Summary of the basic things covered in the lecture (Phy 410) April 3, 2009

1. Planck Distribution function (photon gas)
   Thermal average number of photons and average energy in a single mode of frequency ω and energy $\hbar\omega$ are given by
   $$<s>=\frac{1}{e^{\hbar\omega/\tau}-1}; <\varepsilon>=\frac{\hbar\omega}{e^{\hbar\omega/\tau}-1}$$

2. Counting and summing of modes in a cavity (cube of volume V=L³);
   $\vec{n}=(n_x,n_y,n_z)$

3. The frequency of EM waves in a cavity $\omega_{\bar{n}}=\frac{\pi}{L}cn; n=\sqrt{n_x^2+n_y^2+n_z^2}$

4. Review how you will do calculations for 2 and 1 dimensional photon gas.

5. For photon gas review energy density U/V, spectral density u_ω, entropy, and energy flux density $J_U=\sigma_B T^4$ (Note it is T not τ).

6. Phonons in a solid, Debye law for heat capacity at low T, concept of Debye temperature.
   $$U(\tau)\approx\frac{3\pi^4 N\tau^4}{5(k_B\theta)^3} \text{ for } \tau<<k_B\theta \text{ or } T<<\theta$$
   $$C_V=\frac{12\pi^4 N}{5}\left(\frac{\tau}{k_B\theta}\right)^3; \tau^3 \text{ or } T^3 \text{ law}$$

   In 2-dimensions it is T^2 *law* and in 1-dimension it is T *law*.

7. Chemical potential (Chapter 5)
   $$\mu(\tau,V,N)=\left(\frac{\partial F}{\partial N}\right)_{\tau,V}$$
   For classical ideal 3-dimension gas (no internal degrees of freedom)
   $$\mu=\mu_{ext}=\tau\ln(n/n_Q); n_Q=\left(\frac{M\tau}{2\pi\hbar^2}\right)^{3/2}$$
   When there is internal degrees of freedom
   $$\mu_{tot}=\mu_{ext}+\mu_{int}$$
   For diffusive equilibrium the total chemical potential must be constant.

8. Gibbs factor, Gibbs sum, Grand Sum, Grand Partition function, average number of particles in systems where the particle number can fluctuate.
   $$P(N,\varepsilon_{s(N)})=\frac{e^{[N\mu-\varepsilon_{s(N)}]/\tau}}{\widetilde{Z}}$$
   $$\widetilde{Z}=\sum_N\sum_{s(N)}e^{[N\mu-\varepsilon_{s(N)}]/\tau}=\sum_N\lambda^N\sum_{s(N)}e^{-\varepsilon_{s(N)}/\tau}=\sum_N\lambda^N Z(N,\tau,V)$$

   Average number of particles <N>

$$\tilde{Z} = \sum_N N \sum_{s(N)} e^{[N\mu - \varepsilon_{s(N)}]}$$

$$<N> = \frac{\sum_N N \sum_{s(N)} e^{[N\mu - \varepsilon_{s(N)}]}}{\tilde{Z}} = \tau \frac{\partial \ln \tilde{Z}}{\partial \mu} = \lambda \frac{\partial}{\partial \lambda} \ln \tilde{Z}$$

where absolute activity λ is related to the hemical potential μ and τ

i.e $\lambda = e^{\mu/\tau}$

9. Chapter 6 (Ideal Gas, Fermions, Bosons, Classical regime)

   Fermi Dirac distribution: $f_{FD}(\varepsilon) = \dfrac{1}{e^{(\varepsilon - \mu)/\tau} + 1}$

   Bose Einstein distribution: $f_{BE}(\varepsilon) = \dfrac{1}{e^{(\varepsilon - \mu)/\tau} - 1}$

   For very low densities $n << n_Q$ OR $v >> \lambda_{th}^3$

   $$f_{FD}(\varepsilon) = f_{BE}(\varepsilon) = e^{-(\varepsilon - \mu)/\tau} = \lambda e^{-\varepsilon/\tau}$$

10. Classical ideal gas thermal properties starting from the chemical potential as a function of $N = <N>, V, \tau$

11. Chapter 7 (pages 183-185) Fermions at $\tau = 0$

    $\mu(\tau = 0) = \varepsilon_F$; *Fermi energy*

    $f_{FD}(\varepsilon) = 1$ *for $\varepsilon \le \varepsilon_F$*

    $\qquad = 0$ *for $\varepsilon > \varepsilon_F$*

    $$\varepsilon_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

    $U_0 = \frac{3}{5} N\varepsilon_F$

    $$p_0 = \frac{2}{5}\left(\frac{N}{V}\right)\varepsilon_F$$

12. Thermodynamic identity and use it to calculate changes in enthalpy H and Helmholtz free energy F

    $$\tau\, d\sigma = dU + p\, dV - \mu\, dN$$

    $$H = U + pV; dH = \tau\, d\sigma + V\, dp + \mu\, dN; H(\sigma, p, N)$$

    $$F = U - \tau\sigma; dF = -\sigma\, d\tau - p\, dV + \mu\, dN; F(\tau, V, N)$$