Difficult calculations

The multi-loop and multi-leg calculations are very difficult

but just compare them to the complexity of the sentences that Sarah Palin used in her run for the vice-presidency.

Note: to her, you guys are just a bunch of Southerners
Why stop at NLO?

- NNLO is even better, but also more complicated
- Expect reduced scale dependence
- And in some cases, extra contributions, such as for Higgs production
- Have only been carried out for a few processes to date
- Would really like to have it for inclusive jet production, for example

Figure 16. The single jet inclusive distribution at $E_T = 100\,\text{GeV}$, appropriate for Run I of the Tevatron. Theoretical predictions are shown at LO (dotted magenta), NLO (dashed blue) and NNLO (red). Since the full NNLO calculation is not complete, three plausible possibilities are shown.
Higgs: LO->NLO->NNLO


convergence in going:
LO → NLO → NNLO

Confirmed by the full scale dependence:

\[ \downarrow \]
All-orders approaches

- Rather than systematically calculating to higher and higher orders in the perturbative expansion, can also use a number of all-orders approaches
- In resummation, dominant contributions from each order in perturbation theory are singled out and resummed by use of an evolution equation
- Near boundaries of phase space, fixed order calculations break down due to large logarithmic corrections
- Consider W production
  - one large logarithm associated with production of vector boson close to threshold
  - takes form of $\alpha_s^n \log^{2n-1}(1-z)/(1-z)$ where $z=Q^2/s-1$
  - other large logarithm is associated with recoil of vector boson at very small $p_T$
  - logarithms appear as $\alpha_s^n \log^{2n-1}(Q^2/p_T^2)$

Expression for W boson transverse momentum in which leading logarithms have been resummed to all orders is given by

$$\frac{d\sigma}{dp_T^2} = \sigma \frac{d}{dp_T^2} \exp \left( -\frac{\alpha_s C_F}{2\pi} \log^2 \frac{M_W^2}{p_T^2} \right)$$

Note that distribution goes to zero as $p_T\to0$

Figure 20. The resummed (leading log) W boson transverse momentum distribution.
Parton showers

- A different, but related approach, is provided by parton showering.
- By the use of the parton showering process, a few partons produced in a hard interaction at a high-energy scale can be related to partons at an energy scale close to $\Lambda_{\text{QCD}}$.
- At this lower energy scale, a universal non-perturbative model can then be used to provide the transition to hadrons.
- Parton showering allows for evolution, using DGLAP formalism, of parton fragmentation function.
- Successive values of an evolution variable $t$, a momentum fraction $z$ and an azimuthal angle $\phi$ are generated, along with the flavors of the partons emitted during the parton shower.

Parton Cascade

...plus similar for initial state

Due to successive branching, parton cascade or shower develops. Each outgoing line is source of new cascade, until all outgoing lines have stopped branching. At this stage, which depends on cutoff scale $t_0$, outgoing partons have to be converted into hadrons via a hadronization model.
Parton shower evolution

- On average, emitted gluons have decreasing angles with respect to parent parton directions
  - angular ordering, an aspect of color coherence
- The evolution variable $t$ can be the virtuality of the parent parton [old Pythia and Sherpa], $E^2(1-\cos \theta)$ where $E$ is the energy of the parent parton and $\theta$ is the opening angle between the two partons [Herwig], or the square of the transverse momentum between the two partons [new Pythia]
Sudakov form factors

- Sudakov form factors form the basis for both resummation and parton showering.
- We can write an expression for the Sudakov form factor of an initial state parton in the form below, where $t$ is the hard scale, $t_0$ is the cutoff scale and $P(z)$ is the splitting function.

$$\Delta(t) \equiv \exp \left[ -\int_{t_0}^{t} \frac{dt'}{t'} \int \frac{dz}{z} \frac{\alpha_S}{2\pi} \frac{P(z) f(x/z, t)}{f(x, t)} \right]$$

- Similar form for the final state but without the pdf weighting.
- Sudakov form factor resums all effects of soft and collinear gluon emission, but does not include non-singular regions that are due to large energy, wide angle gluon emission.
- Gives the probability **not** to radiate a gluon greater than some energy.

Figure 21. The Sudakov form factors for initial-state gluons at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

Figure 22. The Sudakov form factors for initial-state gluons at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.
Sudakov form factors: quarks and gluons

so quarks don’t radiate as much; it’s the $C_F$ compared to $C_A$

Figure 21. The Sudakov form factors for initial-state gluons at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

Figure 22. The Sudakov form factors for initial-state gluons at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1, 0.03, 0.01, 0.001 and 0.0001.

Figure 23. The Sudakov form factors for initial-state quarks at a hard scale of 100 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1 and 0.03.

Figure 24. The Sudakov form factors for initial-state quarks at a hard scale of 500 GeV as a function of the transverse momentum of the emitted gluon. The form factors are for (top to bottom) parton $x$ values of 0.3, 0.1 and 0.03.
We can only observe emissions above a certain resolution scale.

Below this resolution scale, singularities cancel, leaving a finite remnant.

(some of) the virtual corrections encountered in a full NLO calculation are included by the use of Sudakov suppression between vertices.

So a parton shower Monte Carlo is not purely a fixed order calculation, but has a virtual component as well.
Merging ME and PS approaches

- Parton showers provide an excellent description in regions which are dominated by soft and collinear gluon emission.
- Matrix element calculations provide a good description of processes where the partons are energetic and widely separated and also take into account interference effects between amplitudes.
  - but do not take into account interference effects in soft and collinear emissions which cannot be resolved, and thus lead to Sudakov suppression of such emissions.
- Hey, I know, let’s put them together, but we have to be careful not to double-count.
  - parton shower producing same event configuration already described by matrix element.
  - Les Houches Accord (which I named) allows the ME program to talk to the PS program.

