PHY 410
HW#2
Assigned 21 Jan 09: Due 26 Jan 09

2.1 Start from the multiplicity function $g(N,s)$ for a N two-state magnet system with spin excess 2s given by

$$g(N,s) = \frac{N!}{(N/2+s)!(N/2-s)!}.$$

Use the Stirling's approximation $\ln N! \approx \frac{1}{2}\ln 2\pi + (N+\frac{1}{2})\ln N - N$ to show that

$$\ln g(N,s) \approx \tfrac{1}{2}\ln(\tfrac{2}{\pi N}) + N\ln 2 - 2s^2/N \qquad (8\,points)$$

$$and\;\; g(N,s) \approx g(N,0)\,e^{-2s^2/N};\, g(N,0) = \sqrt{\tfrac{2}{\pi N}}\,2^N\;(2\,points)$$

2.2 Consider a system consisting of 3 quantum harmonic oscillators (frequency ω) with total number of energy quanta $n = 2$.
Write down all the microstates $(N; s_1, s_2, s_3)$ where s_i is the energy quanta of the ith oscillator. (8 points)
Check that the total number of microstates corresponding to the macrostate $(3,2)$ agree with the multiplicity factor $g(N,n)$ for N oscillators with n quanta. (2 points)

2.3 Consider two binary systems (magnets) $\mathbb{S}1$ and $\mathbb{S}2$. N_1=2 and N_2=2. The spin excess for each system is respectively s_1 and s_2. Now look at the total system as consisting of 4 magnets, $N=N_1+N_2=4$ and spin excess as $2s=2s_1+2s_2$. Check by explicit calculation that the multiplicity of the macrostate $(4,s)$ of the total system satisfies the equation

$$g(4,s) = \sum_{s_1}\sum_{s_2} g_1(2,s_1)\,g_2(2,s_2)$$

Where $g_i(2,s_i)$ is the multiplicity of the macrostate $(2,s_i)$ i^{th} subsystem. (9 points)
You can't use the Stirling's approximation (why?). (1 point)