Summary of the basic things covered in the lecture (Phy 410)
for Exam 2, April 2, 2010

Chapter 4

1. Planck Distribution function (photon gas)
    Thermal average number of photons and average energy in a single mode of
    frequency ω and energy $\hbar\omega$ are given by

    $$<s>=\frac{1}{e^{\hbar\omega/\tau}-1}; <\varepsilon>=\frac{\hbar\omega}{e^{\hbar\omega/\tau}-1}$$

2. Counting and summing of modes in a cavity (cube of volume $V=L^3$);
    $\vec{n}=(n_x,n_y,n_z)$.

3. The frequency of EM waves in a cavity $\omega_{\vec{n}}=\frac{\pi}{L}cn; n=\sqrt{n_x^2+n_y^2+n_z^2}$ (3d)

4. Review how you will do calculations for 2 dimensional photon and phonon gas
    (see 6 below).
    In 2-dimensions one has to sum over $\vec{n}=(n_x,n_y)$ and replace volume V by area

    A= L^2, and $\omega_{\vec{n}}=\frac{\pi}{L}cn; n=\sqrt{n_x^2+n_y^2}$

5. For photon gas in 3d review energy density u=U/V, spectral density u_ω, entropy,
    and energy flux density $J_U=\sigma_B T^4$ (Note it is T not τ).

6. Phonons in a solid. Debye law for heat capacity at low T, concept of Debye
    temperature. In 3-dimensions

    $$U(\tau)\approx\frac{3\pi^4 N\tau^4}{5(k_B\theta)^3} \text{ for } \tau<<k_B\theta \text{ or } T<<\theta$$

    $$C_V=\frac{12\pi^4 N}{5}\left(\frac{\tau}{k_B\theta}\right)^3 ; \tau^3 \text{ or } T^3 \text{ law}$$

    In 2-dimensions it is T^2 law and in 1-dimension it is T law.

Chapter 5

7. Chemical potential

    $$\mu(\tau,V,N)=\left(\frac{\partial F}{\partial N}\right)_{\tau,V}$$

    For classical ideal gas (no internal degrees of freedom), calculate F and obtain

$$\mu = \mu_{ext} = \tau \ln(n/n_Q);$$

$$In\ 3-dim\,ension\ \ n = N/V;\ \ n_Q = \frac{1}{\lambda_{th}^3};$$

$$In\ 2-dim\,ension\ \ n = N/A;\ \ n_Q = \frac{1}{\lambda_{th}^2}$$

Where thermal wave length $\lambda_{th} = \left(\dfrac{2\pi\hbar^2}{M\tau}\right)^{1/2}$

When there is internal degrees of freedom

$$\mu_{tot} = \mu_{ext} + \mu_{int}$$

For diffusive equilibrium the total chemical potential must be constant.

8. For an open system with a given temperature τ and given chemical potential μ. Gibbs factor, Gibbs sum, Grand Sum, Grand Partition function, average number of particles in systems where the particle number can fluctuate.

$$P(N, \varepsilon_{s(N)}) = \frac{e^{[N\mu - \varepsilon_{s(N)}]/\tau}}{\tilde{Z}} = \frac{\lambda^N e^{-\varepsilon_{s(N)}/\tau}}{\tilde{Z}}$$

$$\tilde{Z} = \sum_N \sum_{s(N)} e^{[N\mu - \varepsilon_{s(N)}]/\tau} = \sum_N \lambda^N \sum_{s(N)} e^{-\varepsilon_{s(N)}/\tau} = \sum_N \lambda^N Z(N, \tau, V)$$

$$P(N) = \frac{\lambda^N Z(N, \tau, V)}{\tilde{Z}}$$

Average number of particles <N>

$$< N > = \sum_N NP(N) = \frac{\sum_N N\lambda^N Z(N, \tau, V)}{\tilde{Z}} = \frac{\sum_N N \sum_{s(N)} e^{[N\mu - \varepsilon_{s(N)}]}}{\tilde{Z}} = \tau\frac{\partial \ln \tilde{Z}}{\partial \mu} = \lambda\frac{\partial}{\partial \lambda} \ln \tilde{Z}$$

where absolute activity λ *is related to the hemical potential* μ *and* τ

i.e $\lambda = e^{\mu/\tau}$

Note: The notation for Gibbs Sum $\tilde{Z}$ is different from that used in the lecture and the book, Sorry!)
Review the pressure dependence of binding of atoms to surfaces, myoglobin, hemoglobin etc.

Chapter 6

9. Ideal Gas, Fermions, Bosons, Classical regime

Fermi Dirac distribution: $f_{FD}(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\mu)/\tau} + 1}$

Bose Einstein distribution: $f_{BE}(\varepsilon) = \dfrac{1}{e^{(\varepsilon-\mu)/\tau} - 1}$

For very low densities $n << n_Q$ OR $v >> \lambda_{th}^3$

$$f_{FD}(\varepsilon) = f_{BE}(\varepsilon) = e^{-(\varepsilon-\mu)/\tau} = \lambda e^{-\varepsilon/\tau}$$

10. Obtain classical ideal gas thermal properties starting from the chemical potential as a function of $N = <N>, V, \tau$ and using thermodynamic relations (see below).

11. Thermodynamic identity and how to use it to calculate changes in enthalpy H and Helmholtz free energy F, and F itself.

$$\tau \, d\sigma = dU + p \, dV - \mu \, dN$$

$$H = U + pV; dH = \tau \, d\sigma + V \, dp + \mu \, dN; H(\sigma, p, N)$$

$$F = U - \tau\sigma; dF = -\sigma \, d\tau - p \, dV + \mu \, dN; F(\tau, V, N)$$

$$The \ \ Chemical \, potential \quad \mu(\tau, N, V) = \left(\frac{\partial F}{\partial N}\right)_{\tau, V}$$

$$F(N, \tau, V) = \int_0^N \mu(\tau, N', V) dN'$$

12. For system in 2 and 1 dimensions, one has to extend the thermodynamic identity and the definitions of some of the free energies.

$$\tau \, d\sigma = dU + \pi dA - \mu \, dN \ (2d)$$

$$\tau \, d\sigma = dU + TdL - \mu \, dN (1d)$$

$$H = U + \pi A \ (2d); H = U + TL \ (1d)$$

$$F = U - \tau\sigma$$

In the above equation π is called the spreading pressure and T is called the tension.