Summary of the basic things for the Final Exam (Phy 410)
May 4, 2009
The final exam will be about 75-80* from Chapters 6-9 and 25-20% from the earlier chapters.

1. Counting and summing of modes in 3, 2, and 1 dimensions.
$\vec{n} = (n_x, n_y, n_z)$, $\vec{n} = (n_x, n_y)$, $\vec{n} = (n_x)$
2. Momentum $\vec{p} = \hbar\left(\frac{\pi}{L}\right)\vec{n}$. Energy of photons $\varepsilon_{\vec{p}} = cp$. Energy of nonrelativistic particles $\varepsilon_{\vec{p}} = \frac{p^2}{2m}$, relativistic particles $\varepsilon_{\vec{p}} = \sqrt{m^2c^4 + p^2c^2}$

3. Ideal Gas of Fermions and Bosons, Classical regime
   Fermi Dirac distribution: $f_{FD}(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/\tau} + 1}$
   Bose Einstein distribution: $f_{BE}(\varepsilon) = \frac{1}{e^{(\varepsilon-\mu)/\tau} - 1}$
   For very low densities $n << n_Q \ OR \ v >> \lambda_{th}^3$
   $f_{FD}(\varepsilon) = f_{BE}(\varepsilon) = e^{-(\varepsilon-\mu)/\tau} = \lambda e^{-\varepsilon/\tau}$

4. Fermions at $\tau = 0$ (different dimensions), Concept of density of orbitals (or states)
   $\mu(\tau = 0) = \varepsilon_F;\ Fermi\ energy$
   $$f_{FD}(\varepsilon) = 1 \ for \ \varepsilon \le \varepsilon_F$$
   $$= 0 \ for \ \varepsilon > \varepsilon_F$$
   $$N = \int_0^\infty D(\varepsilon)\, f_{FD}(\varepsilon)\, d\varepsilon$$
   $$U = \int_0^\infty D(\varepsilon)\, \varepsilon\, f_{FD}(\varepsilon)\, d\varepsilon$$
   In 3,2,1 di mensions
   $$D_3(\varepsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}$$
   $$D_2(\varepsilon) = \frac{A}{2\pi}\left(\frac{2m}{\hbar^2}\right)$$
   $$D_1(\varepsilon) = \frac{L}{\pi}\left(\frac{2m}{\hbar^2}\right)^{1'2} \varepsilon^{-1/2}$$

5. Planck Distribution function (photon gas)

Thermal average number of photons and average energy in a single mode of frequency ω and energy $\hbar\omega$ are given by

$$< s >= \frac{1}{e^{\hbar\omega/\tau} - 1}; < \varepsilon >= \frac{\hbar\omega}{e^{\hbar\omega/\tau} - 1}$$

6. Bose Einstein Condensation Temperature(τ_E) is the temperature when the chemical potential becomes zero as one comes down from very high temperature.

$$N = \int_0^\infty D(\varepsilon) \frac{1}{e^{(\varepsilon-\mu)/\tau} - 1}; \mu \le 0.$$

$$U = \int_0^\infty D(\varepsilon)\, \varepsilon \frac{1}{e^{(\varepsilon-\mu)/\tau} - 1}; \mu \le 0.$$

7. Heat Capacity and entropy of Bose gas below the condensation temperature

8. Heat engines (reversible and irreversible), efficiency η; refrigerators and heat pumps, coefficient of performance, COP γ. Engines in tandem.

9. Work done (on or by) a gas (ideal gas of finite mass particles, photon gas) during isothermal and adiabatic processes.

10. Effective work (chemical or electrical) and its relationship with changes in the Gibbs free energy, $dW' = dG$. Physics/Chemistry of electrolysis and heat pump.

11. Boltzman distribution $p(\varepsilon_s) \propto e^{-\varepsilon_s/\tau}$;

12. Gibb's distribution $p(N, \varepsilon_{s(N)}) \propto e^{-(\varepsilon_{s(N)} - \mu N)/\tau} = \lambda^N e^{-(\varepsilon_{s(N)})/\tau}$

13. Partition function (when N is fixed) $Z = \sum_s e^{-\varepsilon_s/\tau}$; $F = -\tau \ln Z$

$$< E >= U = \frac{\sum_s \varepsilon_s e^{-\varepsilon_s/\tau}}{\sum_s e^{-\varepsilon_s/\tau}} \tau^2 \left(\frac{\partial \ln Z}{\partial \tau} \right)$$

14. Gibbs sum $\overline{Z} = \sum_N \sum_{s(N)} \lambda^N e^{-(\varepsilon_{s(N)})/\tau}$;

Calculate <N> from $\overline{Z}$; $< N >= \lambda \left(\frac{\partial \ln \overline{Z}}{\partial \lambda} \right)_{\tau,V}$

15. For black body radiation (in a 3-dimensional cavity)

$$\frac{U}{V} = \frac{\pi^2}{15\hbar^3 c^3}\tau^4$$

$$p = \frac{1}{3}\frac{U}{V}$$

16. Thermodynamic identitites and use it to calculate changes in enthalpy *H* and Helmholtz free energy *F, G*, and other physical quantities.

$$\tau\, d\sigma = dU + p\, dV - \mu\, dN$$

$$H = U + pV; dH = \tau\, d\sigma + V\, dp + \mu\, dN; H(\sigma, p, N)$$

$$F = U - \tau\sigma; dF = -\sigma\, d\tau - p\, dV + \mu\, dN; F(\tau, V, N)$$

$$G = U + pV - \tau\sigma;\ dG = -\sigma\, d\tau + Vdp + \mu\, dN; G(\tau, p, N) = \mu(\tau, p)N$$

$$\text{The Chemical potential} \quad \mu(\tau, P) = G(\tau, p, N)$$

Use these equations to obtain different physical quantities from different free energies under different external (control) constraints.

$$\text{Entropy at constant } V \text{ and } N: \sigma(\tau, V, N) = -\left(\frac{\partial F}{\partial \tau}\right)_{V,N}$$

$$\text{Entropy at constant } p \text{ and } N: \quad \sigma(\tau, p, N) = -\left(\frac{\partial G}{\partial \tau}\right)_{p,N}$$