Figure 26. In the NLO formalism, the same scale, proportional to the hardness of the process, is used for each QCD vertex. For the case of the $W+2$ jet diagram shown above to the left, a scale related to the mass of the $W$ boson, or to the average transverse momentum of the produced jets, is typically used. The figure to the right shows the results of a simulation using the CKKW formalism. Branchings occur at the vertices with resolution parameters $d_i$, where $d_1 > d_2 > d_3 > d_4 > d_5 > d_6$. Branchings at the vertices 1–2 are produced with matrix element information while the branchings at vertices 3–6 are produced by the parton shower.
Merging ME and PS approaches

- A number of techniques to combine, with most popular/correct being CKKW
  - matrix element description used to describe parton branchings at large angle and/or energy
  - parton shower description is used for smaller angle, lower energy emissions
- Division into two regions of phase space provided by a resolution parameter \( d_{\text{ini}} \)
- Argument of \( \alpha_s \) at all of the vertices is taken to be equal to the resolution parameter \( d_i \) (showering variable) at which the branching has taken place
- Sudakov form factors are inserted on all of the quark and gluon lines to represent the lack of any emissions with a scale larger than \( d_{\text{ini}} \) between vertices
  - parton showering is used to produce additional emissions at scales less than \( d_i \)
- For typical matching scale, \( \sim 10\% \) of the n-jet cross section is produced by parton showering from n-1 parton ME

![Diagram](image)

Figure 26. In the NLO formalism, the same scale, proportional to the hardness of the process, is used for each QCD vertex. For the case of the \( W + 2 \) jet diagram shown above to the left, a scale related to the mass of the \( W \) boson, or to the average transverse momentum of the produced jets, is typically used. The figure to the right shows the results of a simulation using the CKKW formalism. Branchings occur at the vertices with resolution parameters \( d_i \), where \( d_1 > d_2 \gg d_3 > d_4 > d_5 > d_6 \). Branchings at the vertices 1–2 are produced with matrix element information while the branchings at vertices 3–6 are produced by the parton shower.

see Alpgen, Madgraph, Sherpa,...
Best of all worlds

from CHS article

trick is to find out what parton shower does at NLO, so as not to double-count with MC@NLO, a small fraction (~10%) of the events have a negative weight (-1)

Several groups have worked on the subject to consistently combine partonic NLO calculations with parton showers.

- Collins, Zu [95, 96].
- Frixione, Nason, Webber (MC@NLO) [97–99].
- Kurihara, Fujimoto, Ishikawa, Kato, Kawabata, Munchisa, Tanaka [100].
- Krämer, Soper [101–103].
- Nagy, Soper [104, 105].

MC@NLO is the only publicly available program that combines NLO calculations with parton showering and hadronization. The HERWIG Monte Carlo is used for the latter. The use of a different Monte Carlo, such as PYTHIA, would require a different subtraction scheme for the NLO matrix elements. The processes included to date are \(W, Z, \gamma^*, H, bb, t\bar{t}, HW, HZ, WW, WZ, ZZ\). Recently, single top hadroproduction has been added to MC@NLO [106]. This is the first implementation of a process that has both initial- and final-state singularities. This allows a more general category of additional processes to be added in the future. Work is proceeding on the addition of inclusive jet production and the production of a Higgs boson via \(WW\) fusion. Adding spin correlations to a process increases the level of difficulty but is important for processes such as single top production.

If, in addition, the CKKW formalism could be used for the description of hard parton emissions, the utility and accuracy of a NLO Monte Carlo could be greatly increased. The merger of these two techniques should be possible in Monte Carlos available by the time of the LHC turn-on.

Powheg is a program/technique that results in no negative weight events; many new processes have been added.

But it’s still very time(theorist)-consuming to add a new process, and most of the processes so far are 2->1, 2->2.
At low $p_T$ for the t-tbar system, the cross section is described (correctly) by the parton shower, which resums the large logs near $p_T \sim 0$.

At high $p_T$, the cross section is described (correctly) by the NLO matrix element.
Hadrons

- The proton is a dynamical object; the structure observed depends on the time-scale ($Q^2$) of the observation.
- But we know how to calculate this variation (DGLAP).
- We just have to determine the starting points from fits to data.

The higher the value of $Q^2$, the more detail we examine.

$$f_i(x, Q^2) = \text{number density of partons } i \text{ at momentum fraction } x \text{ and probing scale } Q^2$$
Parton distribution functions and global fits

- Calculation of production cross sections at the LHC relies upon knowledge of pdf’s in the relevant kinematic region
- Pdf’s are determined by global analyses of data from DIS, DY and jet production
- Two major groups that provide semi-regular updates to parton distributions when new data/theory becomes available
  - MRS->MRST98->MRST99
    ->MRST2001->MRST2002
    ->MRST2003->MRST2004
    ->MSTW2008
  - CTEQ->CTEQ5->CTEQ6
    ->CTEQ6.1->CTEQ6.5
    ->CTEQ6.6->CT09

Figure 27. The CTEQ6.1 parton distribution functions evaluated at a $Q$ of 10 GeV.
Global fits

- With the DGLAP equations, we know how to evolve pdf’s from a starting scale $Q_0$ to any higher scale
- ...but we can’t calculate what the pdf’s are ab initio
  - one of the goals of lattice QCD
- We have to determine them from a global fit to data
  - factorization theorem tells us that pdf’s determined for one process are applicable to another

So what do we need
- a value of $Q_0$ (1.3 GeV for CTEQ, 1 GeV for MSTW) lower than the data used in the fit (or any prediction)
- a parametrization for the pdf’s
- a scheme for the pdf’s
- hard-scattering calculations at the order being considered in the fit
- pdf evolution at the order being considered in the fit
- a world average value for $\alpha_s$
- a lot of data
  - with appropriate kinematic cuts
- a treatment of the errors for the experimental data
- MINUIT
Back to global fits

- **Parametrization: initial form**
  - $f(x) \sim x^{\alpha}(1-x)^{\beta}$
  - estimate $\beta$ from quark counting rules
    - $\beta = 2n_s - 1$ with $n_s$ being the minimum number of spectator quarks
    - so for valence quarks in a proton (qqq), $n_s = 2$, $\beta = 3$
    - for gluon in a proton (qqqg), $n_s = 3$, $\beta = 5$
    - for anti-quarks in a proton (qqqqqbar), $n_s = 4$, $\beta = 7$
  - estimate $\alpha$ from Regge arguments
    - gluons and anti-quarks have $\alpha \sim -1$ while valence quarks have $\alpha \sim 1/2$
  - but at what $Q$ value are these arguments valid?

