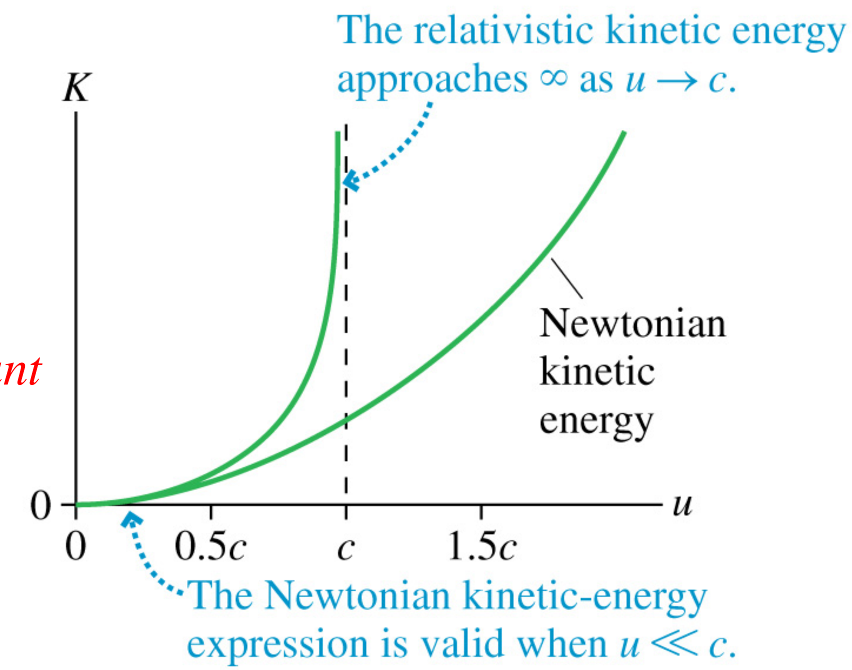
- Multiply by $(m/\Delta\tau)^2$

$$mc^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - \left(\frac{m\Delta x}{\Delta\tau}\right)^2 = mc^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - p^2 = invariant$$

- Relate Δt, time interval in S, to proper time $\Delta\tau$, and then multiply expression by c^2

$$\gamma_p = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \qquad \Delta t = \gamma_p \Delta\tau$$

$$(\gamma_p mc^2)^2 - (pc)^2 = invariant$$



In rest frame

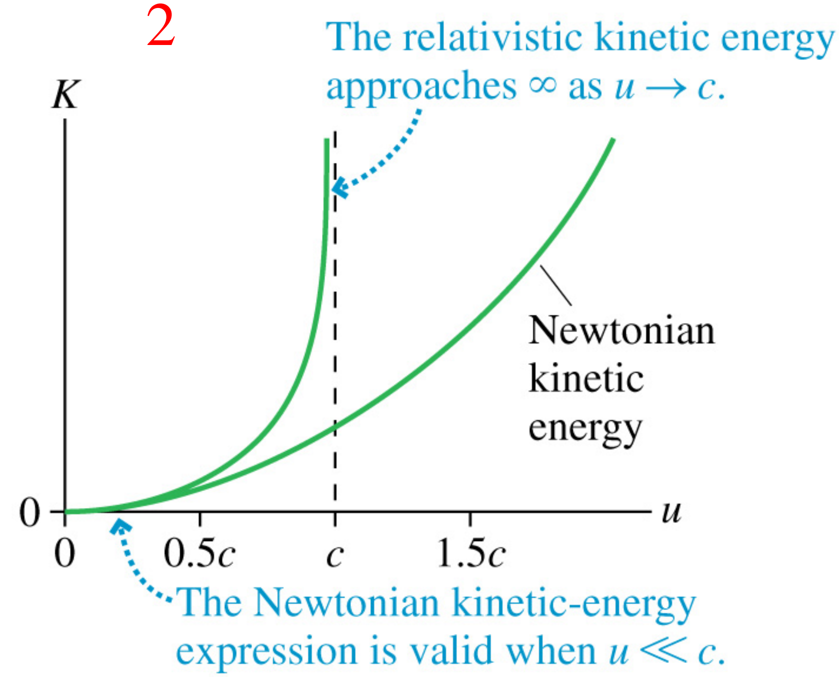
$$(\gamma_p mc^2)^2 - (pc)^2 = (mc^2)^2$$

What is $\gamma_p mc^2$?

energy

$$\gamma_p mc^2 = \frac{mc^2}{\sqrt{1-\frac{u^2}{c^2}}} \approx \left(1+\frac{1}{2}\frac{u^2}{c^2}\right)mc^2 = mc^2 + \frac{1}{2}mu^2$$

- $E=\gamma_p mc^2=E_o+K$
  =rest energy + kinetic energy
- $K=(\gamma_p-1)mc^2$
- Kinetic energy goes to $1/2mu^2$ when $u<<c$
- $E_o=mc^2$ (rest energy)
- $E^2-(pc)^2=E_o^2$



Convenient to quote particle energies in eV

$1\text{ eV} = 1.6 \times 10^{-19}$ J

$m_e=9.11\times10^{-31}$ kg

$m_ec^2=(9.11\times10^{-31}\text{ kg})(3.0\times10^{8}\text{ m/s})^2 = 8.2\times10^{-19}$ J

$=(8.2\times10^{-14}\text{ J})/(1.6\times10^{-19}\text{ J/eV})=0.511$ MeV