PHY 410 – Spring 2008 Exam #3

(1 Hour)

PLEASE WAIT UNTIL YOU ARE TOLD TO BEGIN THE EXAM

While waiting, carefully fill in the information requested below

Your Name:……………………………………………………………….

Your Student Number:…………………………………………………

There are 4 problems. Please answer them all.

USEFUL CONSTANTS AND INTEGRALS

Avogadro's Number N_A= 6.022 x 10^{23}
Boltzmann's constant k = 1.381 x 10^{-23} J/K
= 8.617 x 10^{-5} eV/K
1 eV=1.602 x 10^{-19} J
Gas constant R = kN_A = 8.31 J/mol.K
Planck constant h = 6.626 x 10^{-34} J.s
Electron charge (magnitude) e = 1.602 x 10^{-19} C
Electron mass m = 9.109 x 10^{-31} kg

1 atm = 1.013 bar
1 bar = 10^5 N/m^2
1 eV = 1.602 x 10^{-19} J

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}; \int_0^\infty x^{2n+1} e^{-bx^2} dx = \frac{n!}{2b^{n+1}};$$

$$\int_0^\infty e^{-bx^2} dx = \frac{\sqrt{\pi}}{2b^{1/2}}; \int_0^\infty x^2 e^{-bx^2} dx = \frac{\sqrt{\pi}}{4b^{3/2}}; \int_0^\infty x^4 e^{-bx^2} dx = \frac{3\sqrt{\pi}}{8b^{5/2}}$$

Problem 1 (10 points)

(a) Diamond converts to graphite under high pressure. At 300K this pressure is 15 kbar. If the temperature is increased to 305K the pressure increases to 15.09 kbar. The molar volume increases during this conversion by1.9×10^{-6} m^3. What is the change in molar entropy in going from diamond to graphite? (5 points)

(b) Again look at the diamond to graphite phase transition as a function of pressure at 300K. At p~0, molar Gibbs free energy of diamond is 2.9 kJ higher than that of graphite. Molar volumes of these two are 3.42×10^{-6} m^3 and 5.31×10^{-6} m^3 respectively. If both are incompressible, what is the difference in their Gibbs free energy at 30 kbar? Which one is lower? (5 points)

Problem 2 (15 points)

A simple model to study thermal properties of a magnetic insulator is as follows. It consists of N noninteracting localized magnets. Each magnet can have 6 possible states. The ground state is four-fold degenerate with energy 0. The excited state is 2-fold degenerate with energy ε. The temperature is T.

(a) What is the probability of finding one magnet in any one of the two degenerate excited states? Express your answer in terms of ε and T.
(b) What is the probability of finding one magnet with energy ε?
(c) What is the average energy of the system of N magnets?
(d) What is the heat capacity/magnet C? Draw a qualitative figure of C vs T and discuss how C changes with T as $T \to 0$ and $T \to \infty$.

Problem 3 (10 points)

Argon atoms (mass m) are adsorbed onto a graphite surface and move on the surface as a two-dimensional (2d) ideal gas. They are in equilibrium at temperature T. Their speed distribution is given by Maxwell distribution function (in 2d). (See front page for different integrals).

$$D(v) = C2\pi v\, e^{-\frac{mv^2}{2kT}}$$

(a) What is the average speed?

(b) What is the most probable speed?

(c) What is the root mean square speed as defined by $v_{rms}^2 = \langle v^2 \rangle$? Does it satisfythe equipartition theorem-explain?

Problem 4 (15 points)

(a) Consider a system of 4 quantum particles each of which can occupy single particle states with energies 0, ε, 2ε, 3ε, 4ε, …... (6 points)

(i) What is the ground state energy if the system if they are spinless fermions?

(ii) Suppose the total energy of the 4 fermion system is 9ε. What are the possible microstates associated with this energy?

(iii) Suppose the total energy of the 4 boson system is 3ε. What are the possible microstates associated with this energy?

(b) Consider conduction electrons in aluminium metal. They can be approximated as an ideal gas of fermions (spin ½) moving in 3-dimension. Each Al atom gives 3 conduction electrons and the volume per Al atom is 16.6×10^{-30} m^3. (9 points)

(i) What is the energy of the highest occupied single particle state at 0K?

(ii) What is the energy/electron at 0K?

(iii) What is the pressure of this fermi gas at 0K? What is the physical origin of this pressure?