- **What do we know?**
  1. we know that the sum of the momentum of all partons in the proton is 1 (but see extra slides for modified LO fits)
  2. we know the sum of valence quarks is 3
    - and 2 of them are up quarks and 1 of them is a down quark
    - we know that the net number of anti-quarks is 0, but what about $\bar{d} = \bar{u}$
  3. we know that the net number of strange quarks (charm quarks/bottom quarks) in the proton is 0
    - but we don’t know if $s = \bar{s}$ locally

This already puts a lot of restrictions on the pdf’s
Parametrizations

- That simple parametrization worked for early fits, where the data was not very precise (nor very abundant), but it does not work for modern global fits, where a more flexible form is needed
  - the simple ansatz can be dangerous in that it can (falsely) tie together low x and high x behavior (other than by momentum sum rule)
- In order to more finely tune parametrization, usually multiply simple form by a polynomial in x or some more complicated function

- CTEQ uses for the quark and gluon distributions (CTEQ6.6)

\[ f(x) = x^{(a_1 - 1)}(1 - x)^{a_2} e^{a_3 x} [1 + e^{a_4} x]^{a_5} \]

- For the ratio of dbar/ubar

\[ \frac{d}{u} = e^{a_1} x^{(a_2 - 1)}(1 - x)^{a_3} + (1 + a_4 x)(1 - x)^{a_5} \]

- How do we know this is flexible enough?
  - data is well-described ($\chi^2$/dof ~1 for a NLO fit)
  - adding more parameters just results in those parameters being unconstrained
  - note that with this form, the pdf’s are positive definite (they don’t have to be)
Orders and Schemes

- Fits are available at
  - LO
    - CTEQ6L or CTEQ6L1
      - 1 loop or 2 loop $\alpha_s$
    - in common use with parton shower Monte Carlos
    - poor fit to data due to deficiencies of LO ME’s
  - LO*
    - better for parton shower Monte Carlos (see extra)
  - NLO
    - CTEQ6.1, CTEQ6.6, CT09
    - precision level: error pdf’s defined at this order
  - NNLO
    - more accurate but not all processes known

- At NLO and NNLO, one needs to specify a scheme or convention for subtracting the divergent terms
- Basically the scheme specifies how much of the finite corrections to subtract along with the divergent pieces
  - most widely used is the modified minimal subtraction scheme (or MSbar)
  - used with dimensional regularization: subtract the pole terms and accompanying $\log 4\pi$ and Euler constant terms
  - also may find pdf’s in DIS scheme, where full order $\alpha_s$ correction for $F_2$ in DIS absorbed into quark pdf’s
Scales and Masses

- Processes used in global fits are characterized by a single large scale
  - DIS-$Q^2$
  - lepton pair production-$M^2$
  - vector boson production-$M_V^2$
  - jet production-$p_T^{\text{jet}}$

- By choosing the factorization and renormalization scales to be of the same order as the characteristic scale
  - can avoid some large logarithms in the hard scattering cross section
  - some large logarithms in running coupling and pdf’s are resummed

- Different treatment of quark masses and thresholds
  - zero mass variable flavor number scheme (ZM-VFNS)
  - fixed flavor number scheme (FFNS)
  - variable flavor number scheme (VFNS)
  - see extra slides for more details
Data sets used in global fits (CTEQ6.6)

1. BCDMS $F_2^\text{proton}$ (339 data points)
2. BCDMS $F_2^\text{deuteron}$ (251 data points)
3. NMC $F_2$ (201 data points)
4. NMC $F_2^c/F_2$ (123 data points)
5. $F_2^{(\text{CDHSW})}$ (85 data points)
6. $F_3^{(\text{CDHSW})}$ (96 data points)
7. CCFR $F_2$ (69 data points)
8. CCFR $F_3$ (86 data points)
9. H1 NC e$^-$p (126 data points; 1998-98 reduced cross section)
10. H1 NC e$^-$p (13 data points; high $y$ analysis)
11. H1 NC e$^+$p (115 data points; reduced cross section 1996-97)
12. H1 NC e$^+$p (147 data points; reduced cross section; 1999-00)
13. ZEUS NC e$^-$p (92 data points; 1998-99)
14. ZEUS NC e$^+$p (227 data points; 1996-97)
15. ZEUS NC e$^+$p (90 data points; 1999-00)
16. H1 $F_2^c$ e$^+$p (8 data points; 1996-97)
17. H1 Ro$^b$ for ccbar e$^+$p (10 data points; 1996-97)
18. H1 Ro$^b$ for bbbar e$^+$p (10 data points; 1999-00)
19. ZEUS $F_2^c$ e$^+$p (18 data points; 1996/97)
20. ZEUS $F_2^c$ e$^+$p (27 data points; 1998/00)
21. H1 CC e$^-$p (28 data points; 1998-99)
22. H1 CC e$^+$p (25 data points; 1994-97)
23. H1 CC e$^+$p (28 data points; 1999-00)
24. ZEUS CC e$^-$p (26 data points; 1998-99)
25. ZEUS CC e$^+$p (29 data points; 1994-97)
26. ZEUS CC e$^+$p (30 data points; 1999-00)
27. NuTev neutrino dimuon cross section (38 data points)
28. NuTev anti-neutrino dimuon cross section (33 data points)
29. CCFR neutrino dimuon cross section (40 data points)
30. CCFR anti-neutrino cross section (38 data points)
31. E605 dimuon (199 data points)
32. E866 dimuon (13 data points)
33. Lepton asymmetry from CDF (11 data points)
34. CDF Run 1B jet cross section (33 data points)
35. D0 Run 1B jet cross section (90 data points)

