Phy 410
Quiz #3, Feb 6, 2009

A system has 4 microstates with energies 0, ε, ε, 2ε respectively.

a) What is the partition function at temperature τ ? (3 points)

$$Z = 1 + e^{-\varepsilon/\tau} + e^{-\varepsilon/\tau} + e^{-2\varepsilon/\tau} = 1 + 2e^{-\varepsilon/\tau} + e^{-2\varepsilon/\tau}$$

b) What is the average energy of the system as function of τ ? (3 points)

$$U = \tau^2 \frac{\partial}{\partial \tau} \ln Z = \tau^2 \frac{[-2\varepsilon e^{-\varepsilon/\tau} - 2\varepsilon e^{-2\varepsilon/\tau}](-1/\tau^2)}{1 + 2e^{-\varepsilon/\tau} + e^{-2\varepsilon/\tau}}$$

$$= \frac{[2\varepsilon e^{-\varepsilon/\tau} + 2\varepsilon e^{-2\varepsilon/\tau}]}{1 + 2e^{-\varepsilon/\tau} + e^{-2\varepsilon/\tau}}$$

$$Another\ way\ U = \sum_s \varepsilon_s P(\varepsilon_s) = \frac{0 x 1 + \varepsilon e^{-\varepsilon/\tau} + \varepsilon e^{-\varepsilon/\tau} + 2\varepsilon e^{-2\varepsilon/\tau}}{Z}$$

$$= \frac{[2\varepsilon e^{-\varepsilon/\tau} + 2\varepsilon e^{-2\varepsilon/\tau}]}{1 + 2e^{-\varepsilon/\tau} + e^{-2\varepsilon/\tau}}$$

c) What is the average energy as $\tau \to 0$? (2 points)

$As\ \tau \to 0, e^{-\varepsilon/\tau} and\ e^{-2\varepsilon/\tau} \to e^{-\infty} = 0; U \to 0$ (*only the ground state with energy* 0 *is occupied at* $\tau = 0$

d) What is the average energy as $\tau \to \infty$? (2 points)

$As\ \tau \to \infty, e^{-\varepsilon/\tau} and\ e^{-2\varepsilon/\tau} \to e^{-0} = 1; U \to \dfrac{2\varepsilon + 2\varepsilon}{4} = \varepsilon$ (*The 4 states are equally*

probable $U = 0 x \frac{1}{4} + \varepsilon x \frac{1}{4} + \varepsilon x \frac{1}{4} + 2\varepsilon x \frac{1}{4} = 4\varepsilon x \frac{1}{4} = \varepsilon$