Summary of the basic things covered in the lecture (Phy 410) Feb 18, 2009

1. Microstates and Macrostates (*N* spins and *N* quantum harmonic oscillators)
2. Multiplicity of a given macrostate (*g(N,s)* or *g(N,U)*, or *g(N,M)* for magnets, *g(N,n)* or *g(N,ε)* for oscillators where *U* is the energy, *M* is total magnetic moment).

Spins or magnets with 2 states/magnet

$$g(N,s) = \frac{N!}{(\frac{N}{2}+s)!(\frac{N}{2}-s)!}$$

$$\sum_s g(N,s) = 2^N$$

$$M = 2sm,\ U = -MB = -2smB$$

N quantum harmonic oscillators, each with same frequency ω, energy $\varepsilon_s = s\hbar\omega$. Multiplicity factor of the macrostate with energy $\varepsilon = n\hbar\omega$

$$g(N,n) = \frac{(N-1+n)!}{(N-1)!n!}$$

3. Multiplicity of two systems (N_1 and N_2) in thermal contact with total energy *U* fixed.

$$g(N,U) = \sum_{U_1,U_2;U_1+U_2=U} g_1(N_1,U_1)\bullet g_2(N_2,U_2)$$

For spin systems (2s is the spin excess etc)

$$g(N,s) = \sum_{s_1,s_2;s_1+s_2=s} g_1(N_1,s_1)\bullet g_2(N_2,s_2)$$

4. Concept of entropy, thermal equilibrium and temperature (for system with fixed energy, closed system)

$$\sigma(N,U) = \ln g(N,U)$$

$$\frac{1}{\tau} = \left(\frac{\partial\sigma}{\partial U}\right)_N$$

Equilibrium corresponds to the most probable distribution when the two systems have the same temperature. This corresponds to a particular division of energy between the two systems, $\overline{U}_1$ *and* $\overline{U}_2 = U - \overline{U}_1$.

$$\frac{1}{\tau_1} = \left(\frac{\partial\sigma_1}{\partial U_1}\right)_{N_1,U_1=\overline{U}_1} = \left(\frac{\partial\sigma_2}{\partial U_2}\right)_{N_2,U_2=\overline{U}_2} = \frac{1}{\tau_2}$$

At equilibrium

$$g(N,U) = g_1(N_1,\overline{U}_1)\bullet g_2(N_2,\overline{U}_2)$$

$$\sigma(N,U = \sigma_1(N_1,\overline{U}_1) + \sigma_1(N_2,\overline{U}_2)$$

For two independent systems Multiplicity Factor is a PRODUCT of multiplicity factor for each component. IN CONTRAST Entropy is additive.

5. For thermally open systems (system in equilibrium with a reservoir at temperature τ), probability of a given microstate s with energy ε_s is given by

$$P(\varepsilon_s) = \frac{e^{-\varepsilon_s/\tau}}{Z};$$
$$Z = \sum_s e^{-\varepsilon_s/\tau}; \textit{ Partition function}$$

Two non interacting systems *A* and *B* the partition function $Z_{A+B} = Z_A \bullet Z_B$

For N non interacting systems (spins, osillators, particles etc) $Z_N = Z_1 \bullet Z_1 \bullet Z_1 \bullet Z_1 .... = (Z_1)^N$. Z_1 will also depend on the physical system. Also for indistinguishable particles we need to divide Z_N by N!

6. Helmholtz free energy $F = U - \tau\sigma = -\tau \ln Z$

$$U = \sum_s \varepsilon_s P(\varepsilon_s) = \tau^2 \frac{\partial}{\partial \tau} \ln Z$$
$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N}$$

For two non-interacting systems *A* and *B*, $F_{A+B} = F_A + F_B$

7. Thermodynamic identities (when *N* is constant)

$$\tau\, d\sigma = dU + p\, dV$$
$$dF = -\sigma\, d\tau - p\, dV$$

You can use these identities to calculate temperature, entropy and pressure under various types of external constraints.

8. Maxwell velocity ($\vec{v}$) and speed (v) distribution

$$P(\vec{v}) = \left(\frac{M}{2\pi\tau}\right)^{3/2} e^{-\frac{M\vec{v}^2}{2\tau}}; \; D(v) = \left(\frac{M}{2\pi\tau}\right)^{3/2} 4\pi v^2 e^{-\frac{Mv^2}{2\tau}}$$

9. Equipartition principle in classical systems

10. Thermal properties of black body radiation (Quantum Harmonic Oscillator model). Average total energy *U/V* and energy density u_ω as a function of the radiation frequency ω.