- 2794 data points from DIS, DY, jet production
- All with (correlated) systematic errors that must be treated correctly in the fit
- Note that DIS is the 800 pound gorilla of the global fit with many data points and small statistical and systematic errors
  - and fixed target DIS data still have a significant impact on the global fitting, even with an abundance of HERA data
- To avoid non-perturbative effects, kinematic cuts on placed on the DIS data
  - $Q^2 > 5 \text{ GeV}^2$
  - $W^2 (= m^2 + Q^2 (1-x)/x) > 12.25 \text{ GeV}^2$
Influence of data in global fit

- Charged lepton DIS

\[ F_2(x, Q^2) = x \sum_i e_i^2 [q_i(x, Q^2) + \bar{q}_i(x, Q^2)] \]

- each flavor weighted by its squared charge
- quarks and anti-quarks enter together
- gluon doesn’t enter, in lowest order, but does enter into the structure functions at NLO
- also enters through mixing in evolution equations so gluon contributes to the change of the structure functions as Q^2 increases
- at low values of x

\[ Q^2 \frac{dF_2}{dQ^2} \approx \frac{\alpha_s}{2\pi} \sum_i e_i^2 \int_x^1 dy P_{qg}(y) G(\frac{x}{y}, Q^2) \]

- Q^2 dependence at small x is driven directly by gluon pdf

- At low x, structure functions increase with Q^2; at high x decrease
Influence of data in global fit

**Neutrino DIS**

\[ F_2(x, Q^2) = x \sum_i [q_i(x, Q^2) + \bar{q}_i(x, Q^2)] \]

\[ xF_3(x, Q^2) = x \sum_i [q_i(x, Q^2) - \bar{q}_i(x, Q^2)] \]

- additional structure function allows the separation of quarks and antiquarks but not a complete flavor separation

- caveat: neutrino observables usually obtained using nuclear targets so there is added question of nuclear corrections
Some observations from DIS

- DIS data provide strong constraints on the u and d distributions over the full range of x covered by the data.
- The combination $4\times\bar{u} + \bar{d}$ is well-constrained at small x.
- The gluon is constrained at low values of x by the slope of the $Q^2$ dependence of $F_2$.
  - momentum sum rule connects low x and high x behavior, but loosely.
Inclusive jets and global fits

- We don’t have many handles on the high x gluon distribution in the global pdf fits
- Best handle is provided by the inclusive jet cross section from the Tevatron

At high $E_T$ (high x), gq is subdominant, but there’s a great deal of freedom/uncertainty on the high x gluon distribution

- about 42% of the proton’s momentum is carried by gluons, and most of that momentum is at low x

<table>
<thead>
<tr>
<th>X Bin</th>
<th>Momentum fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^{-1} to 10^{-3}</td>
<td>0.6%</td>
</tr>
<tr>
<td>10^{-3} to 0.01</td>
<td>3%</td>
</tr>
<tr>
<td>0.01 to 0.1</td>
<td>16%</td>
</tr>
<tr>
<td>0.1 to 0.2</td>
<td>10%</td>
</tr>
<tr>
<td>0.2 to 0.3</td>
<td>6%</td>
</tr>
<tr>
<td>0.3 to 0.5</td>
<td>5%</td>
</tr>
<tr>
<td>0.5 to 1.0</td>
<td>1%</td>
</tr>
</tbody>
</table>

TABLE 1. The momentum fraction carried by gluons in a given x bin at a $Q$ value of 5 GeV.

- The inclusion of the CDF/D0 inclusive jet cross sections from Run 1 boosted the high x gluon distribution and thus the predictions for the high $E_T$ jet cross sections

Figure 56. The subprocess contributions to inclusive jet production at the Tevatron for the CTEQ5M and CTEQ6M pdfs. The impact of the larger larger gluon at high x for CTEQ6 is evident.
Global fitting: best fit

- Using our 2794 data points, we do our global fit by performing a $\chi^2$ minimization
  - where $D_i$ are the data points and $T_i$ are the theoretical predictions; we allow for a normalization shift $f_N$ for each experimental data set
    - but we provide a quadratic penalty for any normalization shift
  - where there are $k$ systematic errors $\beta$ for each data point in a particular data set
    - and where we allow the data points to be shifted by the systematic errors with the shifts given by the $s_j$ parameters
    - but we give a quadratic penalty for non-zero values of the shifts $s_j$
  - where $\sigma_i$ is the statistical error for data point $i$

- For each data set, we calculate

$$
\chi^2 = \sum_i \left[ \left( f_N D_i - \sum_{j=1}^k \beta_{ij} s_j \right) - T_i \right]^2 \sigma_i^{-2} + \sum_{j=1}^k s_j^{-2}
$$

- For a set of theory parameters it is possible to analytically solve for the shifts $s_j$, and therefore, continually update them as the fit proceeds

- To make matters more complicated, we may give additional weights to some experiments due to the utility of the data in those experiments (i.e. NA-51), so we adjust the $\chi^2$ to be

$$
\chi^2 = \sum_k w_k \chi^2_k + \sum_k w_{N,k} \left[ \frac{1 - f_N}{\sigma_N^{\text{norm}}} \right]^2
$$

- where $w_k$ is a weight given to the experimental data and $w_{N,k}$ is a weight given to the normalization
Minimization and errors

- Free parameters in the fit are parameters for quark and gluon distributions

\[ f(x) = x^{(a_1 - 1)} (1 - x)^{a_2} e^{a_3 x} [1 + e^{a_4 x}]^{a_5} \]

- Too many parameters to allow all to remain free
  - some are fixed at reasonable values or determined by sum rules

