

Summary of the basic things covered in the lecture (Phy 410) For Exam#1 (Feb 19, 2010)

1. Microstates and Macrostates (N spins and N quantum harmonic oscillators)
2. Multiplicity of a given macrostate ($g(N,s)$ or $g(N,U)$, or $g(N,M)$ for magnets, $g(N,n)$ or $g(N,\varepsilon)$ for oscillators where U is the energy, M is total magnetic moment).

Spins or magnets with 2 states/magnet

$$g(N,s) = \frac{N!}{\left(\frac{N}{2} + s\right)! \left(\frac{N}{2} - s\right)!}$$

$$\sum_s g(N,s) = 2^N$$

$$M = 2sm, U = -MB = -2smB$$

N quantum harmonic oscillators, each with same frequency ω , energy $\varepsilon_s = s\hbar\omega$. Multiplicity factor of the macrostate with energy $\varepsilon = n\hbar\omega$

$$g(N,n) = \frac{(N-1+n)!}{(N-1)!n!}$$

3. Multiplicity of two systems (N_1 and N_2) in thermal contact with total energy U fixed.

$$g(N,U) = \sum_{U_1, U_2, U_1+U_2=U} g_1(N_1, U_1) \cdot g_2(N_2, U_2)$$

For spin systems ($2s$ is the spin excess etc)

$$g(N,s) = \sum_{s_1, s_2, s_1+s_2=s} g_1(N_1, s_1) \cdot g_2(N_2, s_2)$$

4. Concept of entropy, thermal equilibrium and temperature (for system with fixed energy, closed system)

$$\sigma(N,U) = \ln g(N,U)$$

$$\frac{1}{\tau} = \left(\frac{\partial \sigma}{\partial U} \right)_N$$

Equilibrium corresponds to the most probable distribution when the two systems have the same temperature. This corresponds to a particular division of energy between the two systems, \bar{U}_1 and $\bar{U}_2 = U - \bar{U}_1$.

$$\frac{1}{\tau_1} = \left(\frac{\partial \sigma_1}{\partial U_1} \right)_{N_1, U_1 = \bar{U}_1} = \left(\frac{\partial \sigma_2}{\partial U_2} \right)_{N_2, U_2 = \bar{U}_2} = \frac{1}{\tau_2}$$

At equilibrium

$$g(N,U) = g_1(N_1, \bar{U}_1) \cdot g_2(N_2, \bar{U}_2)$$

$$\sigma(N,U) = \sigma_1(N_1, \bar{U}_1) + \sigma_1(N_2, \bar{U}_2)$$

For two independent systems Multiplicity Factor is a PRODUCT of multiplicity factor for each component. IN CONTRAST Entropy is additive.

5. For thermally open systems (system in equilibrium with a reservoir at temperature τ), probability of a given microstate s with energy ϵ_s is given by

$$P(\epsilon_s) = \frac{e^{-\epsilon_s/\tau}}{Z};$$

$$Z = \sum_s e^{-\epsilon_s/\tau}; \text{ Partition function}$$

Two non interacting systems A and B the partition function $Z_{A+B} = Z_A \cdot Z_B$

For N non interacting systems (spins, oscillators, particles etc)

$Z_N = Z_1 \cdot Z_1 \cdot Z_1 \cdot Z_1 \dots = (Z_1)^N$. Z_1 will also depend on the physical system. Also for indistinguishable particles we need to divide Z_N by $N!$

6. Helmholtz free energy $F = U - \tau\sigma = -\tau \ln Z$

$$U = \sum_s \epsilon_s P(\epsilon_s) = \tau^2 \frac{\partial}{\partial \tau} \ln Z$$

$$C_V = \left(\frac{\partial U}{\partial T} \right)_{V,N}$$

For two non-interacting systems A and B , $F_{A+B} = F_A + F_B$

7. Thermodynamic identities (when N is constant)

$$\tau d\sigma = dU + p dV$$

$$dF = -\sigma d\tau - p dV$$

You can use these identities to calculate temperature, entropy and pressure under various types of external constraints.

8. Maxwell velocity (\vec{v}) and speed (v) distribution

$$P(\vec{v}) = \left(\frac{M}{2\pi\tau} \right)^{3/2} e^{-\frac{M\vec{v}^2}{2\tau}}; D(v) = \left(\frac{M}{2\pi\tau} \right)^{3/2} 4\pi v^2 e^{-\frac{Mv^2}{2\tau}}$$

9. Equipartition principle in classical systems