PHY 410

Final Examination, Spring 2009

May 4, 2009 (5:45-7:45 p.m.)

PLEASE WAIT UNTIL YOU ARE TOLD TO BEGIN THE EXAM.
While waiting, carefully fill in the information requested below

Your Name:____

Your Student Number:____

DO NOT TURN THIS PAGE UNTIL THE EXAM STARTS

USEFUL CONSTANTS AND INTEGRALS

Avogadro's Number N_A= 6.022 x 10^{23}
Boltzmann's constant k = 1.381 x 10^{-23} J/K
= 8.617 x 10^{-5} eV/K
Planck constant h = 6.626 x 10^{-34} J.s
Electron charge (magnitude) e = 1.602 x 10^{-19} C
Electron mass m = 9.109 x 10^{-31} kg
Speed of light =2.998 x 10^{8} m/s

1 atm = 1.013 bar
1 bar = 10^{5} N/m^2
1 eV = 1.602 x 10^{-19} J

There are 7 problems. To receive full credit for each answer, *you must work neatly, show your work and simplify your answer to the extent possible.*

Problem 1 (5 points)

A power plant produces 1 GW of electricity at an efficiency of 40%.

(i) What is the rate at which the power plant expels waste heat to the environment?

(ii) What is the rate at which it uses up energy at the hot end?

Problem 2 (7.5 points)

Consider a fuel cell that uses methane (gas) as fuel. The reaction is

$$CH_4 + 2O_2 \rightarrow 2H_2O + CO_2$$

The difference in Gibb's free energy (final – initial) is -800 kJ per mole of CH_4.

(i) Assuming ideal performance, how much electrical work can be produced by the cell for each mole of CH_4.

(ii)The two steps of this reaction are

At –ve electrode $CH_4 + 2H_2O \rightarrow CO_2 + 8H^+ + 8e^-$ and At +ve electrode $2O_2 + 8H^+ + 8e^- \rightarrow 4H_2O$

What is the maximum voltage generated by the cell?

Problem 3 (10 points)

An ideal gas of N bosons of mass M is in a cubical box of volume V. The bosons are nonrelativistic. The density of orbitals (or states) is given by

$$D_3(\varepsilon) = \frac{V}{4\pi^2}\left(\frac{2M}{\hbar^2}\right)^{3/2} \varepsilon^{1/2} \; ; \varepsilon \geq 0.$$

(i)Write down an expression for the number of particles in terms of $D_3(\varepsilon)$ and the Bose-Einstein distribution function $f_{BE}(\varepsilon)$.

(ii) Derive an expression for the BE condensation temperature τ_E in terms M and the density N/V .(Use $\int_0^\infty \frac{x^{1/2}}{e^x - 1} dx = 1.306\sqrt{\pi}$)

(iii) How does τ_E change when the mass of the boson doubles?

Problem 4 (7.5 points)

The measured constant volume heat capacity of a metal at low temperature ($\tau << \tau_F = \varepsilon_F$ *and* $\tau << k_B \theta_{Debye}$) is given by

$C_V = A\tau + B\tau^3$, where *A* and *B* are constants.

(i)What are the physical origins of the above two different contributions to C_V?

(ii) What is the temperature dependence of entropy of the metal?

Problem 5 (10 points)

Consider a defect in a solid which has two energy levels with energy 0 and $\varepsilon > 0$. The degeneracy of the ground level is 2 and that of the excited level is 1. The solid is at a temperature τ.

(i) What is the partition function for this defect?

(ii) What is its average energy $U(\tau)$? Plot U as a function of τ.

(iii) What is $U(\tau)$ at $\tau = \infty$?

(iv) Find the temperature τ_0 when the average energy $U(\tau_0)$ is half the average energy at $\tau = \infty$, $U(\infty)$.

Problem 6 (15 points)

The energy per unit volume of a black body radiation and the radiation pressure p are given by

$$u = \frac{U}{V} = A\tau^4;\ where\ A\ is\ a\,constant, \quad p = \frac{u}{3}$$

A gas of photon is used as the working medium of a heat engine. The gas undergoes an isothermal expansion at temperature τ_h from volume V_1 to $V_2 = 2V_1$.

(i) What is the work done by the photon gas, W_{12}? If instead of a photon gas we use an ideal gas of *N* Ar atoms, what is the work done in this case?

(ii) Now the photon gas undergoes an adiabatic expansion from (τ_h, V_2) to (τ_l, V_3) ($where\ \tau_l = \tau_h/2, V_3 = 2V_2$), and the work done by the photon gas is W_{23}? What is W_{23}/W_{12}?

Problem 7 (15 points)

Consider an ideal 2-dimensional gas of non-relativisitic electrons (spin ½) of density n=N/A at <u>zero temperature</u>. The density of states for this system is given by $D_2(\varepsilon)=\frac{A}{2\pi}\left(\frac{2m}{\hbar^2}\right);\varepsilon\geq 0.$

(i) Derive an equation relating the Fermi energy ε_F as a function of n, mass m, and other fundamental constant(s).

(ii) Calculate the average energy (energy/particle) in terms of the Fermi energy ε_F .

(iii) What is the Fermi energy ε_F (in electron volts) if $n=10^{18}\ m^{-2}$