- 20 free parameters for CTEQ6.1, 22 for CTEQ6.6, 24 for CT09

- Result is a global \( \chi^2/\text{dof} \) on the order of 1
  - for a NLO fit
  - worse for a LO fit, since the LO pdf’s cannot make up for the deficiencies in the LO matrix elements
PDF Errors: old way

- Make plots of lots of pdf’s (no matter how old) and take spread as a measure of the error
- Can either underestimate or overestimate the error
- Review sources of uncertainty on pdf’s
  - data set choice
  - kinematic cuts
  - parametrization choices
  - treatment of heavy quarks
  - order of perturbation theory
  - errors on the data
- There are now more sophisticated techniques to deal with at least the errors due to the experimental data uncertainties
PDF Errors: new way

- So we have optimal values (minimum $\chi^2$) for the $d=20$ (22,24) free pdf parameters in the global fit
  - $\{a_\mu\}, \mu=1,\ldots,d$
- Varying any of the free parameters from its optimal value will increase the $\chi^2$
- It’s much easier to work in an orthonormal eigenvector space determined by diagonalizing the Hessian matrix, determined in the fitting process

$$H_{\mu\nu} = \frac{1}{2} \frac{\partial \chi^2}{\partial a_\mu \partial a_\nu}$$

To estimate the error on an observable $X(a)$, due to the experimental uncertainties of the data used in the fit, we use the Master Formula

$$(\Delta X)^2 = \Delta \chi^2 \sum_{\mu,\nu} \frac{\partial X}{\partial a_\mu} \left( H^{-1} \right)_{\mu\nu} \frac{\partial X}{\partial a_\nu}$$
PDF Errors: new way

- Recap: 20 (22,24) eigenvectors with the eigenvalues having a range of $>1\times10^6$
- Largest eigenvalues (low number eigenvectors) correspond to best determined directions; smallest eigenvalues (high number eigenvectors) correspond to worst determined directions
- Easiest to use Master Formula in eigenvector basis

To estimate the error on an observable $X(a)$, from the experimental errors, we use the Master Formula

$$\left(\Delta X\right)^2 = \Delta \chi^2 \sum_{\mu,\nu} \frac{\partial X}{\partial a_\mu} \left(H^{-1}\right)_{\mu,\nu} \frac{\partial X}{\partial a_\nu}$$

where $X_i^+$ and $X_i^-$ are the values for the observable $X$ when traversing a distance corresponding to the tolerance $T(=\sqrt{\Delta \chi^2})$ along the $i$th direction.
PDF Errors: new way

- What is the tolerance $T$?
- This is one of the most controversial questions in global pdf fitting?
- We have 2794 data points in the CTEQ6.6 data set (on order of 2000 for CTEQ6.1)
- Technically speaking, a 1-sigma error corresponds to a tolerance $T(=\sqrt{\Delta\chi^2})=1$
- This results in far too small an uncertainty from the global fit
  - with data from a variety of processes from a variety of experiments from a variety of accelerators
- For CTQE6.1, we chose a $\Delta\chi^2$ of 100 to correspond to a 90% CL limit
  - with an appropriate scaling for the larger data set for CTEQ6.6
- MSTW has chosen a $\Delta\chi^2$ of 50 for the same limit so CTEQ errors will be larger than MSTW errors

\[ \Delta X^+_{\text{max}} = \sqrt{\sum_{i=1}^{N} [\max(X_i^+ - X_0, X_i^- - X_0, 0)]^2}, \]

\[ \Delta X^-_{\text{max}} = \sqrt{\sum_{i=1}^{N} [\max(X_0 - X_i^+, X_0 - X_i^-, 0)]^2}. \]

Figure 29. The pdf errors for the CDF inclusive jet cross section in Run 1 for the 20 different eigenvector directions. The vertical axes show the fractional deviation from the central prediction and the horizontal axes the jet transverse momentum in GeV.
What do the eigenvectors mean?

- Each eigenvector corresponds to a linear combination of all 20 (22,24) pdf parameters, so in general each eigenvector doesn’t mean anything?
- However, with 20 (22,24) dimensions, often eigenvectors will have a large component from a particular direction
- Take eigenvector 1 (for CTEQ6.1); error pdf’s 1 and 2
- It has a large component sensitive to the small $x$ behavior of the $u$ quark valence distribution
- Not surprising since this is the best determined direction
What do the eigenvectors mean?

- Take eigenvector 8 (for CTEQ6.1); error pdf’s 15 and 16
- No particular direction stands out
What do the eigenvectors mean?

- Take eigenvector 15 (for CTEQ6.1); error pdf's 29 and 30
- Probes high x gluon distribution

creates largest uncertainty for high $p_T$ jet cross sections at both the Tevatron and LHC

Figure 29. The pdf errors for the CDF inclusive jet cross section in Run 1 for the 20 different eigenvector directions. The vertical axes show the fractional deviation from the central prediction and the horizontal axes the jet transverse momentum in GeV.
Aside: PDF re-weighting

- Any physical cross section at a hadron-hadron collider depends on the product of the two pdf’s for the partons participating in the collision convoluted with the hard partonic cross section.

- Nominally, if one wants to evaluate the pdf uncertainty for a cross section, this convolution should be carried out 41 times (for CTEQ6.1); once for the central pdf and 40 times for the error pdf’s.

- However, the partonic cross section is not changing, only the product of the pdf’s.

- So one can evaluate the full cross section for one pdf (the central pdf) and then evaluate the pdf uncertainty for a particular cross section by taking the ratio of the product of the pdf’s (the pdf luminosity) for each of the error pdf’s compared to the central pdf’s.

\[
\sigma_{AB} = \int dx_a dx_b \ f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab\rightarrow X}
\]

\[
\frac{\int f^i_{a/A}(x_a, Q^2) f^i_{b/B}(x_b, Q^2)}{\int f^0_{a/A}(x_a, Q^2) f^0_{b/B}(x_b, Q^2)}
\]

- \(f^i\) is the error pdf and \(f^0\) the central pdf.

- This works exactly for fixed order calculations and works well enough (see later) for parton shower Monte Carlo calculations.

