# PHY 410

Final Examination, Spring 2008

April 30, 2008 (5:45-7:45 p.m.)

**PLEASE WAIT UNTIL YOU ARE TOLD TO BEGIN THE EXAM.**
While waiting, carefully fill in the information requested below

Your Name:________________________________________________

Your Student Number:______________________________________

**DO NOT TURN THIS PAGE UNTIL THE EXAM STARTS**

**USEFUL CONSTANTS AND INTEGRALS**

Avogadro's Number $N_A = 6.022 \times 10^{23}$
Boltzmann's constant $k = 1.381 \times 10^{-23}$ J/K
$= 8.617 \times 10^{-5}$ eV/K
Gas constant $R = kN_A = 8.31$ J/mol.K
Planck constant $h = 6.626 \times 10^{-34}$ J.s
Electron charge (magnitude) $e = 1.602 \times 10^{-19}$ C
Electron mass $m = 9.109 \times 10^{-31}$ kg
Speed of light $=2.998 \times 10^{8}$ m/s

1 atm = 1.013 bar
1 bar $= 10^{5}$ N/m$^{2}$
1 eV $= 1.602 \times 10^{-19}$ J

This exam is worth 75 points. Each problem on the exam has its point value listed. Take a moment and look over the exam before you begin. When problem 7 is counted, the maximum number of points is 85. If you do it, it will add to the total. However, maximum score is capped at 75 points.

To receive full credit for each answer***, you must work neatly, show your work and simplify your answer to the extent possible.***

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1. Consider a refrigerator operating between a cold temperature $T_c$ and hot temperature $T_h$. Amount of heat $Q_c$ is extracted at $T_c$ and $Q_h$ is rejected at $T_h$. The work done by the power source is $W$. (10 points)

a) What is the coefficient of performance of an ideal refrigerator?

b) If $T_c$=100K, $T_h$=300K, and 1 kW of power is supplied by the power source how much heat is extracted from the cold source in 1 sec for an ideal refrigerator?

2. Consider an ideal 3-dimensional gas of electrons of density $N/V = 10^{27}$ $m^{-3}$. (15 points)

   a) What is the fermi momentum?

   b) The electrons are extremely relativistic and the energy of an electron with momentum $\vec{p}$ is given by $\varepsilon_{\vec{p}} = cp$, where $c$ is the speed of light. What is the fermi energy? Express your answer in electron volts.

   c) What is the energy per electron at $T$=0 in eV.

3. 1 mole of water at 50°C is added to 1 mole of water at 10°C. The heat capacity of water $C_P$ is 75.29 J/Kmole and assume it to be a constant.(10 points)

   a) What is the final temperature?

   b) What is the net change in entropy in this mixing process? Is it positive, negative or zero?

4. Consider an atom with spin *S*=1 in a crystalline solid at temperature *T*. The three microstates of the atom are given by $m = 1,0,-1$. In the presence of both crystalline field and an external magnetic field *B,* the energy levels are given by $E(m) = \Delta m^2 - \mu m B$, where $\Delta$ is a constant energy depending on the crystal, and $\mu m$ is the magnetic moment associated with the state $m$. (15 points)

   a) Write down the partition function.

   b) What is the average magnetic moment *M* of the atom? Express your answer as a function of $\mu, B, \Delta, T$.

   c) When the magnetic field is very small, show that $M$ reduces to the form $M = \chi B$, where $\chi$ is called the magnetic susceptibility. Find $\chi$.

   d) What happens to $\chi$ when the crystalline field is turned off. Does it increase or decrease?

5. For blackbody radiation at temperature $T$ contained in volume $V$, the spectrum (energy density per unit photon energy) $u(\varepsilon) = A\frac{\varepsilon^3}{e^{\beta\varepsilon} - 1}, A = \frac{8\pi}{(ch)^3}, \beta = \frac{1}{kT}$ .(15 points)

   a) Plot qualitatively $u(\varepsilon)$ as a function of the photon energy $\varepsilon$, focusing on the small and large $\varepsilon$ regions.

   b) Show that the total energy density is given by $\frac{U}{V} = \alpha T^4$ and calculate $\alpha$.

   (Use $\int_0^\infty \frac{x^3}{e^x - 1} dx = \frac{\pi^4}{15}$)

   c) What is the heat capacity per unit volume?

   d) How does the entropy of the blackbody radiation change with temperature and volume?

6. Consider a one-dimensional quantum harmonic oscillator (QHO), modeling one of the 3N vibrational modes of a solid, with frequency $\nu$ ( 1 energy quantum $\varepsilon = h\nu$). (10 points)

   a) What are the energy levels of the QHO?

   b) What is the average energy *U* of the QHO as a function of $\varepsilon$ and *T*?

   c) If $\varepsilon/kT = 0.1$, what is the approximate value of *U*? Compare your result with that of a classical harmonic oscillator at the same temperature and explain your result physically.

7. (Optional Problem) A solid has two structures, 1 and 2. At $T$=0, structure 1 has larger energy than structure 2 by 50 kJ/mole. Entropy of both the structures increase linearly with $T$ starting from 0; i.e $S=AT$. $A_1$=200 J/K$^2$ and $A_2$=150 J/K$^2$. Assuming that there is no volume difference between these two structures what is the temperature when there is a phase transition between the two. Which structure is stable above this temperature?(10 points)