PHY410

HW #1

1.1

(a) $2^4 = 16$ possible microstates

$$(\uparrow_1 + \downarrow_1)(\uparrow_2 + \downarrow_2)(\uparrow_3 + \downarrow_3)(\uparrow_4 + \downarrow_4) = \uparrow_1\uparrow_2\uparrow_3\uparrow_4 + \uparrow_1\uparrow_2\uparrow_3\downarrow_4 + \uparrow_1\uparrow_2\downarrow_3\uparrow_4$$

$$+ \uparrow_1\downarrow_2\uparrow_3\uparrow_4 + \downarrow_1\uparrow_2\uparrow_3\uparrow_4 + \uparrow_1\uparrow_2\downarrow_3\downarrow_4 + \uparrow_1\downarrow_2\uparrow_3\downarrow_4 + \downarrow_1\uparrow_2\uparrow_3\downarrow_4$$

$$+ \uparrow_1\downarrow_2\downarrow_3\uparrow_4 + \downarrow_1\uparrow_2\downarrow_3\uparrow_4 + \downarrow_1\downarrow_2\uparrow_3\uparrow_4 + \uparrow_1\downarrow_2\downarrow_3\downarrow_4 + \downarrow_1\uparrow_2\downarrow_3\downarrow_4 + \downarrow_1\downarrow_2$$

$$+ \downarrow_1\downarrow_2\downarrow_3\uparrow_4 + \downarrow_1\downarrow_2\downarrow_3\downarrow_4$$

(b)

Macrostates (N,s)		Probility
4 magnets up ($s=2$)	: $\uparrow_1\uparrow_2\uparrow_3\uparrow_4$	$\frac{1}{16}$
3 magnets up ($s=1$)	: $\uparrow_1\uparrow_2\uparrow_3\downarrow_4$ $\uparrow_1\uparrow_2\downarrow_3\uparrow_4$ $\downarrow_1\uparrow_2\uparrow_3\uparrow_4$ $\uparrow_1\downarrow_2\uparrow_3\uparrow_4$	$\frac{4}{16}$
2 magnets up ($s=0$)	$\uparrow_1\uparrow_2\downarrow_3\downarrow_4$ $\uparrow_1\downarrow_2\uparrow_3\downarrow_4$ $\downarrow_1\uparrow_2\uparrow_3\downarrow_4$ $\uparrow_1\downarrow_2\downarrow_3\uparrow_4$ $\downarrow_1\uparrow_2\downarrow_3\uparrow_4$ $\downarrow_1\downarrow_2\uparrow_3\uparrow_4$	$\frac{6}{16}$
1 magnets up ($s=-1$)	$\uparrow_1\downarrow_2\downarrow_3\downarrow_4$ $\downarrow_1\uparrow_2\downarrow_3\downarrow_4$ $\downarrow_1\downarrow_2\uparrow_3\downarrow_4$ $\downarrow_1\downarrow_2\downarrow_3\uparrow_4$	$\frac{4}{16}$
0 magnets up ($s=-2$)	$\downarrow_1\downarrow_2\downarrow_3\downarrow_4$	$\frac{1}{16}$

(c)

$\underline{s=2}$, $g(4,2) = \dfrac{4!}{(2+2)!\,(2-2)!} = 1$ ✓

$\underline{s=1}$ $g(4,1) = \dfrac{4!}{(2+1)!\,(2-1)!} = 4$ ✓

$\underline{s=0}$ $g(4,0) = \dfrac{4!}{(2+0)!\,(2-0)!} = 6$ ✓

$\underline{s=-1}$ $g(4,-1) = \dfrac{4!}{(2-1)!\,(2+1)!} = 4$ ✓

$\underline{s=-2}$ $g(4,-2) = \dfrac{4!}{(2-2)!\,(2+2)!} = 1$ ✓

$\boxed{1.2}$

(a) possible outcomes : $2^N = 2^{20} = 1\,048\,576$

(b) probability to get exactly that order :

$$P = \frac{1}{2^{20}} = 2^{-20} \simeq 9.537 \cdot 10^{-7}$$

(c) 12 heads 8 ~~8 tails~~ : $N_\uparrow - N_\downarrow = 12-8 = 4 = 2s \Rightarrow s = 2$

the probability $P = \dfrac{g(20,2)}{2^{20}} = \dfrac{20!}{(10+2)!\,(10-2)!\,2^{20}}$

$$P \simeq 0.12$$

1.3

$\underline{N=4}$

- Stirling's approximation $N! \approx N^N e^{-N} \sqrt{2\pi N}$

$$\delta = \frac{N! - \left(N^N e^{-N}\sqrt{2\pi N}\right)}{N!} = \frac{40! - \left(40^{40} e^{-40}\sqrt{2\pi \cdot 40}\right)}{40!} = 0.208\,\%$$

- approximation $\log N! \cong \frac{1}{2}\log 2\pi + \left(N+\frac{1}{2}\right)\log N - N$

$$\delta = \frac{\log N! - \left(\frac{1}{2}\log 2\pi + \left(N+\frac{1}{2}\right)\log N - N\right)}{\log N!} \qquad (N = 40)$$

$$= 0.0019\,\%$$

1.4

(a) Half dipoles point "up" and half point "down"

$$\Leftrightarrow \quad N_\uparrow - N_\downarrow = 0 = 2s \Rightarrow s = 0$$

Number of microstates is $g(N,0)$

$$g(N,0) = \left(\frac{2}{\pi N}\right)^{1/2} 2^N = \left(\frac{2}{\pi 10^{23}}\right)^{1/2} 2^{10^{23}}$$

$$= \sqrt{\frac{1}{5\pi}} \cdot 10^{-11} \cdot 2^{10^{23}}$$

$$= 0.252 \times 10^{-11} \times 2^{10^{23}}$$

$$\approx 2^{-39} \cdot 2^{10^{23}}$$

$$\approx \boxed{2^{10^{23}}} \quad \left(10^{23} \gg 39\right)$$

(b)

1 second $\longrightarrow$ 10^9 changes

10×10^9 years $= 10\times10^9\times365\times24\times60\times60$ seconds $= 3.15\times10^{17}$

Then the number of microstates will be explored :

$$10^9\times3.15\times10^{17} = \boxed{3.15\times10^{26}} \text{ (states)}$$

still tiny compared to $g(N,0) = 2^{10^{23}}$ states