- Most experiments now have code to easily do this…

- and many programs will do it for you (MCFM)
A very useful tool

Allows easy calculation and comparison of pdf's
Let's try it out

Up and down quarks dominate at high $x$, gluon at low $x$. As $Q^2$ increases, note the growth of the gluon distribution, and to a lesser extent the sea quark distributions.
Uncertainties

Uncertainties get large at high $x$.

Uncertainty for gluon larger than that for quarks.

pdf's from one group don't necessarily fall into uncertainty band of another.

...would be nice if they did.
Uncertainties and parametrizations

- Beware of extrapolations to x values smaller than data available in the fits, especially at low $Q^2$.
- Parameterization may artificially reduce the apparent size of the uncertainties.
- Compare for example uncertainty for the gluon at low x from the recent neural net global fit to global fits using a parametrization.

$Q^2=2 \text{ GeV}^2$

Note: gluon can range negative at low x.
Correlations

- Consider a cross section $X(a)$
- $i^{th}$ component of gradient of $X$ is
  \[ \frac{\partial X}{\partial a_i} \equiv \partial_i X = \frac{1}{2}(X_{i}^{(+)} - X_{i}^{(-)}) \]
- Now take 2 cross sections $X$ and $Y$
  - or one or both can be pdf's
- Consider the projection of gradients of $X$ and $Y$ onto a circle of radius 1 in the plane of the gradients in the parton parameter space
- The circle maps onto an ellipse in the $XY$ plane
- The angle $\phi$ between the gradients of $X$ and $Y$ is given by
  \[ \cos \phi = \frac{\nabla X \cdot \nabla Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X \Delta Y} \sum_{i=1}^{N} \left( X_{i}^{(+)} - X_{i}^{(-)} \right) \left( Y_{i}^{(+)} - Y_{i}^{(-)} \right) \]
- The ellipse itself is given by
  \[ \left( \frac{\delta X}{\Delta X} \right)^2 + \left( \frac{\delta Y}{\Delta Y} \right)^2 - 2\left( \frac{\delta X}{\Delta X} \right)\left( \frac{\delta Y}{\Delta Y} \right) \cos \varphi = \sin^2 \varphi \]

- If two cross sections/pdf's are very correlated, then $\cos \phi \approx 1$
- …uncorrelated, then $\cos \phi \approx 0$
- …anti-correlated, then $\cos \phi \approx -1$
Correlations

- Consider a cross section \( X(a) \)
- \( i^{th} \) component of gradient of \( X \) is

\[
\frac{\partial X}{\partial a_i} = \partial_i X = \frac{1}{2}(X_i^+ - X_i^-)
\]

- Now take 2 cross sections \( X \) and \( Y \)
  - or one or both can be pdf's
- Consider the projection of gradients of \( X \) and \( Y \) onto a circle of radius 1 in the plane of the gradients in the parton parameter space
- The circle maps onto an ellipse in the \( XY \) plane
- The angle \( \phi \) between the gradients of \( X \) and \( Y \) is given by

\[
\cos \phi = \frac{\vec{\nabla}X \cdot \vec{\nabla}Y}{\Delta X \Delta Y} = \frac{1}{4\Delta X \Delta Y} \sum_{i=1}^{N} \left( X_i^+ - X_i^- \right) \left( Y_i^+ - Y_i^- \right)
\]

- The ellipse itself is given by

\[
\left( \frac{\delta X}{\Delta X} \right)^2 + \left( \frac{\delta Y}{\Delta Y} \right)^2 - 2 \left( \frac{\delta X}{\Delta X} \right) \left( \frac{\delta Y}{\Delta Y} \right) \cos \phi = \sin^2 \phi
\]
Correlations between pdf’s

Correlations between $f(x_1, Q)$ and $f(x_2, Q)$ at $Q = 85$ GeV

Can you guess which PDF’s these are?

Homework assignment: which pdf’s and why?
Correlations between $f_1(x_1, Q)$ and $f_2(x_2, Q)$ at $Q = 85$ GeV

Sometimes there is a clear physics reason behind the correlation (e.g., sum rules or assumed Regge-like behavior); sometimes not
Lecture 3
Tevatron data

- Wealth of data from the Tevatron, both Run 1 and Run 2, that allows us to test/add to our pQCD formalism
- Consider for example W/Z production
  - cross section increases with center-of-mass energy as expected
- We’ve already seen that the data is in reasonable agreement with the theoretical predictions
Rapidity distributions

- Effect of NNLO is basically a normalization shift from NLO
- Data is in good agreement
- May be able to provide some further constraints on future pdf fits
Transverse momentum distributions

- Soft (and hard) gluon effects cause W/Z bosons to be produced at non-zero transverse momentum
- Well-described by ResBos and parton shower Monte Carlos
  - although latter need to have non-perturbative $k_T$ added in by hand
High $p_T$ region is due to hard gluon(s) emission, but is also well-described by predictions such as ResBos.

If we look at average transverse momentum of Drell-Yan pairs as a function of mass, we see that there is an increase that is roughly logarithmic with the mass:

- as expected from the logs that we saw accompanying soft gluon emission.
Inclusive jet production

- This cross section/measurement spans a very wide kinematical range, including the highest transverse momenta (smallest distance scales) of any process.
- Note in the cartoon to the right that in addition to the 2→2 hard scatter that we are interested in, we also have to deal with the collision of the remaining constituents of the proton and anti-proton (the “underlying event”).
- This has to be accounted for/subtracted for any comparisons of data to pQCD predictions.

Figure 43. Schematic cartoon of a 2 → 2 hard-scattering event.

Figure 44. The inclusive jet cross section from CDF in Run 2.
Study of inclusive jet events

- Look at the charged particle transverse momenta in the regions transverse to the dijet direction.
- Label the one with the larger amount of transverse momenta the max direction and the one with the smaller amount the min direction.
- The momenta in the max direction increases with the $p_T$ of the lead jet, while the momenta in the min cone is constant and is approximately equal to that in a minimum bias event.
- “Tunes” to the underlying event model in parton shower Monte Carlos can correctly describe both the max and min regions and can be used for the correct subtraction of UE energy in jet measurements.

