PHY410
HW#1

Assigned 12 Jan 09: Due 21 Jan 09

Please do all the problems (Total points 35)

1.1 A system consists of 4 elementary magnets at fixed sites on a line (see Fig 1.3)

(a) Make a list of all the possible microstates (2 points)

(b) Make a list of all the different “macrostates” (N,s) and their probabilities (4 points)

(c) Compute the multiplicity of each macrostate g(N,s) using the combinatorial formula g(N,s) = $\frac{N!}{\left(\frac{N}{2}+s\right)!\left(\frac{N}{2}-s\right)!}$ and check that these results agree with what you got by brute-force counting. (4 points)

1,2 Now instead of system of magnets consider a system of N fair coins. Head (H) corresponds to “up” and tail (T) corresponds to “down”. Suppose you flip 20 (N=20) fair coins.

(a) How many possible outcomes (microstates) are there? (2 points)

(b) What is the probability of getting the sequence (HTHHTTTHTHHHTHHHHTHT) (in exactly that order) (2 points)

(c) What is the probability of getting 12 heads and 8 tails (in any order)? (4 points)

1.3 Use a pocket calculator to check the accuracy of Stirling’s approximation ($N! \approx N^N e^{-N}\sqrt{2\pi N}$) for N=40. Also check the accuracy of the approximation $log N! \cong \frac{1}{2} log 2\pi + (N+\frac{1}{2}) log N - N$. (10 points)

1.4 Consider a real physical system: a two-state paramagnetic system (I will explain why I call this a paramagnetic system later) with 10^{23} elementary magnetic dipoles, with the total energy fixed at zero so that exactly half the dipoles point “up” and half point “down”.

(a) How many microstates are “accessible” to this system? (Use Eq. 36 of Chapter 1) (3 points)

(b) Suppose that the microstate of this system changes a billion times per second. How many microstates will it explore during the age of the universe (about 10 billion years)?(4 points)