Figure 45. Definition of the ‘toward’, ‘away’ and ‘transverse’ regions.

Figure 46. The sum of the transverse momenta of charged particles inside the TransMAX and TransMIN regions, as a function of the transverse momentum of the leading jet.
Corrections

- Hadron to parton level corrections
  - subtract energy from the jet cone due to the underlying event
  - add energy back due to hadronization
  - partons whose trajectories lie inside the jet cone produce hadrons landing outside

...partially cancel, but UE correction is larger for cone of 0.7

Figure 48. Fragmentation and underlying event corrections for the CDF inclusive jet result, for a cone size $R = 0.7$. 
Hadronization corrections

- Can do a back-of-the-envelope calculation with a Field-Feynman-like model
  - and find on the order of 1 GeV/c (see extra slides)
- Or can study a parton shower Monte Carlo with hadronization on/off
  - and find on the order of 1 GeV/c (for a cone of radius 0.7 at the Tevatron)
- NB: hadronization correction for NLO (at most 2 partons in a jet) = the correction for parton showers (many partons in a jet) to the extent that the jet shapes are the same at the NLO and parton shower level

- What is the dependence of the hadronization corrections (also called splashout) on jet transverse momentum?
  - not so much (as Borat might say)
- This may seem surprising (that the correction does not increase with the jet $p_T$)
- But jets get narrower as the $p_T$ increases (see later), so the parton level energy in the outermost annulus of the jet (where the splashout originates) is fairly constant as a function of jet $p_T$
Corrections

- Hadron to parton level corrections
  - subtract energy from the jet cone due to the underlying event
  - add energy back due to hadronization
    ▲ partons whose trajectories lie inside the jet cone produce hadrons landing outside

- Result is in good agreement with NLO pQCD predictions using CTEQ6.1 pdf’s
  - pdf uncertainty is similar to experimental systematic errors

Figure 48. Fragmentation and underlying event corrections for the CDF inclusive jet result, for a cone size $R = 0.7$.

Figure 49. The inclusive jet cross section from CDF in Run 2 compared on a linear scale to NLO theoretical predictions using CTEQ6.1 and MRST2004 pdfs.
Inclusive jet cross section

- New physics tends to be central.
- PDF explanations are universal.
- Crucial to measure over a wide rapidity interval.
Full disclosure for experimentalsists

- Every cross section should be quoted at the hadron level with an explicit correction given between the hadron and parton levels.

---

TABLE IX: Measured inclusive jet cross sections as a function of $p_T$ for jets in the region $0.1 < |y| < 0.7$ together with the statistical (stat.) and systematic (sys.) uncertainties. The bin-by-bin parton-to-hadron-level ($C_{p\rightarrow h}$) corrections are also shown.

<table>
<thead>
<tr>
<th>$p_T$ (GeV/c)</th>
<th>$\sigma \pm (\text{stat.}) \pm (\text{sys.})$ [nb/(GeV/c)]</th>
<th>$C_{p\rightarrow h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>62 – 72</td>
<td>$(6.28 \pm 0.04^{+0.09}_{-0.50}) \times 10^9$</td>
<td>1.072 ± 0.108</td>
</tr>
<tr>
<td>72 – 83</td>
<td>$(2.70 \pm 0.02^{+0.28}_{-0.29}) \times 10^9$</td>
<td>1.055 ± 0.088</td>
</tr>
<tr>
<td>83 – 96</td>
<td>$(1.15 \pm 0.01^{+0.11}_{-0.11}) \times 10^9$</td>
<td>1.041 ± 0.071</td>
</tr>
<tr>
<td>96 – 110</td>
<td>$(4.88 \pm 0.03^{+0.51}_{-0.41}) \times 10^{-1}$</td>
<td>1.030 ± 0.057</td>
</tr>
<tr>
<td>110 – 127</td>
<td>$(2.07 \pm 0.01^{+0.22}_{-0.21}) \times 10^{-1}$</td>
<td>1.022 ± 0.045</td>
</tr>
<tr>
<td>127 – 146</td>
<td>$(8.50 \pm 0.04^{+0.31}_{-0.31}) \times 10^{-2}$</td>
<td>1.015 ± 0.035</td>
</tr>
<tr>
<td>146 – 169</td>
<td>$(3.30 \pm 0.01^{+0.11}_{-0.11}) \times 10^{-2}$</td>
<td>1.010 ± 0.027</td>
</tr>
<tr>
<td>169 – 195</td>
<td>$(1.24 \pm 0.01^{+0.17}_{-0.15}) \times 10^{-2}$</td>
<td>1.006 ± 0.020</td>
</tr>
<tr>
<td>195 – 224</td>
<td>$(4.55 \pm 0.05^{+0.67}_{-0.61}) \times 10^{-3}$</td>
<td>1.003 ± 0.014</td>
</tr>
<tr>
<td>224 – 259</td>
<td>$(1.56 \pm 0.04^{+0.35}_{-0.34}) \times 10^{-3}$</td>
<td>1.002 ± 0.010</td>
</tr>
<tr>
<td>259 – 298</td>
<td>$(4.94 \pm 0.06^{+0.91}_{-0.86}) \times 10^{-4}$</td>
<td>1.001 ± 0.006</td>
</tr>
<tr>
<td>298 – 344</td>
<td>$(1.42 \pm 0.02^{+0.35}_{-0.32}) \times 10^{-4}$</td>
<td>1.000 ± 0.003</td>
</tr>
<tr>
<td>344 – 396</td>
<td>$(3.53 \pm 0.08^{+0.73}_{-0.68}) \times 10^{-5}$</td>
<td>1.001 ± 0.001</td>
</tr>
<tr>
<td>396 – 457</td>
<td>$(6.87 \pm 0.25^{+1.95}_{-1.84}) \times 10^{-6}$</td>
<td>1.001 ± 0.000</td>
</tr>
<tr>
<td>457 – 527</td>
<td>$(1.22 \pm 0.13^{+0.46}_{-0.34}) \times 10^{-6}$</td>
<td>1.003 ± 0.001</td>
</tr>
<tr>
<td>527 – 700</td>
<td>$(7.08 \pm 1.97^{+3.09}_{-2.54}) \times 10^{-8}$</td>
<td>1.005 ± 0.001</td>
</tr>
</tbody>
</table>
Jet Shapes

- Jets get narrower as the jet $p_T$ increases
  - smaller rate of hard gluon emission as $\alpha_s$ decreases
    (can be used to try to determine $\alpha_s$)
  - jets switch from being gluon-induced to quark-induced
Jet Shapes: quark and gluon differences

- Pythia does a good job of describing jet shapes
  - parton showering + hadronization + multiple parton interactions
- If effects of the underlying event are subtracted out, NLO (where a jet is described by at most two partons) also describes the jet shapes well
Quark/gluon shape differences

- Quarks and gluons radiate proportional to their color factors

\[ r \equiv \frac{\langle n_g \rangle}{\langle n_q \rangle} \equiv \frac{\langle \text{gluon jet multiplicity} \rangle}{\langle \text{quark jet multiplicity} \rangle} \]

- At leading order

\[ r = \frac{\langle C_A \rangle}{\langle C_F \rangle} = \frac{9}{4} = 2.25 \]

- With higher order corrections, \( r \sim 1.5 \)
Jet shapes

- Look at the fraction of jet energy in cone of radius 0.7 that is outside the “core” (0.3)
- Gluon jets are always broader than quark jets, but both get narrower with increasing jet $p_T$
- How to correct for the jet energy outside the prescribed cone?
  - a NLO calculation “knows” about the energy outside the cone, so no correction is needed/wanted
  - for LO comparisons, can correct based on Monte Carlo simulations

at small $p_T$, jet production dominated by $gg$ and $gq$ scattering due to large gluon distribution at low $x$
Back to jet algorithms

- For some events, the jet structure is very clear and there's little ambiguity about the assignment of towers to the jet.
- But for other events, there is ambiguity and the jet algorithm must make decisions that impact precision measurements.
- If comparison is to hadron-level Monte Carlo, then hope is that the Monte Carlo will reproduce all of the physics present in the data and influence of jet algorithms can be understood.
  - more difficulty when comparing to parton level calculations.

CDF Run II events
Jets in real life

- Jets don’t consist of 1 fermi partons but have a spatial distribution
- Can approximate this as a Gaussian smearing of the spatial distribution of the parton energy
  - the effective sigma ranges between around 0.1 and 0.3 depending on the parton type (quark or gluon) and on the parton $p_T$
- Note that because of the effects of smearing that
  - the midpoint solution is (almost always) lost
    ▲ thus region II is effectively truncated to the area shown on the right
  - the solution corresponding to the lower energy parton can also be lost
    ▲ resulting in dark towers

Figure 52. A schematic depiction of the effects of smearing on the midpoint cone jet clustering algorithm.

Figure 50. An example of a Monte Carlo inclusive jet event where the midpoint algorithm has left substantial energy unclustered.
Jets in real life

- In NLO theory, can mimic the impact of the truncation of Region II by including a parameter called $R_{\text{sep}}$
  - only merge two partons if they are within $R_{\text{sep}}*R_{\text{cone}}$ of each other
    - $R_{\text{sep}} \approx 1.3$
  - $\approx 4\text{-}5\%$ effect on the theory cross section; effect is smaller with the use of $p_T$ rather than $E_T$ (see extra slides)
  - really upsets the theorists (but there are also disadvantages)
- Dark tower effect is also on order of few ($<5\%$) effect on the (experimental) cross section

Figure 22. The parameter space ($d, z$) for which two partons will be merged into a single jet.
Comparison of $k_T$ and cone results

- Remember
  - at NLO the $k_T$ algorithm corresponds to Region I (for $D=R$); thus at parton level, the cone algorithm is always larger than the $k_T$ algorithm

- Let's check this out with CDF results after applying hadronization corrections

- Nice confirmation of the perturbative picture

![Figure 22](image.png)

The parameter space $(d,Z)$ for which two partons will be merged into a single jet.
**k_T/midpoint ratios for all rapidities**

**FIG. 17:** The ratios of the inclusive jet cross sections measured using the \(k_T\) algorithm with \(D = 0.7\) [9] to those measured using the Midpoint jet finding algorithm with \(R_{\text{cone}} = 0.7\) in this paper (points). The systematic uncertainty on the ratio is given as the yellow band. The predictions from NLO pQCD (solid lines) and PYTHIA (dashed lines) for this ratio are also shown.
The SISCone jet algorithm developed by Salam et al is preferred from a theoretical basis, as there is less IR sensitivity from not requiring any seeds as the starting point of a jet.

So far, at the Tevatron, we have not explicitly measured a jet cross section using the SISCone algorithm, although studies are underway, but we have done some Monte Carlo comparisons for the inclusive cross sections.

Less contribution from UE for SISCone algorithm.

Differences of the order of a few percent at the hadron level reduce to <1% at the parton level.
**New k_T algorithm**

- k_T algorithms are typically slow because speed goes as O(N^3), where N is the number of inputs (towers, particles,…)
- Cacciari and Salam (hep-ph/0512210) have shown that complexity can be reduced and speed increased to O(N) by using information relating to geometric nearest neighbors
  - should be useful for LHC
  - already implemented in ATLAS and CMS
- Optimum is if analyses at LHC use **both** cone and k_T algorithms for jet-finding
  - universal benchmark
  - need experience now from the Tevatron

\[\sim 10000 \text{ particles}\]

Clustering takes \(\sim 20\) minutes with old methods.
0.6s with FastJet.